THE SUPERCRITICAL OSCILLATORY MODE IN He II

R. A. BABLIDZE

Physics Institute, Academy of Sciences, Georgian S.S.R.

Submitted to JETP editor July 20, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 2086-2093 (December, 1962)

A study of the supercritical oscillatory mode has been carried out using the attenuation of second sound. The onset and stability of the quantized Onsager-Feynman vortices have been determined from the additional attenuation they produce in the second-sound wave. The excess attenuation coefficient γ'_m has been measured as a function of the amplitude and frequency of the oscillations of a resonant cavity. It was found that γ'_m depends upon the maximum linear velocity V_0 at the periphery of the cavity. An empirical expression for this relation is derived. It is shown that the second-sound propagation velocity does not vary as the cavity oscillates. The critical velocities and relaxation times associated with vortex formation are evaluated. The decay law for the quantized turbulence is established, indicating that two different types of vortices occurred in our experiments, having different half-lives: $\tau_1 = 14.20 \pm 0.86$ sec, and $\tau_2 = 2.10 \pm 0.28$ sec. An expression is also derived for the rate of decay of the quantized turbulence.

FOLLOWING the well-known work of Andronikashvili[1] it became quite clear that only the normal component of He II takes part in the oscillations of the disk, while the superfluid remains at rest. On extending Andronikashvili's experiments to higher amplitudes, however, Hollis-Hallett[2] found that, beginning at some critical amplitude Φ_c, the superfluid component of the He II is drawn into motion as well. This implies that as the amplitude Φ_c is reached, an interaction arises between the two components of He II. It is well known that interaction between the normal and superfluid components of He II is attributable to scattering of the elementary excitations—the rotons and phonons constituting the normal component—by the Onsager-Feynman vortices which develop in the superfluid within the supercritical flow regime.

In a series of experiments by Hall and Vinen[3], Andronikashvili, et al[4], and Lane's group[5], it has been demonstrated that in He II under uniform rotation phenomena arise whose quantitative description leads to good agreement with the theory of Feynman[6]. According to this theory, a regular system of quantized vortices is formed in rotating He II, extending through the liquid in a direction parallelling the axis of rotation. Due to the presence of this vortex system, rotating He II imitates reasonably well the rotation of classical viscous fluids.

On the other hand, Vinen[7], in a study of heat flow in He II, showed that the destruction of ideal superfluid flow in He II, which occurs when a certain critical velocity is attained, is likewise associated with the generation of a complex, irregular system of quantized Onsager-Feynman vortices, i.e., it is attributable to turbulence in the superfluid component.

Finally, it is quite evidently natural and logical to hypothesize that the supercritical phenomena observed in oscillatory experiments[8-10] are also associated with quantized Onsager-Feynman vortices. In all probability, owing to the non-stationary nature of the flow, the system of vortex lines appearing in these experiments cannot be strictly regular, as in the case of uniform rotation, but will have the form of a badly tangled knot. A similar conclusion has been reached by Hall in his survey[11].

The fundamental problem which must be resolved, either experimentally or theoretically, before a picture of quantum turbulence can be finally established, concerns the kinetics of the accumulation and the stability of the quantized vortices which form and decay as the vessel undergoes rotational acceleration and deceleration.

The present experimental work is devoted to such a study.

The basis for the investigation was provided by an experiment in which a hollow cylindrical second sound resonator filled with He II undergoes forced harmonic torsional oscillations about its axis of symmetry. We drew our conclusions regarding the number of vortices produced under these conditions and their stability from the increased attenuation of the second sound waves caused by their presence. When the frequency of the oscillations fell below the critical value, no additional attenuation was observed at all.

EXPERIMENTAL METHOD

The mechanism used for exciting the torsional oscillations (1 in Fig. 1) made it possible to vary the period and amplitude of the resonator oscillations within the limits $T = 2.8 - 12.5$ sec and $\varphi = 0.06 - 2$ radians. This allows coverage of a range of maximum velocities from 0.5 to 90 mm/sec at the periphery of the resonator. The second-sound generator and detector were cemented to the cover and the bottom of the resonator, respectively ($\varphi = 40$ mm, $l = 137$ mm). At $T = 1.6°K$, the Q of the resonator reached 1500 in He II at rest. The work was carried out over the frequency range from 60 to 3000 cps. The diffuse heat current density in the resonator never exceeded 5×10^{-4} W/cm.

