GRAVITATIONAL INTERACTION OF FERMIONS

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Gravitational interaction of spin-1/2 particles is considered in the linear approximation. It is shown that if gravitational interaction is taken into account, the question whether a free neutrino is two- or four-component acquires a physical meaning. The vertex part for the interaction between fermions and the gravitational field is shown to possess properties analogous to those of the electrodynamic vertex described by the Ward theorem. Observable effects due to spins are considered.

THE present work is devoted to the question of how the spin of the fermion influences its interaction with a gravitational field. We consider the corresponding spin terms in the gravitational interaction of particles with spin-1/2 and clarify how the presence of other particle interactions (strong, weak, and electromagnetic) influences the form of these terms. We also discuss possible experimental manifestations of the spin effects considered here.

1. LINEAR APPROXIMATION

We carry out the analysis within the framework of the linear approximation to the general theory of relativity, formulated by Gupta [1]. In this approximation, the interaction between two particles is described by a diagram of the Moller type. The amplitude corresponding to this diagram is

$$4\pi\varkappa^2\Gamma_{ik}D_{ik,\,mn}\Gamma_{mn}, \qquad (1)$$

where $\kappa = \sqrt{k} = 8 \times 10^{-19}/m_p$ (m_p is the proton mass; we put everywhere $\hbar = c = 1$), the propagation function of a graviton with four-momentum q has the form

$$D_{ik,\,mn} = q^{-2}(\delta_{im}\delta_{kn} + \delta_{in}\delta_{km} - \delta_{ik}\delta_{mn}), \qquad (2)$$

and the graviton vertex Γ_{ik} is determined by a relation which is a mathematical expression of the equivalence principle:

$$\int \langle 2 | \theta_{ik}(x) e^{iqx} | 1 \rangle d^4x = (2\pi)^4 \delta(p_1 - p_2 - q)\,\Gamma_{ik}(p_1, p_2, q). \qquad (3)$$

Here $\langle 2|$ and $|1\rangle$ are physical states of a given particle (in the Schrödinger representation) and θ_{ik} is the symmetrized energy-momentum tensor [2], corresponding to the total Lagrangian L of the interacting elementary particles:

$$L = L_f + L_s + L_e + L_w + L_x, \qquad (4)$$

where L_f is the free Lagrangian of the elementary particles, L_s, L_e, and L_w are the Lagrangians of the strong, electromagnetic, and weak interactions, and L_x the Lagrangian of interactions still unknown to us, if they exist.

From the conservation of the energy-momentum tensor

$$\partial\theta_{ik}/\partial x_k = 0 \qquad (5)$$

it follows that the gravitational vertex $\Gamma_{ik}(q)$ satisfies the transversality condition

$$q_k\Gamma_{ik} = 0. \qquad (6)$$

2. "BARE" VERTICES

If we "turn off" L_s, L_e, L_w, and L_x we readily obtain the gravitational vertices for "bare" particles. For a meson with zero spin (which we will call arbitrarily a π meson) we have

$$\Gamma_{ik}^{\pi} = \varphi_2^*\,[2p_ip_k - \tfrac{1}{2}(q_iq_k - q^2\delta_{ik})]\,\varphi_1, \qquad (7)$$

where $p = (p_1 + p_2)/2$. For a fermion with spin-1/2 (we call it a proton)

$$\Gamma_{ik}^{p} = \tfrac{1}{2}\bar{u}_2(\gamma_ip_k + \gamma_kp_i)\,u_1 = \tfrac{1}{2}\bar{u}_2\{\gamma_ip_k\}\,u_1. \qquad (8)$$

The braces $\{\ldots\}$ denote symmetrization with respect to the indices i and k. We note that Γ_{ik}^{p} does not contain the proton mass, but only its four-momentum and has the property of γ_5-invariance.

3. "INERTIAL" NEUTRINOS

For the neutrino the graviton vertex is

$$\Gamma_{ik}^{\nu} = \tfrac{1}{2}\bar{u}_2\{\gamma_ip_k\}(1 + \gamma_5)\,u_1, \qquad (9)$$

if the free Lagrangian of the neutrino is of the two-component type, and

$$\Gamma^{\nu}_{ik} = \frac{1}{2}\bar{u}_2\{\gamma_i p_k\} u_1, \quad (10)$$

if the free Lagrangian is of the four-component type. In the latter case the virtual gravitons could be converted into pairs of neutrinos and antineutrinos with "irregular" helicity (left $\bar{\nu}$ and right ν). Such anomalous neutrinos and antineutrinos could not be emitted, scattered, and absorbed through weak interactions, but would be deflected by gravitational fields and would themselves be sources of such fields.

