CURRENT FLUCTUATIONS IN SEMICONDUCTORS NEAR A NON-EQUILIBRIUM STATIONARY STATE

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A method is proposed for theoretical investigation of current fluctuations in a semiconductor close to a stationary non-equilibrium state produced by a strong electric field. Current fluctuations are studied by this method in atomic semiconductors where the interaction between the current carriers and acoustic phonons is important. It has been found that the fluctuations in the low-frequency range are proportional to the square root of the electric field. The spectral density of longitudinal (along the direction of the electric field) fluctuations of the current exhibits an appreciable dispersion (dependence on the frequency ω) in the radio frequency range. The spectral density of transverse current fluctuations is not dispersive in the low frequency region. In the high-frequency region, the fluctuations are proportional to the $^3/_2$ power of the field and inversely proportional to the square of the frequency.

THE problem of current fluctuations in a semiconductor in a state of thermodynamic equilibrium was first studied by Nyquist in 1928 [1] (see also [2]). Subsequently it has been shown that the results of Nyquist are a special case of a very general theorem on the connection of fluctuations of physical quantities with the dissipative properties of the system in the case of external influences upon it. This connection was established by Callen and Welton. [3] This theorem makes it possible, for example, to reduce the problem of current fluctuations in a system in a state of thermodynamic equilibrium to the problem of the calculation of the conductivity tensor of such a system (with account of its dispersion).

There are no theorems of such a type applicable to a system in a strongly non-equilibrium state (for example, a semiconductor in a strong electric field). To the contrary, it can be established that, in contrast with the case of the equilibrium state, the current fluctuations in such a system are not connected by some exact general relation with its reaction to the effect of a weak electromagnetic field of variable frequency. Therefore, the fluctuations in such systems require special consideration for each case. Here we shall consider one such case—current fluctuations in a semiconductor located in a strong electric field, which produces appreciable deviations from the equilibrium electron distribution accompanied as a rule by departures from Ohm's law.[1)]

The experimental study of these fluctuations makes it possible to obtain a series of interesting facts concerning the semiconductor itself and also the character of the non-equilibrium state near which the fluctuations are taking place. For example, one can determine the mean energy of the conduction electrons. The latter is especially interesting in those extraordinary cases in which Ohm's law is satisfied in spite of the fact that the electron distribution departs considerably from equilibrium with the lattice temperature. Such a situation can take place in ionic semiconductors. [8,9] It is then possible to establish the mean energy of the electrons only by indirect evidence, inasmuch as under such conditions the conductivity does not depend on the electric field while, for example, the current fluctuations can increase sharply with increase in the field.

1. GENERAL CONSIDERATIONS

Let the electronic [2)] system be characterized by a one-particle distribution function $\overline{F}_p$, where p is the quasi-momentum of the electrons. $\overline{F}_p$ is determined from the kinetic equation

[1)] A series of theoretical researches (see the review of Lax,[4] where there is an extensive bibliography) have been concerned with problems of fluctuations close to the stationary state for various cases. Note should also be made of recent researches[5,6] which have been devoted to fluctuations in a non-equilibrium plasma, and to the study of fluctuations in semiconductors in a strong electric field in the limiting case of low frequencies.[7]

[2)] In what follows, for definiteness, we shall be speaking of electrons in the conduction band, although our discussions apply in equal measure to holes in the valence band.

$$e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial \mathbf{p}}\bar{F}=-\hat{S}\bar{F}, \qquad (1)^*$$

where $\hat{S}$ is the collision operator, e the electron charge, v its velocity, **E** and **H** the intensities of the electric and magnetic fields, respectively, and c the velocity of light. For simplicity we consider in what follows a case (which is the most interesting from the experimental viewpoint) in which the effects associated with the Fermi degeneracy of the electrons are negligibly small. Then $\hat{S}$ is a linear operator.

It is assumed that the electric field **E** does not depend on time, i.e., that the external conditions are stationary. The electrons give up the energy obtained from the field **E** to various scatterers (for example, phonons). We assume that the state of the scatterers can be regarded as unchanging during a sufficiently long interval of time. Only in this case can Eq. (1) have a solution which is time-independent, i.e., a stationary state can actually be established. Then, by knowing the stationary distribution $\bar{F}$, we can find the mean current density $\bar{\mathbf{J}}$, the relation of which to **E** is generally nonlinear.

At each instant of time t, the electron distribution function (together with the current density $\mathbf{J}=\bar{\mathbf{J}}+\delta\mathbf{J}$) fluctuates, taking on some value $F_p = \bar{F}_p + \delta F_p(t)$. As a result of these fluctuations, the correlators of the type $\overline{\delta J_i(\mathbf{r}, t+\tau)\,\delta J_k(\mathbf{r}', t)}$ are shown to be different from zero. They also characterize the current fluctuations in the system. The bar here denotes an averaging over all instants of time for a fixed value of τ. The argument t over which the averaging takes place will frequently be omitted.