The second sound signals were fed into a tuned amplifier[1)] with a gain of $\sim 10^7$ and, after rectification, were applied to an ÉPP-09 chart recorder which recorded the time dependence of the second-sound resonance amplitude in both the subcritical and the supercritical regimes. The experimental apparatus has already been described in a more detailed fashion[12].

The second-sound attenuation coefficient in the oscillating resonator may be written down in the form

$$\gamma = \gamma_0 + \gamma'(\omega, \varphi), \tag{1}$$

where γ_0 refers to the stationary resonator and $\gamma'(\omega, \varphi)$ characterizes the additional attenuation due to the Onsager-Feynman vortices within the resonator as it oscillates with frequency ω and amplitude φ. Using the relation between the attenuation coefficient γ and the Q of the resonator, we obtain

$$\gamma'(\omega, \varphi) = \frac{\pi \nu_0}{u_2 Q_0}\left(\frac{1}{Q_{\text{rel}}} - 1\right), \tag{2}$$

where $Q_{\text{rel}} = Q/Q_0$, Q_0 and Q are the values for the resonator at rest and in rotation, and ν_0 and u_2 are the resonance frequency and the velocity of the second sound.

[1)]This amplifier was developed by the Institute for Physics Problems, U.S.S.R. Academy of Sciences. We take this opportunity to express our deep gratitude to A. N. Vetchinkin for much valued advice, and for his assistance in the construction of the amplifier.

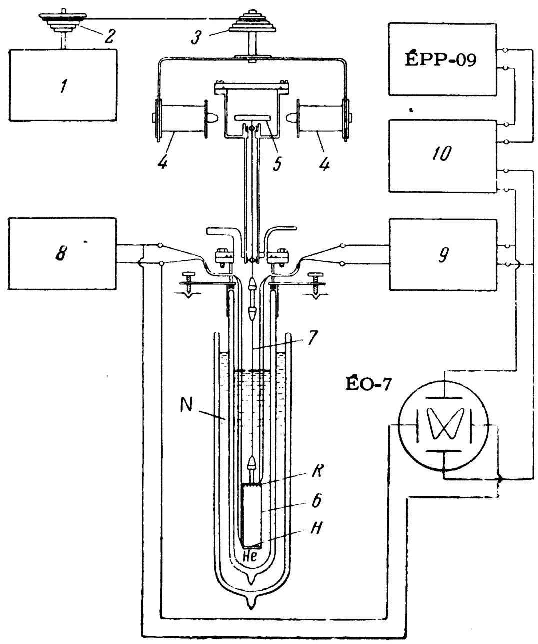

FIG. 1. Block diagram of the experimental apparatus: 1 – mechanism for exciting sinusoidal torsional oscillations, 2, 3 – pulleys, 4 – electromagnets, 5 – magnetic armature, 6 – second sound resonator, 7 – glass rod, 8 – audio-frequency generator, 9 – tuned amplifier, 10 – rectifier, H – constantan heater, the second sound source, R – phosphor bronze thermometer, the second sound detector.

All of the experiments were carried out at a temperature of 1.6°K, at which $\partial u_2/\partial T \approx 0$. The excess attenuation coefficient $\gamma'(\omega, \varphi)$ was measured to an accuracy of 12–14%.

EXPERIMENTAL RESULTS

It was found in the preliminary experiments[12,13] that to an accuracy of 0.15% the oscillations of the resonator exert no influence upon the velocity of propagation of second sound. This implies that the two-fluid model of He II remains correct even in supercritical oscillatory regimes. It was shown at the same time that the excess second sound attenuation coefficient in the oscillating resonator is independent of the amplitude and frequency of the second sound over the whole range of frequencies employed (60–3000 cps), and of the power applied to the heater ($2 \times 10^{-4} - 5 \times 10^{-3}$ W).

