SPIRALITY INVERSION IN NUCLEAR REACTIONS

N. P. KLEPIKOV and Ya. A. SMORODINSKIĬ

Joint Institute for Nuclear Research and Moscow State University

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It is shown that for arbitrary spin there is a phase transformation which leaves invariant all those observable quantities which are unchanged when the spiralities of all the particles participating in the reaction are changed. For the special case of spin ½, the transformation is related to the Minami ambiguity. Multiple scattering destroys the invariance because of the Thomas precession of the spin.

In the well known paper of Minami [1] (cf. [2]) it was shown that the cross section for scattering of a spin-½ particle by a spinless target is invariant under interchange of phases of all pairs of states corresponding to the same value of the total angular momentum:

$$\delta\,(l=j-{}^1/_2)\rightleftarrows\delta(l=j+{}^1/_2). \tag{1}$$

However, the Minami transformation changes the sign of the polarization, and so the resultant two-valuedness is removed if this sign is known. In a paper by Puzikov, Ryndin, and one of the present authors, [3] it was noted that interchange of phases with a simultaneous change of their signs,[1)]

$$\delta\,(l=j-{}^1/_2)\rightleftarrows-\delta\,(l=j+{}^1/_2). \tag{2}$$

changes neither the cross section nor the polarization, and can lead to an ambiguity in the phase shift analysis. Such an ambiguity is eliminated, for example, by rotating the polarization by means of a magnetic field or by measuring the energy dependence of the phases at low energies.

In a later paper, Zastavenko [5] discussed[2)] the problem of generalizing the transformation (1) to the case of higher spins and to relativistic particles. In all of this work the physical meaning of the transformation (2) was not made clear. It turns out that this transformation expresses a simple type of symmetry.

Transformation (2) is simply the change in sign of the spirality of all particles (the projection of the polarization of the particle along the direction of its velocity). The simple fact that in experiments on scattering which do not use external fields (or polarized targets), one cannot measure the sign of the longitudinal polarization leads to an ambiguity. In this form the statement is easily generalized to arbitrary spins. A more unexpected result is the generalization to relativistic systems, where a "relativistic rotation of the spin" destroys the symmetry and, at sufficiently low energy, results in a "fine structure" of the cross sections for multiple scatterings, computed on the basis of a "complete experiment."

The transformation (1) can be written in matrix form:

$$M\rightarrow(\sigma\mathbf{n}_f)\,M(\sigma\mathbf{n}_i). \tag{3}$$

Here M is the scattering matrix (in spin space), $\mathbf{n}_i$ and $\mathbf{n}_f$ are unit vectors along the directions of the incident and scattered particle in the cms. Since $\sigma\cdot\mathbf{n}_{i,f}=\exp[i(\sigma\cdot\mathbf{n}_{i,f})\pi/2]=U(\mathbf{n}_{i,f})$ is the operator which rotates the spin through an angle π about the axis $\mathbf{n}_{i,f}$, Eq. (3) can be rewritten as

$$M\sim U^+(\mathbf{n}_f)\,MU(\mathbf{n}_i). \tag{4}$$

It is obvious without calculation that such a transformation changes the sign of the transverse polarization of the particles. Transformation (4) anticommutes with the space reflection,

$$P\,(\sigma\mathbf{n})=-(\sigma\mathbf{n})\,P \tag{5}$$

(where $\mathbf{n}$ is either of the vectors $\mathbf{n}_i$ or $\mathbf{n}_f$); it therefore changes the parity of the state.

It is easy to see that the substitution (4) does not violate the unitarity condition

$$i\,(\mathbf{M}^+-\mathbf{M})=2k\,\mathbf{M}^+\mathbf{M}. \tag{6}$$

Furthermore it is clear that condition (6) is not violated by the substitution

$$\mathbf{M}\rightarrow-\mathbf{M}^+. \tag{7}$$

This follows from the fact that $MM^+=M^+M$ be-

[1)]This substitution has been used recently in a paper of Nauenberg and Pais.[4]

[2)]See also [6]. In [7] an incorrect derivation is given.

cause of the unitarity of the matrix $S = 1 + 2ikM$.

Transformation (7) consists of a change of the sign, transition to the Hermitian conjugate matrix in spin space and interchange of final and initial momenta. The signs of all spin components change. This transformation causes a change in sign of all phases, as can be easily seen by writing the scattering matrix in the JM-representation, in which it is symmetric (because of the symmetry in time).

The successive application of the two transformations: rotation of all spins through π about their momenta and the transformation (7) gives inversion of the spirality of all the particles.

