

CLUSTER STRIPPING REACTIONS

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A theoretical formula for the stripping cross section of a nucleon cluster is deduced by taking into account the internal structure of the nuclei and clusters participating in the reaction. The theoretical angular distribution curve for the reaction  $Li^6(Li^7, t)B^{10}$  is compared with the experimental data.

THE question of the existence of substructures (clusters) in nuclei has been frequently raised in the recent literature. The effect of clustering of nucleons would be expected to be particularly clearly pronounced in various nuclear reactions with emission or capture of clusters. Among the entire aggregate of nuclear reactions, a special place is occupied by direct stripping and pickup reactions, since they relate the ground and low-lying excited states of the nuclei, that is, precisely the region in which one can speak of individual properties of a nuclear level. So far only one paper has been published [1] on the theoretical interpretation of stripping of clusters in the  $Be^9(Li^6, \alpha)B^{11}$  reaction. This paper makes use of ordinary theory of the stripping of a nucleon from a deuteron with all the specific "deuteron" simplifications of this theory. The internal structure of the cluster is not considered.

The purpose of the present paper is to present a theoretical derivation of a formula for the stripping cross section of any cluster from any incident nucleus (including a heavy ion), and take correct account of the internal structure of the nucleus and of the cluster in accordance with various model representations. A more complete reduction of the experimental data with the aid of the formula obtained will be undertaken in the future.

1. We shall agree to assign the incident nucleus the index P; the target nucleus will be designated T; the fragment of nucleus P which is emitted as a result of the reaction will be designated by e, and the second fragment of the P nucleus by t; the nucleus formed as a result of the reaction will be designated by R.

In the Born approximation [2], the amplitude of the reaction  $T(P, e)R$  is [2]

$$T = (2\pi)^{1/2} \mu_{eR} \hbar^{-2} [\langle \psi_f^\dagger | V_{et} | \psi_i \rangle + \langle \psi_f^\dagger | V_{eT} + (V_{eT} + V_{et}) G (V_{eT} + V_{tT}) | \psi_i \rangle],$$

where  $\psi_i$  and  $\psi_f$  are the wave functions of the initial and final states;  $\mu_{eR} = M_e M_R / (M_e + M_R)$  is the reduced mass;  $V_{ab}$  is the interaction between nuclei a and b; G is the resolvent,  $G = 1 / (H - E - i\epsilon)$  (H is the total Hamiltonian of the reaction and E is the corresponding eigenvalue of H).

The main contribution to the amplitude of the stripping reaction is made by the first term of the sum; the second term, as was shown by Fulton and Owen [2], causes the scattering and the reaction through the compound nucleus. We therefore assume for the amplitude of the stripping reaction the expression

$$T = (2\pi)^{1/2} \mu_{eR} \hbar^{-2} \langle \psi_f^\dagger | V_{et} | \psi_i \rangle.$$

Describing the motion of the nuclei P, T, e, and R in accordance with the Born approximation by means of plane waves we obtain after some manipulation [3]

$$T = \frac{\mu_{eR}}{\hbar^2 (2\pi)^{11/2}} C \int \psi_R^*(R_{tT}, x, \zeta) e^{ik_1 R_{tT}} \psi_e^*(y) \psi_T(\zeta) \times \psi_P(R_{et}, x, y) e^{-ik_2 R_{et}} dR_{tT} dR_{et} dx dy d\zeta$$

$$C = E_{c.m.} - \epsilon_P - \frac{\hbar^2}{2} \left[ \frac{k_P^2}{\mu_{tT}} + \frac{k_e^2}{\mu_{et}} - \frac{2(k_P k_e)}{M_t} \right],$$

where  $E_{c.m.}$  is the total kinetic energy of the system in the c.m.s.,  $\epsilon_P$  the binding energy of the e and t clusters in the nucleus P,  $k_P$  and  $k_E$  the wave vectors of the nuclei P and e,  $x$  the internal coordinates of the nucleons in the nucleus t,  $y$  the internal coordinates of the nucleons in the nucleus e,  $\zeta$  the internal coordinates of the nucleons in the nucleus T,  $R_{tT}$ , and  $R_{et}$  the radius vectors joining the respective mass centers of nuclei t and T or e and t,  $\psi_n$  the wave function of the nucleus n,  $\hbar k_1 = \hbar(k_P - k_e M_T / M_R)$  the momentum of the cluster t relative to the center of mass of the nucleus R, and  $\hbar k_2 = \hbar(k_e - k_T M_e / M_T)$  the momentum of the cluster e relative to the center of mass of the nucleus P.