Figure 2 shows a typical curve obtained on the ÉPP-09 recorder. As is evident from Fig. 2, the excess attenuation does not appear immediately after the resonator is set into oscillation, but only after a certain delay time $\xi = t_2 - t_1 = t_{\text{rel}} + t_c$, equal to the sum of the relaxation time for vortex formation, t_{rel}, and a time, t_c, required for the resonator to reach the critical velocity for forma-

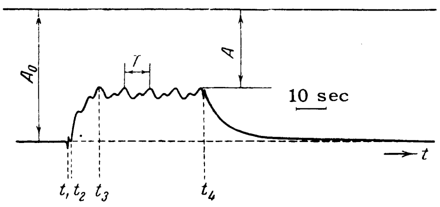

FIG. 2. Time dependence of the second sound resonance amplitude at a temperature of 1.6° K, φ = 1.527 rad, T = 8.64 sec, and V_0 = 22.78 mm/sec. Here, t_1 is the starting time, and t_4 the termination time of the torsional oscillations of the resonator, A_0 is the second sound resonance amplitude before the resonator is set into oscillation, and A is the minimum amplitude of the second sound resonance in the supercritical oscillatory regime.

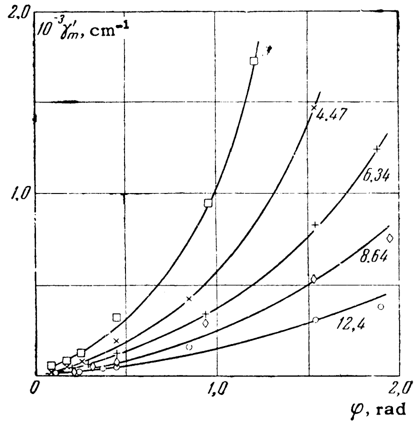

FIG. 4. Dependence of the coefficient γ'_m upon the amplitude φ of the oscillations for constant period T (the numbers on the curves are the values of T in seconds).

tion of the vortices. From the curves giving the measurements of the time ξ [12] it was possible to conclude with confidence that the relaxation times associated with vortex formation are certainly less than 1 sec. The data of Mamaladze[14] are in full accordance with this conclusion.

As the characteristic describing the attenuation of second sound in the supercritical oscillatory regime we selected the excess attenuation coefficient $\gamma'_m(\omega, \varphi)$, corresponding to the maximum attenuation in the interval $t_3 - t_4$ of Fig. 2 (for a detailed study of this interval, see below).

The coefficient $\gamma'_m(\omega, \varphi)$ was measured in two ways: at constant amplitude, as a function of the resonator oscillation frequency, $\gamma'_m = \gamma'_m(\omega, \varphi = \text{const})$, and at constant frequency as a function of amplitude, $\gamma'_m = \gamma'_m(\omega = \text{const.}, \varphi)$. Equation (2) was used for the calculations. The results thus obtained are presented in Figs. 3 and 4. Calculations show that both families of curves, $\gamma'_m(\omega, \varphi = \text{const})$ and $\gamma'_m(\omega = \text{const}, \varphi)$, fall closely along the common curve $\gamma'_m = \gamma'_m(V_0)$ given in Fig. 5. Analysis of this curve demonstrates that its functional form may be described by the equation

$$\gamma'_m(\omega, \varphi) = C(V_0 - A)^B, \tag{3}$$

where A, B, and C are constants. The constant A has a physical significance, and represents the critical velocity V_c.

Reduction of the experimental results by the method of least squares yields the following values for the constants:

$$A = V_c = 1.28 \pm 0.29 \text{ mm/sec} \qquad B = 1.510 \pm 0.011,$$
$$C = (5.00 \pm 0.21) \cdot 10^{-7} \text{ sec}^{3/2} \text{ mm}^{-5/2}$$

The errors indicated here are probable errors in the respective quantities. The value obtained for the critical velocity, V_c = (1.28 ± 0.29) mm/sec, is found to be in good agreement with the results of other authors[2,10].

The comparisons which we have made[12] indicate a close correspondence between the numerical values of the excess attenuation coefficient obtained by us and by Hall and Vinen[3].