It follows from the foregoing that the question of whether the neutrino is really a two-component particle is a physical problem. In principle, this problem could be solved by measuring the flux of the neutrinos in cosmic space by two different methods: first by its gravitational effect and then by the weak interactions due to this flux [1]. Comparing the results of these measurements, one could establish whether anomalous inertial neutrinos exist in nature. [2]

4. WHEN DOES GRAVITATIONAL INTERACTION BECOME STRONG?

It follows from (1), (2), and (8) that the gravitational interaction of two particles increases with increasing energy like E^2, where E is the total c.m.s. energy. Indeed, a gravitational amplitude of the scattering of two particles with four-momenta p and q is proportional to

$$\varkappa^2[2(pk)^2 - p^2k^2]q^{-2}. \quad (11)$$

This expression must be compared with the electromagnetic amplitude of the Moller scattering

$$e^2pk/q^2. \quad (12)$$

The contributions of both amplitudes become comparable when

$$2pk \sim e^2/\varkappa^2. \quad (13)$$

Thus, when $E \approx \sqrt{2pk} \approx \kappa^{-1} \approx 10^{+19}m_p$ the gravitational interaction becomes strong. Corresponding to this energy is a length on the order of 10^{-32} cm. That the gravitational interaction becomes strong at this length was first pointed out by Landau [5].

[1]Estimates of the maximum neutrino density in the universe are contained in the paper of Pontecorvo and Smorodinskiĭ.[3]

[2]The question of gravitational interaction of the neutrino was considered previously by Brill and Wheeler.[4] These authors, however, assumed tacitly that there are no inertial neutrinos in nature.

5. CLASSICAL EFFECTS

If the graviton vertex is known for a particle, then the Lagrangian of its interaction with an external weak gravitational field γ_{ik} can be written in the form

$$L_g = \frac{1}{2}\gamma_{ik}\Gamma_{ik}, \quad (14)$$

where the external field is connected with the metric tensor g_{ik} by the well known relation

$$g_{ik} = \delta_{ik} + \gamma_{ik}. \quad (15)$$

To the contrary, the field produced by a particle in the linear approximation is

$$\gamma_{ik} = 8\pi\varkappa^2 D_{ik,\,mn}\Gamma_{mn}. \quad (16)$$

The amplitude (14), if we regard it as an effective Lagrangian of gravitational interaction of two "particles" (for example, the sun and a photon), makes it possible to describe such classical effects of the general theory of relativity as the red shift and the deflection of light by the sun (but not the rotation of the mercury perhelion, which is a second-order effect with respect to the gravitational field). Here it is necessary, in view of the very meaning of the classical effects, to change over to the x-representation [3].

6. STATIC APPROXIMATION

In order to clarify the physical meaning of the proton vertex (8), let us consider it in the static approximation, when $p_1 = (m, 0, 0, 0)$, $p_2 = p_1 + q$, and q has only small spatial components: $q = (0, \mathbf{q})$. We shall include the zeroth and first-order terms in $\mathbf{q}$.

Using the relation

$$m\bar{u}_2\gamma_i u_1 \rightarrow \bar{u}_2(p_i + \frac{1}{2}\sigma_{ik}q_k)u_1,$$

we rewrite (8) in the form

$$\Gamma^{p}_{ik} = \bar{u}_2 m^{-1}[p_i p_k + \frac{1}{4}\{\sigma_{ir}q_r p_k\}]\dot{u}_1. \quad (17)$$

If we now change over to a nonrelativistic notation and discard the longitudinal terms, which make no contribution to the physical processes, we readily find that in the zeroth approximation and in the approximation linear in q only two terms remain in Γ_{ik}. The first is

$$\Gamma_{00} = m, \quad (18)$$

and the other

$$\Gamma_{0\alpha} = \frac{1}{4}[\sigma\mathbf{q}]_\alpha, \quad (19)^*$$

[3]The three classical effects were recently derived in the classical linearized gravitational theory by Bludman.[6]

*$[\sigma\mathbf{R}] = \sigma \times \mathbf{R}$.

where σ is the three-dimensional proton spin vector, and $\alpha = 1, 2, 3$.