For a particle with spin $1/2$, spirality inversion reduces to the substitution (2). The arguments given above are immediately generalized to the case of arbitrary spin. Only the form of the operators changes. In particular, the rotation operator $U(\mathbf{n})$ for a particle with spin S will be $\exp[i(\mathbf{S}\cdot\mathbf{n})\pi]$, and its commutation relation with the space inversion operator has the form

$$UP = (-1)^{2S} PU. \tag{8}$$

In the case of systems of several particles, the spirality inversion must be carried out on each particle. In this case the rotation operator splits up into a product of operators acting on each of the particles. Thus for a system with integral spin the Minami transformation and spirality inversion does not change the parity of the states, while for systems with half-integral spin the parity changes.

If we consider any process occurring during the collision of an arbitrarily polarized beam with an arbitrarily polarized target, it is clear from the discussion that if one measures only the absolute value of the components of the polarization (in the initial and final states), there are four sets of amplitudes which satisfy all the experimental results. If the sign of the transverse polarization is determined in the experiments, only two sets are left.[3)]

To eliminate the remaining ambiguity one must measure the sign of the longitudinal polarization. To do this one must measure the pseudoscalar $\mathbf{S}\cdot\mathbf{n}$. This can be done by switching on a magnetic field [measuring the scalar $(\mathbf{S}\cdot\mathbf{n})(\mathbf{H}\cdot\mathbf{n})$], or an electric field—measuring the scalar $(\mathbf{S}\cdot\mathbf{n})(\mathbf{E}\cdot\mathbf{n}_i \times \mathbf{n}_f)$—or, finally, by studying the dependence of the effects on energy [$(\mathbf{S}\cdot\mathbf{n})$ is a scalar under time reversal].

Polarized beams are usually prepared by scattering on targets, for various planes of scattering. If the particles are nonrelativistic, the polarization direction does not change in going from one system to another, and multiple scattering does not alter the conclusions given above. This is immediately clear if we note that the result of an n-fold scattering which starts with the scattering of an unpolarized particle, is described by the trace of the density matrix

$$\rho = \underbrace{MM\ldots M}_{n}\ \underbrace{M^{+}\ldots M^{+}M^{+}}_{n} \tag{9}$$

and that the direction of the incident particle coincides with the direction of the scattered particle from the preceding process.

The situation changes if the particles are relativistic. Stapp[8] pointed out the role of this effect in multiple scattering processes. The polarization of the scattered particle which results from the first scattering (by a target at rest) is rotated through an angle Ω (in the negative direction of the scattering angle) in going to the cms of the second scattering (cf. the Appendix). Since the rotation through angle Ω occurs about an axis normal to the plane of scattering, it does not commute with the spirality inversion, so this effect removes all the ambiguities considered above. In this sense the Lorentz transformation plays a role analogous to that of a magnetic field switched on between successive scatterings.

For spinless particles a second scattering gives no new information beyond that from the first scattering and does not remove the ambiguity in the sign of the phases which is connected with the transformation (7). For particles of spin $1/2$, the polarization resulting from scattering of an unpolarized beam is perpendicular to the plane of scattering. Thus the invariance of the asymmetry of the second scattering with respect to the transformation $M \to -U^{+}MU$ is not removed by the relativistic rotation of the spin. Breakdown of this invariance begins with the asymmetry of the third scattering.

For particles with spin greater than $1/2$, the vector part of the polarization after the first scattering is again not changed by a relativistic rotation around the normal, but the tensor part of the polarization is not invariant under rotation. Thus in this case the invariance is already destroyed by the asymmetry of the second scattering.[4)]

[3)]The sign of the polarization is determined usually if one knows the levels of the target nucleus (for example, in the scattering of nucleons by helium).

[4)]This was discovered by Zastavenko[5] by a direct calculation.

We note that the invariance of observable quantities which we have described does not occur at all if conditions are imposed on the scattering matrix which are incompatible with reversal of the spirality of all the particles. Thus, if in the matrix for scattering of particles with spin 0, ½, we assume that an odd number of the lowest phases are different from zero, the corresponding transformation of phases cannot be carried out, and all quantities observed in a second scattering are different for all 2^n sets of phases [9] (where n is the number of phases in the sequence $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}, \ldots$), which are compatible with the cross section for scattering of an unpolarized beam.

APPENDIX I

RELATIVISTIC ROTATION OF THE SPIN

Relativistic rotation of the spin was treated in a paper of Stapp. [8] This kinematic effect is nothing other than the "Thomas precession," and is most simply described in Lobachevskiĭ space, [10] as was pointed out in a recent paper of Wick. [11] The effect becomes completely elementary if we use the operation of parallel displacement in this space. [12]

Because of the relation $v^2 = (p/m)^2 = 1$, the ends of the velocity four-vectors lie on the surface of a hyperboloid in a four-dimensional velocity space with a pseudo-Euclidean metric. The metric on the surface itself is defined so that if a is the "length" of the geodesic between two points, $\cosh a = \gamma$ is the Lorentz factor corresponding to the relative velocity of these points.