The phenomena occurring in the interval $t_2 - t_3$ of Fig. 2 may be given the following interpretation: immediately following the appearance of the first vortices, at the point t_2, they begin to decay, and thereafter the processes of vortex formation and decay operate simultaneously. The shape of the curve is governed by the relation between the rates of the two processes. The rate of vortex formation depends upon the velocity with which the resonator moves, while the rate with which the turbulence decays depends, naturally, upon the nature of the vortices, and upon their number at a given instant.

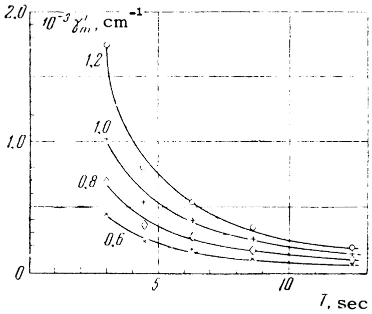

FIG. 3. Dependence of the coefficient γ'_m upon the period T of the resonator oscillations for constant amplitude φ (the numbers on the curves are the values of φ in radians).

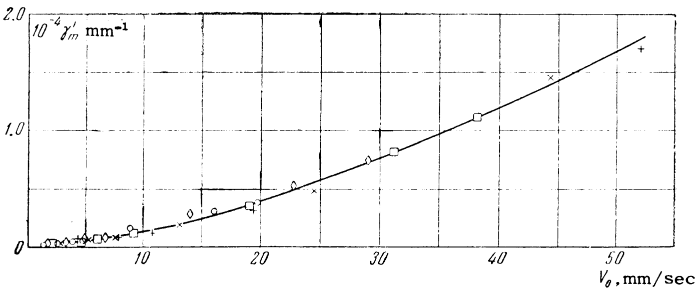

FIG. 5. Dependence of the coefficient γ'_m upon the maximum linear velocity V_0 at the periphery of the resonator. The solid curve corresponds to the relation $\gamma'_m = C(V_0 - A)^B$, where $A = V_c$ = 1.28 mm/sec, B = 3/2, and $C = 5 \times 10^{-7}$ $\text{sec}^{3/2} \text{mm}^{-5/2}$.

Clearly, if the rate of formation of the vortices is the greater of the two, the second sound attenuation will increase, while if vortex decay predominates, it will decrease.

Thus, starting at the time t_2, the accumulation of the vortices is accompanied by an increase in their decay rate until, finally, at t_3 a state of statistical equilibrium is reached between the vortices being formed and those decaying. Since at this point the rate of vortex formation (at V_0) has its maximum value, a further increase in the number of vortices is in principle impossible.

Let us now investigate the interval $t_3 - t_4$ (Fig. 2). To simplify matters let us turn to Fig. 6, in which is represented the simplest case, for which at the moment rotation is begun (t_1) the resonator is at one of its extreme positions. Here, the upper sinusoidal curve represents a graph of the linear velocity at the periphery of the resonator. In the lower graph, the excess second sound attenuation coefficient γ' is plotted along the y axis against time on the x axis. The upper limit of γ', for the given value of the velocity V_0, is designated by γ'_m.

We shall attempt to explain the succession of large and small maxima in the excess attenuation curve. Having attained its peak value γ'_m at the point t_3, the attenuation begins to decrease due to the reduction in the velocity of the resonator. Over the interval $t_3 - t_4$ the decrease in γ' proceeds under the conditions of competition between the processes of vortex decay and formation, but at the point t_4, at which the resonator falls below the critical velocity, vortex formation ceases and only pure vortex decay ensues. At t_5 the resonator again attains the critical velocity; but vortex formation begins, not immediately, but only after a time t_{rel}. At this instant, the vortex formation process begins. After a certain interval, the rates of vortex formation and decay become equal, and the excess attenuation curve passes through a minimum (t_6).