If we substitute (18) and (19) in (16) and change over to the x-representation, then (18) yields the usual static field

$$\gamma_{00} = -\gamma_{\alpha\alpha} = 2\varkappa^2 m/R, \qquad (20)$$

and (19) yields the field produced by the spin moment of the proton

$$\gamma_{0\alpha} = \varkappa^2 [\sigma \mathbf{R}] R^{-3}. \qquad (21)$$

The latter expression is analogous to the well known expression for the field produced by the mechanical moment of a rotating massive body[7],

$$\gamma_{0\alpha} = 2\varkappa^2 [\mathbf{M}\mathbf{R}] R^{-3}. \qquad (22)$$

7. "DRESSED" VERTICES

So far we have considered only gravitational vertices of free particles. Let us turn on now all the possible interactions between them [Lagrangian (4)]. Then, as usual, "virtual clouds" will be produced around the particles and their gravitational interaction is no longer pointlike. From the transversality condition it follows that the Γ_{ik} for the pion and proton have in this case the form

$$\Gamma_{ik}^{\pi} = \varphi_2^* [g_1 p_i p_k + g_2 (q_i q_k - q^2 \delta_{ik})] \varphi_1, \qquad (23)$$

$$\Gamma_{ik}^{p} = \bar{u}_2 m^{-1} [f_1 p_i p_k + \tfrac{1}{4} f_2 \{\sigma_{ir} q_r p_k\} + f_3 \gamma_5 \{\sigma_{ir} q_r p_k\}$$
$$+ f_4 \gamma_5 \{(\gamma_i q^2 - 2mq_i) p_k\} + f_5 (q_i q_k - q^2 \delta_{ik})$$
$$+ f_6 \gamma_5 (q_i q_k - q^2 \delta_{ik})] u_1. \qquad (24)$$

Here { } denotes as before symmetrization over the indices i and k, and all the f and g are scalar functions of q^2.

The discussion of the properties of the graviton vertex Γ_{ik}^{p} is best carried out using the analogy with the Hofstadter vertex Γ_i^p, which describes the electromagnetic interaction between a proton and a photon:

$$\Gamma_i^p = \bar{u}_2 m^{-1} [e_1 p_i + e_2 \sigma_{ir} q_r + e_3 \sigma_{ir} q_r \gamma_5$$
$$+ e_4 (\gamma_i q^2 - 2mq_i) \gamma_5] u_1. \qquad (25)$$

Here e_1 is the electric form factor, e_2 the magnetic form factor, the term proportional to e_3 is the electric dipole, and the term proportional to e_4 is the anapole [8].

It is obvious that the terms f_3 and f_4, in analogy with the terms e_3 and e_4, can result only from parity nonconservation. In addition, for f_3 and e_3, nonconservation of combined parity is also necessary.

It is well known that the conservation of the electromagnetic field

$$\partial j_i / \partial x_i = 0 \qquad (26)$$

leads to the relation

$$e_1 (q^2 = 0) = 1, \qquad (27)$$

which denotes the equality of the "bare" and "dressed" proton charges (with accuracy to the photon polarization of the vacuum). An analogous relation holds also for the graviton vertex:

$$f_1 (q^2 = 0) = g_1 (q^2 = 0) = 1. \qquad (28)$$

This relation denotes the equality of the gravitating and inertial masses for "dressed" particles.

It is well known that $e_2(0)$ is generally speaking not equal to unity, which corresponds to the proton having an anomalous magnetic moment. Unlike in electrodynamics, there should be no "anomalous gravitational magnetic moment," and the following relation should hold true

$$f_2 (0) = 1. \qquad (29)$$

The physical meaning of this relation is clear from the interpretation given the term $g_{0\alpha}$ in the preceding section. Inasmuch as the total momentum of the particle and the virtual cloud is equal to the spin of the bare particle when the interaction is turned on, the gravitational field produced by this momentum should equal the field produced by the spin of the "bare" particle.

The particle can acquire an electric dipole moment (e_3 term) if there exists a CP-noninvariant interaction (L_X) which shifts the center of the distribution of the electric charge in the virtual cloud of the particle relative to the center of mass of this cloud. In the case of a gravitational vertex, the role of the electric charges is assumed by the masses of the virtual particles, and by definition the center of this distribution is the center of mass. Therefore, unlike the electric dipole moment, the "gravitational dipole moment" should vanish even if a CP-noninvariant interaction L_X exists:

$$f_3 (q^2 = 0) = 0. \qquad (30)$$

The terms proportional to f_4, f_5, and f_6, like the anapole term e_4, vanish when $q^2 = 0$ and do not lead to any long-range forces whatever.