The transformation of the spin from one coordinate system to another can be described as a parallel displacement of the three-dimensional spin vector along a geodesic on the hyperboloid passing through the two corresponding points. [12] Thus one can immediately show that a rotation of the spin in the lab system differs from the rotation in the cms by a rotation which is related to the parallel displacement along a hyperbolic triangle with vertices corresponding to the 4-velocity of the particle before scattering R_1, the 4-velocity of the center of mass C and the 4-velocity of the particle after scattering R_2.

This rotation is equal to the hyperbolic defect Ω (or the area of the triangle, if the curvature of the Lobachevskiĭ space—the velocity of light, is set equal to unity):

$$\varphi_{\text{lab}} = \varphi_{\text{c.m.}} - |\Omega|.$$

The angle of rotation Ω can be written in vector form:*

$$|\sin\Omega| = \omega\,|\varepsilon_{\alpha\beta\gamma\delta}R_{1\beta}C_\gamma R_{2\delta}|,$$
$$\omega = \frac{1+\operatorname{ch}a+\operatorname{ch}b+\operatorname{ch}c}{8\operatorname{ch}^2(a/2)\operatorname{ch}^2(b/2)\operatorname{ch}^2(c/2)}.$$

We note that

$$\sin^2\Omega/\omega^2 = 1 + 2(R_1C)(CR_2)(R_2R_1) - (R_1C)^2 - (CR_2)^2$$
$$- (R_2R_1)^2 = 1 + 2\operatorname{ch}a\operatorname{ch}b\operatorname{ch}c - \operatorname{ch}^2 a - \operatorname{ch}^2 b - \operatorname{ch}^2 c$$

(a, b, c are the sides of the triangle).

The vector of the rotation in each of the three systems is parallel to the three-dimensional vector product of the space parts of the two 4-velocities, differing from it by a factor ω. This is the same as Stapp's result.

In describing multiple scattering, it is most convenient to perform the calculations in the lab system; then in formula (9) one should insert the matrix for rotation through angle Ω between each pair of matrices M.

APPENDIX II

TRANSFORMATION OF THE SCATTERING MATRIX AND OBSERVABLES

We have considered the three transformations:

$$M \to -M^+, \tag{a}$$
$$M \to U^+MU, \tag{b}$$
$$M \to -U^+M^+U. \tag{c}$$

Let us examine the effect of these transformations for the three systems.

1) Spinless particle. We write the scattering matrix in the form

$$M = e^{i\alpha}\sqrt{\sigma}.$$

Transformation (a) gives the relations

$$\sigma \to \sigma, \qquad \alpha \to \pi - \alpha, \qquad \delta_l \to -\delta_l.$$

2) Spin ½. The coefficients in the scattering matrix

$$M = a + b\,(\boldsymbol{\sigma}\mathbf{n})$$

are

$$a = \tfrac{1}{2}\sqrt{\sigma}e^{i\alpha}\left(e^{-i\beta/2}\sqrt{1+P} + e^{i\beta}\sqrt{1-P}\right),$$
$$b = \tfrac{1}{2}\sqrt{\sigma}e^{i\alpha}\left(e^{-i\beta/2}\sqrt{1+P} - e^{i\beta/2}\sqrt{1-P}\right)$$

(where σ is the differential cross section and P is the polarization), and transform according to the formulas

*ch = cosh.

$$a \to -a^*, \quad b \to b^*, \quad \sigma \to \sigma, \quad P \to -P, \quad \alpha \to \pi - \alpha$$

under transformation (a),

$$a \to a\cos\theta + ib\sin\theta, \quad b = -ia\sin\theta - b\cos\theta,$$
$$\sigma \to \sigma, \quad P \to -P, \quad \alpha \to \alpha, \quad \beta \to 2\theta - \beta$$

under transformation (b), and

$$a \to -a^*\cos\theta + ib^*\sin\theta, \quad b = ia^*\sin\theta - b^*\cos\theta,$$
$$\sigma \to \sigma, \quad P \to P, \quad \alpha \to \pi - \alpha, \quad \beta \to 2\theta - \beta$$

under transformation (c).

Under transformation (c) the transverse-longitudinal components of the tensor relating the components of the spin (in the plane of scattering) before and after scattering change sign. The other components do not change.

