PRODUCTION OF SLOW π MESONS IN PION-NUCLEON AND NUCLEON-NUCLEON COLLISIONS

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It is shown that in the extreme case of production of "slow" π mesons in πN and NN collisions, invariance under time reversal leads to a number of consequences for observable quantities which are independent of the dynamics. Thus, for example, the polarization of recoil nucleons during the production of "slow" π^0 mesons in πN collisions is found to be zero.

1. The invariance of the S matrix under time reversal imposes rather sharp restrictions on the elastic scattering amplitude (see, for example, [1]). In the general case of inelastic processes, this invariance leads only to a relation between the amplitude of direct and inverse processes. We consider the production of pions in πN and NN collisions and show that when the energy of the produced pion can be neglected, T-invariance allows us to obtain a number of useful consequences for the observed quantities. Processes of the production of "slow" pions have been considered by Nambu and Lurié,[2] who, for the model with conserved axial current, obtained limiting theorems relating the elastic scattering amplitude to the amplitude for the production of a massless pion of zero energy. Our considerations are based only on the requirement of invariance, but the results obtained by us are likewise valid only in the limiting case k → 0 (k is the 4-momentum of the produced meson). Since real pions have finite mass, the relations considered below cannot, strictly speaking, be fulfilled. Nevertheless, it should be expected that they are approximately fulfilled at sufficiently high energies when the energy of the additional meson can be neglected in comparison with the energy of the remaining particles.

2. We consider first the production of a π^0 meson in the reaction

$$\pi + p \to \pi + p + \pi^0. \quad (1)$$

The matrix element of this process can be written in the form

$$\langle p'q'k \,|S\,|\, pq\rangle = -(2\pi)^4\, i\,(m^2/p_0'p_0 2q_0' 2q_0\, 2k_0)^{1/2}$$
$$\times\, \bar{u}\,(p')M\,(p'q'k;\, pq)\; u\,(p)\; \delta\,(p' + q' + k - p - q). \quad (2)$$

Here p, p′ and q, q′ are the momenta of the nucleon and meson, k is the momentum of the π^0 meson, m is the nucleon mass, and $p_0' = -ip_4'$ etc.

As is known (see, for example, [3]), the emission of a particle of momentum −k is equivalent to the absorption of an anti-particle of momentum k, i.e.,

$$M\,(p',\, q',\, -k;\, pq) = M\,(p'q';\, kpq). \quad (3)$$

This relation connects the amplitude for process (1) in the unphysical region with the amplitude for the absorption of the π^0 (π^0 is a real neutral particle) in the reaction

$$\pi^0 + \pi + p \to \pi + p. \quad (4)$$

We can now use the invariance of the S matrix under time reversal and connect the matrix element of process (4) with the matrix element of the inverse process (1). We obtain

$$\langle p'q' \,|\, S \,|\, pqk\rangle = -\,{}_t\langle pqk \,|\, S\,|p'q'\rangle_t, \quad (5)$$

where the state $|p'q'\rangle_t$, for example, describes a meson of momentum $-q'^*$ and a nucleon of momentum $-p'^*$ and opposite spin. The minus sign in (5) is due to the transformation properties of the π^0 meson under time reversal. From (2), (3), (5), and the invariance under space inversion, it follows that

$$M\,(p',\, q',\, -k;\, pq) = [U^+M\,(pqk;\, p'q')\, U]^T. \quad (6)$$

Here U is a unitary matrix satisfying the condition

$$U^+\gamma_\mu U = \gamma_\mu^T,$$

and the symbol T denotes transposition.

Relation (6) is analogous to the known relation for crossing symmetry and connects the amplitude of the inelastic process (1) with the amplitude of the same process in the unphysical region. In the limiting case of the production of a massless π^0 meson with zero energy ($k \to 0$), we obtain from (6) the following restriction on the amplitude of the process in the physical region:

$$M(p'q'0;\ pq) = [U^+ M(pq0;\ p'q')\, U]^T. \tag{7}$$

In the real case of a π^0 meson with a finite mass, we can expect that this relation will be fulfilled only when the π^0-meson c.m.s. energy is close to the rest mass μ, while the total energy of the system and the momentum transfer is much greater than μ. In invariant form, these conditions can be written as follows:

$$-(Pk)/\sqrt{s} \sim \mu, \quad s \gg \mu^2, \quad |t| \gg \mu^2, \tag{8}$$

where $P = p + q$ is the total 4-momentum of the system, $s = -P^2$, and $t = -(p' - p)^2$.