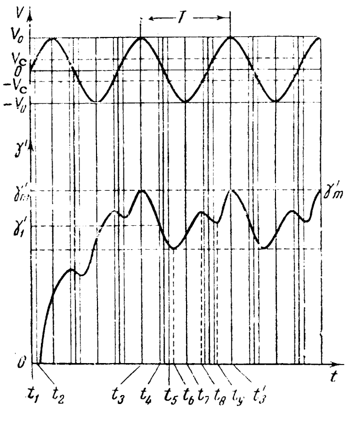

FIG. 6. Time dependence of the excess attenuation coefficient γ': V_0 is the maximum linear velocity at the periphery of the resonator, V_c is the critical velocity for vortex formation, γ'_m is the upper limit of γ', and t_1 is the time at which the resonator is set into oscillation.

Over the time interval $t_3 - t_6$ the excess second sound attenuation decreases to such an extent that the coefficient γ' cannot attain its peak value γ'_m at the time t_7, but reaches it only after a half-period, at t'_3. It is evident from Fig. 6 that the coefficient γ' could attain its maximum value γ'_m at the time t_7 only in the event that the minimum at the point t_6 were to occur somewhat above the value γ'_1, as is the case in the next half-period (the points t_3 and t'_3). To the right of t'_3, the above-described pattern is repeated during each succeeding oscillation period of the resonator. The principal maxima γ'_m are repeated exactly after each period of oscillation of the resonator, and coincide in time with the velocity maxima V_0 (the points t_3 and t'_3), while the minima (at the points t_6 and t_9) and the maximum occurring between them (at t_8) are for well-understood reasons displaced to the right of the corresponding minima and maximum in the velocity of the resonator. The frequency of the oscillations in γ' is double that of the oscillations of the resonator. This is explained simply by the fact that the maximum velocity V_0 is reached twice during each period of the resonator.

Thus, by considering a superposition of the processes of vortex decay and formation, taking into account the existence of critical velocities and the delay times associated with them ($\xi = t_c + t_{rel}$), we are able to explain the shape of the excess attenuation curve over the interval $t_1 - t_4$ of Fig. 2, with its characteristic alternation and distribution of large and small maxima.

At the instant t_4 (Fig. 2), the torsional oscillations of the resonator are stopped. While to the left of the point t_4 the entire excess second sound attenuation curve is the result of competition between the processes of vortex decay and formation, to its right only the decay process is operative. From a systematic study of the processes involved in pure vortex decay (to the right of t_4) the law describing the time dependence of the excess attenuation coefficient γ' was found to have the form of a sum of two exponential functions:

$$\gamma'(t) = M \exp\left(-\frac{0.693}{\tau_1}\, t\right) + K \exp\left(-\frac{0.693}{\tau_2}\, t\right) \qquad (4)$$

with half-lives

$$\tau_1 = 14.20 \pm 0.86 \text{ sec}, \qquad \tau_2 = 2.10 \pm 0.28 \text{ sec}$$

which were universal throughout all of our experiments. As before, the probable errors are taken as indicative of the accuracy. Here and below, time is measured from the instant at which the resonator stops.

Equation (4) is strongly reminiscent of the radioactive decay law for a mixture of two genetically unrelated substances (see, for example, [15]). This leads to the concept that in the process just considered for the decay of quantum turbulence there are present two different types of vortices, with differing decay half-lives τ_1 and τ_2. It should at this point be stated that the reason for the occurence of vortices of different types is as yet unclear. The suggestion that this might be associated with the two different directions of rotation of the resonator was refuted by specially-designed experiments, in which the resonator made only one half-oscillation in a single direction. These experiments with unidirectional motion of the resonator gave results in full agreement with Eq. (4).

The coefficients M and K in (4) determine the initial value of $\gamma'(t)$:

$$\gamma'(0) = M + K. \qquad (5)$$

Dividing Eq. (4) by $\gamma'(0)$, we obtain

$$\frac{\gamma'(t)}{\gamma'(0)} = P \exp\left(-\frac{0{,}693}{\tau_1} t\right) + (1-P) \exp\left(-\frac{0{,}693}{\tau_2} t\right), \qquad (6)$$

where $P = M/(M+K)$.

In contrast to the decay half-lives τ_1 and τ_2, the coefficient P, which establishes the ratio between the initial numbers of vortices of each sort, ranges in the various experiments from 0.2 to 0.5. The source of this non-reproducibility in P from experiment to experiment may lie in the decelerations due to the braking moments exerted on the resonator (at t_4 in Fig. 2), which are not precisely identical for the various experiments.