3) Nucleon-nucleon scattering. The formulas analogous to the preceding ones have the form

$$M = \tfrac{1}{2}a\,[1 + (\sigma_1 n)(\sigma_2 n)] + \tfrac{1}{2}b\,[1 - (\sigma_1 n)(\sigma_2 n)]$$
$$+ \tfrac{1}{2}c\,[(\sigma_1 m)(\sigma_2 m) + (\sigma_1 l)(\sigma_2 l)] + \tfrac{1}{2}d\,[(\sigma_1 m)(\sigma_2 m)$$
$$- (\sigma_1 l)(\sigma_2 l)] + \tfrac{1}{2}e\,[(\sigma_1 n) + (\sigma_2 n)];$$
$$a = 2^{-3/2}\sqrt{\sigma}e^{i\alpha}\,(e^{-i\beta}\sqrt{1 + D + K + Q + 4P}$$
$$+ e^{i\beta}\sqrt{1 + D + K + Q - 4P}),$$
$$b = 2^{-1/2}\sqrt{\sigma}e^{i(\alpha+\gamma)}\sqrt{1 + D - K - Q},$$
$$c = 2^{-1/2}\sqrt{\sigma}e^{i(\alpha+\gamma)}\sqrt{1 - D + K - Q},$$
$$d = 2^{-1/2}\sqrt{\sigma}e^{i(\alpha+\varepsilon)}\sqrt{1 - D - K + Q},$$
$$e = 2^{-3/2}\sqrt{\sigma}e^{i\alpha}\,(e^{-i\beta}\sqrt{1 + D + K + Q + 4P}$$
$$- e^{i\beta}\sqrt{1 + D + K + Q - 4P}).$$

Here P is the polarization of an unpolarized beam, D is the depolarization in the forward hemisphere, K is the depolarization in the backward hemisphere, and Q is the normal-normal component of the polarization correlation.

For transformation (a) we have:

$$a \to -a^*, \quad b \to -b^*, \quad c \to -c^*, \quad d \to -d^*, \quad e \to e^*,$$
$$\alpha \to \pi - \alpha, \quad P \to -P, \quad \gamma \to -\gamma, \quad \delta \to -\delta, \quad \varepsilon \to -\varepsilon;$$

for transformation (b):

$$a \to a\cos 2\theta + ie\sin 2\theta, \quad e \to -ia\sin 2\theta - e\cos 2\theta,$$
$$\beta \to 2\theta - \beta, \quad P \to -P;$$

for transformation (c):

$$a \to -a^*\cos 2\theta + ie^*\sin 2\theta, \quad e \to ia\sin 2\theta - e^*\cos 2\theta,$$
$$b \to -b^*, \quad c \to -c^*, \quad d \to -d^*,$$
$$\alpha \to \pi \to \alpha, \quad \beta \to 2\theta - \beta, \quad \gamma \to -\gamma,$$
$$\delta \to -\delta, \quad \varepsilon \to -\varepsilon.$$

Under transformations (b) and (c), the longitudinal-transverse and the transverse-longitudinal components of the tensors change sign (coefficients which have not been written explicitly do not change).

In triplet states the transformation (c) is associated with the transformation of the phase matrix:

$$S_{11}^j \to (2j+1)^{-2}\{S_{11}^j + 4j(j+1)S_{22}^j + 4\sqrt{j(j+1)}S_{12}^j\},$$
$$S_{12}^j \to 2\sqrt{j(j+1)}(2j+1)^{-2}(S_{11}^j - S_{22}^j)$$
$$+ [4j(j+1) - 1](2j+1)^{-2}S_{12}^j,$$
$$S_{22}^j \to (2j+1)^{-2}\{4j(j+1)S_{11}^j + S_{22}^j - 4\sqrt{j(j+1)}S_{12}^j\}.$$

Its elements are described by the phases and mixing parameter:

$$S_{11}^j = \cos^2\varepsilon_j \exp(2i\delta_j^+) + \sin^2\varepsilon_j \exp(2i\delta_j^-),$$
$$S_{22}^j = \sin^2\varepsilon_j \exp(2i\delta_j^+) + \cos^2\varepsilon_j \exp(2i\delta_j^-),$$
$$S_{12}^j = 1/2\sin 2\varepsilon_j\,[\exp(2i\delta_j^-) - \exp(2i\delta_j^+)].$$

Transformation (c) reduces to

$$\delta^+ \to -\delta^-, \quad \varepsilon \to -\varepsilon + \cot^{-1} 2\sqrt{j(j+1)}.$$

The phases of singlet states and states with $j = l$ do not change under transformation (c) and change under (a) and (b).

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