We see thus that if (28), (29), and (30) are satisfied, the graviton vertices of the "dressed" and "bare" particles should essentially be the same at small values of q. A check on this prediction is of considerable interest.

8. CONCLUDING REMARKS

A check on relation (28) on the one hand and relations (29) and (30) on the other are problems of entirely differing difficulty. Relation (28) was essentially verified in the well known experiments of Eotvos. It has been confirmed with very high accuracy (see, for example, [9]). As regards relations (29) and (30), their verification is for the time being beyond the capabilities of the experiment, since the corresponding effects are very small.

In a static gravitational field the "gravitational magnetic moment" of the particle can manifest itself only if the particle moves. (The corresponding effects for a classical top were considered by Schiff [10]).

The existence of a "gravitational dipole moment" of a particle (f_3 term) would lead to precession of its spin in a gravitational field. Thus, if we assume that for a proton $f_3 = 1$, it turns out that the angular velocity of precession is $\omega = 2g/c \approx (1/3)10^{-7}$ sec^{-1} (where $g = 980$ cm/sec^2 and $c = 3 \times 10^{10}$ cm/sec), i.e., the proton should make one revolution every 4π years.

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APPENDIX

Let us trace the connection between the statement of Sec. 7 and the general formula (3) for Γ_{ik} 4). For this purpose we consider Γ_{ik} for small spatial **q** and expand $\exp(iq_i x_i)$ in powers of qx. We furthermore separate in explicit form the dependence of the states $|1\rangle$ and $\langle 2|$ on the time:

$$|1\rangle = |1,0\rangle \exp(-iE_1 t), \quad \langle 2| = |2,0\rangle \exp(iE_2 t).$$

We now use the condition of invariance under spatial rotations, the transversality and symmetry conditions for Γ_{ik}, and the circumstance that for a proton we have the vector σ and its dual tensor $\sigma_{\alpha\beta} = \epsilon_{\alpha\beta\gamma}\sigma_\gamma$. It is then easy to obtain that only the following terms can differ from zero:

$$\Gamma_{00} = \langle 1,0|\theta_{00} d^3x|0,1\rangle = m_p,$$
$$\Gamma'_{00} = \langle 1,0|\theta_{00} x_\gamma d^3x|0,1\rangle q_\gamma,$$
$$\Gamma'_{0\alpha} = \tfrac{1}{2}\langle 1,0|(\theta_{0\alpha} x_\gamma - \theta_{0\gamma} x_\alpha) d^3x|0,1\rangle q_\gamma.$$

4)The idea of the following discussion belongs to L. D. Landau.

We consider first $\Gamma'_{0\alpha}$. The quantity

$$\langle 1,0|(\theta_{0\alpha} x_\gamma - \theta_{0\gamma} x_\alpha) d^3x|0,1\rangle$$

is the mean value of the angular momentum operator over the physical proton. Since the total momentum of the proton does not change when the interaction is turned on, we have

$$\langle 1,0|(\theta_{0\alpha} x_\gamma - \theta_{0\gamma} x_\alpha) d^3x|0,1\rangle = \tfrac{1}{2}\sigma_{\alpha\gamma},$$
$$\Gamma'_{0\alpha} = \tfrac{1}{4}\sigma_\alpha,$$

We consider now Γ'_{00}. The vector R_γ

$$R_\gamma = \langle 1,0|\theta_{00} x_\gamma d^3x|0,1\rangle$$

can be directed only along σ_γ, so that $\Gamma'_{00} = 0$ if parity is conserved.

We now consider the general case. The formula for R_γ is invariant under translation, since when the origin is changed, $x_\gamma \to x_\gamma + l_\gamma$, the vector R_γ goes over into $R_\gamma + m_p l_\gamma$. This means that in this approximation there should exist additional conditions which fix the origin. It is likely that the origin can be chosen in the center of mass of the state $|1\rangle$. But then, by definition, $R_\gamma = 0$ and $\Gamma'_{00} = 0$.

Comparing the obtained results with general expressions (23) and (24) for Γ^{π}_{ik} and $\Gamma^{(p)}_{ik}$ we readily verify that (28), (29), and (30) are correct.

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