The second exponential in Eq. (4), with the lesser decay half-life τ_2, falls off considerably faster than the first; after a certain interval, therefore, the decay of the turbulence follows a simple exponential law. An exponential term also appears in the formula for the decay of quantum turbulence found by Vinen in his experiments with heat flow in He II (see, for example, Eq. (38) in his survey article [17]).

Finally, from Eq. (4) it is possible to obtain an expression for the rate of the turbulence decay process:

$$\frac{\partial\gamma'}{\partial t} = -\left[\frac{0{,}693}{\tau_1} M \exp\left(-\frac{0{,}693}{\tau_1} t\right) + \frac{0{,}693}{\tau_2} K \exp\left(-\frac{0{,}693}{\tau_2} t\right)\right]. \qquad (7)$$

In conclusion, it should be noted that postulating a priori the possibility of the generation of vortices in superfluid helium, all of the phenomena observed in our experiments may be considered to be in qualitative agreement with the Onsager-Feynman vortex concept. The absence of any detailed theory of vortex flow in He II in supercritical periodic processes makes quantitative comparisons impossible.

I feel obliged to express my sincere gratitude to Prof. É. L. Andronikashvili for suggesting the theme of the present investigation and for directing its fulfillment. I wish to thank Yu. G. Mamaladze and G. A. Gamtsemlidze for their aid in discussing a number of problems arising in the course of this study, and G. G. Bezhashvili and K. A. Turkadze, and all of the technical personnel of the Tbilisi State University cryogenics laboratory, for their help in this work.

The preliminary experiments were carried out at the Institute for Physics Problems of the U.S.S.R. Academy of Sciences, with the kind permission of Academician P. L. Kapitza and Prof. V. P. Peshkov. I also wish to thank K. N. Zinov'eva for much helpful advice.

[1] É. L. Andronikashvili, JETP **16**, 780 (1946).

[2] A. C. Hollis-Hallett, Proc. Roy. Soc. **A210**, 404 (1952).

[3] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. **A238**, 204, 215 (1956).

[4] Andronikashvili, Mamaladze, Matinyan, and Tsakadze, UFN **73**, 3 (1961), Soviet Phys. Uspekhi **4**, 1 (1961).

[5] Reppy, Depatie, and Lane, Phys. Rev. Lett. **5**, 541 (1960).

[6] R. P. Feynman, Phys. Rev. **94**, 262 (1954); Progr. in Low Temp. Physics **1**, Amsterdam, 1955.

[7] W. F. Vinen, Proc. Roy. Soc. **A240**, 114, 128 (1957); **A242**, 493 (1957); **A243**, 400 (1957).

[8] C. B. Benson and A. C. Hollis-Hallett, Can. J. Phys. **34**, 668 (1956).

[9] R. J. Donnelly and O. Penrose, Phys. Rev. **103**, 1137 (1956).

[10] G. A. Gamtsemlidze, JETP **37**, 950 (1959), Soviet Phys. JETP **10**, 809 (1960).

[11] H. E. Hall, Adv. in Phys. (Phil. Mag. Suppl.) **9**, 89 (1960).

[12] R. A. Bablidze, Trudy Instituta Fiziki AN SSSR (Trans. Physics Institute Acad. Sci. U.S.S.R.), in press.

[13] É. L. Andronikashvili and R. A. Bablidze, Proc. VII Internat. Conf. on Low Temp. Physics, Toronto, 1961; p. 555.

[14] Yu. G. Mamaladze, Soobshcheniya AN SSSR (Communications Acad. Sci. U.S.S.R.) **22**, 665 (1959).

[15] M. Curie, Radioactivite, Hermann, Paris, 1935 (Russian transl., Fizmatgiz, Moscow, 1960, Sec. 63).

[16] H. E. Hall, Phil. Trans. Roy. Soc. London **A250**, 359 (1957).

[17] W. F. Vinen, Progr. in Low Temp. Physics **3**, Amsterdam, 1961; p. 1.

Translated by S. D. Elliott
358