We separate in the wave function  $\psi_R(\mathbf{R}_{tT}, \mathbf{x}, \boldsymbol{\zeta})$  a part  $\varphi_R(\mathbf{R}_{tT})$  corresponding to the relative motion of two clusters in the nucleus, and a part  $\chi_R(\mathbf{x}, \boldsymbol{\zeta})$ , corresponding to the internal motion of the nucleons in the clusters:

$$\psi_R = \varphi_R \chi_R.$$

Assuming the interaction potential of the clusters in the nucleus to be spherically symmetrical, we can write for  $\varphi_R$

$$\varphi_R(\mathbf{R}_{tT}) = u_R^{L_1}(R_{tT}) Y_{M_1}^{L_1}(\Omega_{R_{tT}}),$$

where  $\Omega_{R_{tT}}$  are the angular coordinates of the vector  $\mathbf{R}_{tT}$ . Thus

$$\psi_R = u_R^{L_1} Y_{M_1}^{L_1} \chi_R.$$

Analogously,  $\psi_P = u_P^{L_2} Y_{M_2}^{L_2} \chi_P$ . Now the reaction amplitude assumes the form

$$T = \frac{\mu_e R C}{\hbar^2 (2\pi)^{1/2}} I_1^* I_2 \Lambda, \quad (1)$$

where

$$I_1 = \int u_R^{L_1}(R_{tT}) Y_{M_1}^{L_1}(\Omega_{R_{tT}}) e^{-ik_1 R_{tT}} R_{tT}^2 dR_{tT} d\Omega_{R_{tT}},$$

$$I_2 = \int u_P^{L_2}(R_{et}) Y_{M_2}^{L_2}(\Omega_{R_{et}}) e^{-ik_2 R_{et}} R_{et}^2 dR_{et} d\Omega_{R_{et}},$$

$$\Lambda = \int \chi_R^*(\mathbf{x}, \boldsymbol{\zeta}) \psi_e^*(\mathbf{y}) \chi_P(\mathbf{x}, \mathbf{y}) \psi_T(\boldsymbol{\zeta}) d\mathbf{x} d\mathbf{y} d\boldsymbol{\zeta}.$$

In accordance with the stripping approximations [4] we introduce into the integrals  $I_1$  and  $I_2$  the cutoff radii  $R_1$  and  $R_2$ , which are respectively those minimum values of  $R_{tT}$  and  $R_{et}$ , starting with which we can neglect the influence of the interactions  $V_{tT}$ ,  $V_{et}$ , and  $V_{eT}$ . The introduction of the cutoff radii is justified by the fact that in an approximation more accurate than the Born approximation the plane waves in  $I_1$  and  $I_2$  will be replaced by more accurate wave functions, which take into account the interaction in the initial and final states and which decrease with decreasing  $R_{tT}$  and  $R_{et}$ . In addition, the wave functions  $u_R^{L_1}$  and  $u_P^{L_2}$  will be maximal at the surface of the R and P nuclei, since the probability of cluster formation in the nucleus is maximal at its boundary and minimal in the central region [5]. Thus, the main contribution to the integrals  $I_1$  and  $I_2$  will be made by the region near the surface of the nuclei R and P. This indeed constitutes the "surface" character of the stripping reaction.

If we put in (1)  $R_2 = 0$  and  $R_{et} = 0$ , we arrive at the case of stripping from a deuteron. Such simplifications can be made for a deuteron, since we can use there the approximation of zero action radius for the nuclear forces (see, for example, [6]), by

putting  $V_{et} \sim \delta(R_{et})$ . The interaction  $V_{eT}$  is in the deuteron case negligibly small (for heavy nuclei P, comparable in dimension with the nucleus R, the influence of  $V_{eT}$  increases [7]). In addition, the wave function of the deuteron is strongly "smeared" over the entire space, and the main contribution to  $I_2$  is made by its exponential part at large  $R_{et}$ . Therefore  $u_P^{L_2}$  can be taken in asymptotic form and integrated in  $I_2$  over all of space [4].