We now consider the consequences to which relation (7) leads as regards the observed quantities.[1)] From the usual requirements of invariance, we obtain the following expression for the production amplitude of the "soft" π^0 meson:

$$M(p'q'0;\ pq) = a\gamma_5 + b\gamma_5(\gamma, q + q'), \tag{9}$$

where a and b are functions of the invariants s and t. Using (7), we find that $b = 0$, and in the amplitude there remains only the term

$$M(p'q'0;\ pq) = a\gamma_5. \tag{10}$$

Such a simple form of the amplitude allows us to draw conclusions on the polarization phenomena in the "slow" π^0-meson production process, independently of the dynamics. The polarization of the recoil protons turns out to be

$$\xi_\mu = 2(\varkappa\xi^0)\varkappa_\mu - \xi_\mu^0, \tag{11}$$

where ξ_μ^0 is the proton polarization in the initial state and κ_μ is a unit 4-vector in the direction of the momentum transfer $p' - p$ (we use covariant notation for the polarization [4]). The polarization ξ_μ vanishes at all angles if the initial protons are unpolarized. It is interesting to note that if the π^0 meson is scalar, then condition (7) is automatically satisfied and the polarization does not vanish identically. The experimental confirmation of the vanishing of the polarization can therefore serve as an additional argument in favor of the pseudoscalarity of the π^0 meson. [5]

The preceding considerations can, of course, be directly applied to the production of a π^0 meson in a reaction of the type

$$a + b \to a + b + \pi^0,$$

where a and b are any particles.

Thus, for example, for the production of π^0 mesons in pp collisions, expression (7) should be replaced by the following relation:

$$M(p_1'p_2'0;\ p_1p_2) = [U_1^+ U_2^+ M(p_1p_20;\ p_1'p_2')\, U_2 U_1]^T, \tag{12}$$

where p_1, p_2 and p_1', p_2' are the momenta of the nucleons before and after the collision, and U_1 and U_2 are matrices acting on the spin variables of the first and second nucleons ($U^+\gamma^{(i)}U_i = \gamma^{(i)T}$). From the requirements of invariance together with (12) and the Pauli principle, we obtain

$$M(p_1'p_2'0;\ p_1p_2) = A\,[\gamma_5^{(1)} + \gamma_5^{(2)}] + B\,[\gamma_5^{(1)}(\gamma^{(2)}, p_1 + p_1') + \gamma_5^{(2)}(\gamma^{(1)}, p_2 + p_2')]. \tag{13}$$

The polarization arising in collisions of unpolarized particles differs from zero in this case. The correlation of the polarizations C_{nn} in the direction normal to the scattering plane turns out to be zero. For polarized incident protons, relation (12) leads, as in the case of elastic scattering, [1,6] to the following expression for the reaction cross section

$$\sigma = \sigma_0[1 + (\xi_0\xi)]. \tag{14}$$

Here ξ_0 is the polarization of the incident protons, ξ is the polarization arising in collisions of unpolarized particles. We note that (12) is quite similar to the relation for the scattering amplitude resulting from T invariance in the case of pp elastic collisions. For this very reason, we also obtain expression (14) for the cross section for the production of "slow" π^0 mesons by polarized protons. Correspondingly, (14) is also valid in the more general case of the production of "slow" π^0 mesons in collisions of polarized protons with unpolarized particles of arbitrary spin (cf. [1]).

3. We now consider the production of charged mesons. If in this case we bring the pion from the initial state to the final state, we go over to a meson of opposite charge. Therefore, under time reversal, we actually go over to another process and thus obtain relations between the amplitudes of different processes. We consider, for example, the reaction

$$\pi^+ + p \to \pi^+ + \pi^+ + n, \tag{15}$$

in which one of the π^+ mesons in the final state is slow. Proceeding in the same way as in the production of neutral mesons, we obtain the relation

$$M_+(p'q'0;\ pq) = [U^+ M_-(pq\,0;\ p'q')\, U]^T, \tag{16}$$

1) While neglecting the dependence on the momentum k in the amplitude, we retain k in the statistical weight.

where M_+ is the amplitude of reaction (15) and M_- is the amplitude for the production of a "slow" π^- in the reaction

$$\pi^+ + n \rightarrow \pi^- + \pi^+ + p, \tag{17}$$

or, owing to charge symmetry, the amplitude for the production of a "slow" π^+ meson in the reaction

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n. \tag{18}$$

On the basis of the requirements of invariance, we obtain

$$M_+ (p'q'0;\ pq) = a_+\gamma_5 + b_+\gamma_5(\gamma,\ q + q'),$$
$$M_- (pq0;\ p'q') = a_-\gamma_5 + b_-\gamma_5(\gamma,\ q + q'). \tag{19}$$

From (16) it follows that

$$a_+ = a_-, \qquad b_+ = -b_-. \tag{20}$$

This means that the differential cross sections of reactions (15) and (17) or (18) are equal, and the polarizations of the recoil nucleons are equal in magnitude and opposite in sign.

In a similar way, we can show that the differential cross sections for the production of "slow" mesons in the reactions

$$p + p \rightarrow n + p + \pi^+, \qquad n + p \rightarrow p + p + \pi^- \tag{21}$$

are equal. In some cases, owing to charge symmetry, we can also obtain a restriction of the type (7) for the production amplitude of charged mesons. Thus, for example, the production amplitude for a "slow" π^- meson in the reaction

$$\pi^- + p \rightarrow \pi^- + \pi^+ + n \tag{22}$$

is subject to this requirement and, consequently, the polarization of the recoil neutron vanishes.

We note, in conclusion, that the production of soft γ rays can be considered in a similar way. As is known (see, for example, [7]), in this case the amplitude of the radiation is represented in the form of a product of the amplitude for a nonradiative process and the amplitude for the radiation of a dipole quantum with a given momentum transfer. A condition of type (6) then reduces to the usual requirement of T invariance for the amplitude of the nonradiative process.

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