Returning to the consideration of formula (1), let us expand, as is customary in stripping theory, the plane waves in terms of spherical functions, and let us integrate in  $I_1$  and  $I_2$  with account of the cutoff radii. The amplitude of the stripping reaction will now have the form

$$T = \frac{\mu_e R \hbar^2 (-1)^{L_2} i^{L_1+L_2}}{(2\pi)^{1/2} \mu_{et} \mu_{tT} C} R_1 R_2 u_R^{L_1}(R_1) u_P^{L_2}(R_2) \times W^{L_1}(k_1 R_1) W^{L_2}(k_2 R_2) Y_{M_1}^{L_1} Y_{M_2}^{L_2} \Lambda, \quad (2)$$

where

$$W^L(kR) = \left[ \frac{dj_L(kr)}{dr} - j_L(kr) \frac{d}{dr} \ln u_n^L(r) \right] \Big|_{r=R}, \quad n = R, P;$$

$j_L(kr)$ —the spherical Bessel function.

It must be noted that the factor  $W^{L_2}(k_2 R_2)$  which is contained in formula (2) has for  $L_2 \neq 0$  a maximum at a scattering angle  $\theta \neq 0^\circ$ . This can appreciably influence the angular distribution of the stripping reaction.

2. Let us determine now how the reaction amplitude is influenced by the structure of the nuclei that participate in it. For this purpose we transform the integral  $\Lambda$ , using model representations and the concept of reduced width of the reaction.

In analogy with the single-nucleon case [8,9], we represent the reduced widths  $\theta_R^2$  and  $\theta_P^2$  in the form of a product of two factors

$$\theta^2 = S \theta_0^2.$$

In our case

$$\theta_{0R}^2 = [u_R^{L_1}(R_1)]^2 R_1^3/3, \quad \theta_{0P}^2 = [u_P^{L_2}(R_2)]^2 R_2^3/3$$

will be the analogs of the single-particle reduced width. They determine the probability of the breakup of the nucleus R or P into component clusters. The factors  $S^R$  and  $S^P$ —the spectroscopic factors—will determine the probability of formation of clusters in the nuclei R and P. The magnitude of the spectroscopic factor depends on the chosen nuclear model. To describe the stripping of the clusters in the region of light nuclei we can employ three models.

In the simplest cluster model, a particular case of which is the alpha-particle model, the wave function of the compound nucleus is written simply in the form of the product of the wave functions of the internal motion of the nucleons in the cluster and the wave function of the relative motion of the centers of mass of the clusters<sup>1)</sup>:

$$\begin{aligned}\psi_R(x, \zeta, \mathbf{R}_{tT}) &= \psi_T(\zeta) \psi_t(x) \varphi_R(\mathbf{R}_{tT}), \\ \psi_P(x, y, \mathbf{R}_{et}) &= \psi_t(x) \psi_e(y) \varphi_P(\mathbf{R}_{et}).\end{aligned}$$

It is clear that for this model  $\Lambda = 1$  and  $S^P = S^R = 1$ .

In the more complicated model of the clusters [10-14] the wave function of the compound nucleus is written with account of the antisymmetrization over all the nucleons of the nucleus (and not only over the cluster nucleons, as in the preceding model). The quantitative treatment of the reduced width has not yet been developed, but one can expect [15]  $S^P$  and  $S^R$  to be close to unity in this model.

We can, finally, calculate the probability of cluster formation in nuclei within the framework of the shell model, using methods developed in [16,17]. In this case (for the LS coupling scheme) we have

$$\begin{aligned}V\overline{S^R} &= \langle \beta_R l^n [f_R] T_R S_R L_R \rangle \\ &\times \beta_T l^{n-m} [f_T] T_T S_T L_T; \beta_t l^m [f_t] T_t S_t L'_1 \rangle K_{L'_1 L_t}, \\ V\overline{S^P} &= \langle \beta_P l_1^p [f_P] T_P S_P L_P \rangle \\ &\times \beta_e l_1^{p-m} [f_e] T_e S_e L_e; \beta_t l_1^m [f_t] T_t S_t L'_2 \rangle K_{L'_2 L_t},\end{aligned}$$

where  $\langle \ell^n | \ell^{n-m}; \ell^m \rangle$  is the symbol for the fractional parentage coefficient for the individual  $m$  nucleons out of a system of  $n$  nucleons;  $n$ ,  $m$ , and  $p$  are respectively the number of nucleons in the outer shell of the nuclei  $R$ ,  $t$ , and  $P$ ;  $\ell$  is the orbital momentum of the individual nucleons;  $T_X$ ,  $S_X$ ,  $L_X$ , and  $[f_X]$  are the isotopic spin, spin, orbital momentum, and the Young tableau ascribed to the given state of the nucleus  $x$ ;  $L'_1$  and  $L'_2$  are the orbital momenta of the configuration of the  $m$  nucleons of shells  $\ell$  and  $\ell_1$ ;  $\beta_X$  is the additional quantum number, necessary for a complete classification of the states;

$$K_{L'_1 L_t} = \left\langle \beta_t l^m [f_t] T_t S_t L'_1 \left| \frac{\varphi_{N_1 L_1}}{\varphi_{00}} \right| \beta_t l^m [f_t] T_t S_t L_t \right\rangle,$$

$$K_{L'_2 L_t} = \left\langle \beta_t l_1^m [f_t] T_t S_t L'_2 \left| \frac{\varphi_{N_2 L_2}}{\varphi_{00}} \right| \beta_t l^m [f_t] T_t S_t L_t \right\rangle,$$

where  $|\rangle$  is the symbol for the wave function of

the nucleon configuration in the independent particle shell model;  $\text{pN}_L$  is the wave function of the motion of the center of mass of the cluster  $t$  in the oscillator potential<sup>2)</sup> with quantum numbers  $N$  and  $L$ .

In final form the amplitude of the reaction will be

$$\begin{aligned}T &= \frac{\mu_{eR} \hbar^2 (-1)^{L_1} i^{L_1+L_2}}{(2\pi)^{3/2} \mu_{et} \mu_{tT} C} \frac{3\theta_{0R} \theta_{0P}}{\sqrt{R_1 R_2}} \\ &\times \sum Y_{M_1}^{L_1} Y_{M_2}^{L_2} W^{L_1} W^{L_2} \Gamma V \overline{S^P S^R},\end{aligned}\quad (3)$$

where the summation is over  $M_{S_t}$ ,  $M_{L_t}$ ,  $M_{L_1}$ ,  $M_{L_2}$ ,  $I_Z$ ,  $I_{Z'}$ ,  $M_{I_Z}$ ,  $M_{I_{Z'}}$ ,  $L'_1$ ,  $L'_2$ ,  $M_{L'_1}$ , and  $M_{L'_2}$ , while the statistical multiplier appears as a result of the vector addition of various momenta:

$$\begin{aligned}\Gamma &= (I_T S_t M_{I_T} M_{S_t} | I_Z M_{I_Z}) (L_t L_1 M_{L_t} M_{L_1} | L'_1 M_{L'_1}) \\ &\times (I_Z L'_1 M_{I_Z} M_{L'_1} | I_R M_{I_R}) (I_e S_t M_{I_e} M_{S_t} | I_{Z'} M_{I_{Z'}}) \\ &\times (L_t L_2 M_{L_t} M_{L_2} | L'_2 M_{L'_2}) (I_{Z'} L'_2 M_{I_{Z'}} M_{L'_2} | I_P M_{I_P}) \\ &\times (T_T T_t M_{T_T} M_{T_t} | T_R M_{T_R}) (T_t T_e M_{T_t} M_{T_e} | T_P M_{T_P}) U \\ &\times (S_R L_T I_T L_1 : L_R I_Z) U (L_T S_T I_Z S_t : S_R I_T) \\ &\times U (S_P L_e I_e L_2 : L_P I_{Z'}) \\ &\times U (L_e S_e I_{Z'} S_t : S_P I_e) (-1)^{2(S_P+S_R)};\end{aligned}$$

(... | ...) is the Clebsch-Gordan coefficient, and  $I_X$  the spin of the nucleus  $x$ ;  $I_Z$  and  $I_{Z'}$  are quantities that are labeled by the channel spins:

$$I_Z = S_R + L_T, \quad I_{Z'} = S_P + L_e;$$

$U(abcd:ef)$  is the Racah function, connected with the Racah coefficients  $W(abcd:ef)$  by the relation

$$U(abcd:ef) = \sqrt{(2e+1)(2f+1)} W(abcd:ef).$$

The differential cross section of the reaction will have the form

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{k_e M_P}{k_P M_e} \frac{9\mu_{eR}^2 \hbar^4}{(2\pi)^2 \mu_{et}^2 \mu_{tT}^2 C^2} \frac{[W^{L_1}(k_1 R_1) W^{L_2}(k_2 R_2)]^2}{(2I_T+1)(2I_P+1)} \\ &\times \sum \left| \sum \Gamma Y_{M_1}^{L_1} Y_{M_2}^{L_2} \frac{\theta_R \theta_P}{\sqrt{R_1 R_2}} \right|^2\end{aligned}\quad (4)$$

(summation over  $M_{I_P}$ ,  $M_{I_R}$ ,  $M_{I_e}$ ,  $M_{I_T}$ ).

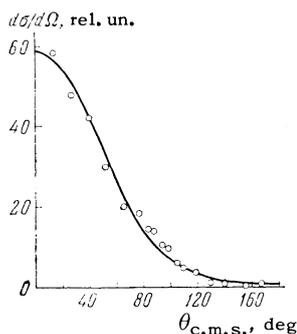
3. The absence of systematic experimental data on the stripping reactions of nucleon clusters does

<sup>2)</sup> $K_{L'_1 L_t}$  appears when the internal motion of  $m$  nucleons of the cluster  $t$  is separated from the motion of the center of mass of the entire cluster  $t$ . Such a separation can be carried out only for an oscillator potential. This limitation is not unrealistic, for the potential for the region of light nuclei under consideration is quite close to an oscillator potential.

<sup>1)</sup>To simplify the notation we do not write out the coefficients of vector addition of the cluster momenta.

not enable us to calculate as yet the reduced widths of the reactions and to estimate the region of applicability of the nuclear models listed above. In the present investigation we only calculated the angular distribution of the  $\text{Li}^6(\text{Li}^7, t)\text{B}^{10}$  reaction and carried out the comparison with Morrison's experimental data<sup>[18]</sup>. After making the necessary summation in (4) and leaving only the angle-dependent terms that influence the angular distribution, we obtain

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\theta} \sim [W^{L_1}(k_1 R_1) W^{L_2}(k_2 R_2) / C]^2.$$



Angular distribution of the reaction  $\text{Li}^6(\text{Li}^7, t)\text{B}^{10}$  (0.72 MeV). The continuous line denotes the theoretical curve while the circles mark the experimental points from<sup>[15]</sup>.

In place of the exact form of the function  $W^L$  we used the approximate form  $W^L(kR) \approx \sqrt{2\mu\epsilon/\hbar^2} R j_L(kR)$ , proposed by Banerjee<sup>[7]</sup>, which is particularly well justified for the case of stripping of nucleon clusters.

The reaction  $\text{Li}^6(\text{Li}^7, t)\text{B}^{10}$ , which goes to the ground state  $0^+$  of the  $\text{B}^{10}$  nucleus, gives an isotropic angular distribution, due obviously to the mechanism of the compound nucleus. We therefore consider a reaction that goes to the excited (0.72 MeV) state  $1^+$  of the  $\text{B}^{10}$  nucleus. In accordance with<sup>[10,11]</sup>,  $\text{Li}^7$  in the ground state was considered as a system of an alpha particle and a triton, moving with relative momentum  $L_2 = 1$ . The level  $1^+$  (0.72 MeV) of  $\text{B}^{10}$  is regarded as a system made up of an  $\text{Li}^6$  cluster and an alpha cluster in ground states. From the spin conservation law it follows that the capture of an alpha cluster can occur only with momentum  $L_1 = 0$ .

The calculation of the theoretical angular distribution curve was made for different values of the cutoff radii  $R_1$  and  $R_2$ . The best agreement with the experimental points was obtained for  $R_1 = R_2 = 3.7 \times 10^{-13}$  cm (see the figure<sup>3)</sup>). However,

<sup>3)</sup>In the comparison with experiment we have subtracted the isotropic background due to the reaction that proceeds via the compound nucleus.

variation of  $R_1$  and  $R_2$  in the range  $\pm 0.5 \times 10^{-13}$  cm changes the form of the theoretical curve only slightly.

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