In final analysis, we are interested in the problem of the fluctuations of the total current flowing in the circuit. Therefore, we can neglect the spatial correlations of current fluctuations which take place at very small distances [cf. [10]] and assume that $\overline{\delta J_i(\mathbf{r}, t+\tau)\,\delta J_k(\mathbf{r}', t)} \sim \delta^3(\mathbf{r}-\mathbf{r}')$. It is then appropriate to introduce the notation

$$\overline{\delta J_i(\mathbf{r}, t+\tau)\,\delta J_k(\mathbf{r}', t)} = \overline{\delta J_i(\tau)\,\delta J_k}\,\delta^3(\mathbf{r}-\mathbf{r}');$$

$$\overline{\delta J_i(\tau)\,\delta J_k} = \int_{-\infty}^{\infty} (\delta J_i\,\delta J_k)_\omega e^{-i\omega\tau}\,d\omega. \qquad (2)$$

Conversely,

$$(\delta J_i\,\delta J_k)_\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau}\overline{\delta J_i(\tau)\,\delta J_k}. \qquad (2a)$$

If the stationary state is a stable one (which is assumed), then $\overline{\delta J_i(\tau)\,\delta J_k}$ falls off sufficiently rapidly at large τ and the integral of (2a) converges.

Our purpose is to derive an equation of the kinetic type with the help of which one can consider current fluctuations close to a stationary state. For this purpose, we assume that at the instant of time t the electron distribution function of the k-th component of the current vector differs from the mean value by a quantity δJ_k (as a result of fluctuations). Then, even at the next moment, the distribution function will differ from its stationary value. This difference is connected with two circumstances. First, the system must during some instant of time "maintain a memory" of the fluctuation which took place, by assumption, at the time t. Second, new fluctuations will take place in it, the occurrence of which is a random process.

We assume that the change with time of the distribution function in which we are interested is associated with only the first circumstance. For this case, it is sufficient to study the correlator in terms of which (2) is directly expressed,

$$\overline{\delta F_p(t+\tau)\,\delta J_k(t)} = \int_{-\infty}^{\infty} e^{-i\omega\tau}(\delta F_p\,\delta J_k)_\omega\,d\omega, \qquad (3)$$

as a function of τ. Actually, as a result of the averaging over t, the effect of the second circumstance will be eliminated, since it is a random phenomenon.[3)] As a net result, for example at $\tau > 0$, the expression (3) must describe a diffusion of the fluctuations which had a certain definite value at $\tau = 0$.[4)] This process can be studied with the aid of a kinetic equation which is valid for $\tau > 0$:

$$\frac{\partial}{\partial\tau}\overline{\delta F_p(\tau)\,\delta J_k} + e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial \mathbf{p}}\overline{\delta F_p(\tau)\,\delta J_k} = -\sum_{p'} S_{pp'}\,\overline{\delta F_{p'}(\tau)\,\delta J_k}, \qquad (4)$$

where $S_{pp'}$ are the matrix elements of the collision operator. We also introduce the quantity

$$\gamma_p^k(\omega) = \frac{1}{2\pi}\int_0^{\infty} d\tau e^{i\omega\tau}\overline{\delta F_p(\tau)\,\delta J_k}. \qquad (5)$$

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$

[3)] Actually, this is an assumption on the absence of time correlation between random forces acting on an electron system in a stationary state.

[4)] We note that the correlator (3) can be represented in the form

$$\overline{\delta F_p(\tau)\,\delta J_k} = e\sum_{p'} v_{p'}^k\,\overline{\delta F_p(\tau)\,\delta F_{p'}} = e\sum_{p'} v_{p'}^k\,\overline{F_p(\tau)\,F_{p'}} - \bar{F}_p\,\bar{J}_k.$$

Correlators of the type $\overline{\delta F_p(\tau)\,\delta F_{p'}}$ for systems in stationary states were considered by Lax[4] (compare also with [11]).

Multiplying (4) by $e^{i\omega t}$ and integrating, we find that it satisfies the following equation:

$$-i\omega\gamma_{\mathbf{p}}^{k}(\omega)+e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}\gamma_{\mathbf{p}}^{k}(\omega)+\sum_{\mathbf{p}'}S_{\mathbf{pp}'}\gamma_{\mathbf{p}'}^{k}(\omega)=\frac{1}{2\pi}\overline{\delta F_{\mathbf{p}}\,\delta J_{k}}\ . \tag{6}$$

In order to compute the average $\overline{\delta F_{\mathbf{p}}\delta J_{\mathbf{k}}}$, we represent it in the form [5)]

$$\overline{\delta F_{\mathbf{p}}\,\delta J_{k}}=e\sum_{\mathbf{p}''}\overline{\delta F_{\mathbf{p}}\,\delta F_{\mathbf{p}''}}\,v_{\mathbf{p}''}^{k}. \tag{7}$$

Inasmuch as we neglect every interaction between the electrons, we have $\overline{F_{\mathbf{p}}F_{\mathbf{p}''}}=\overline{F}_{\mathbf{p}}\overline{F}_{\mathbf{p}''}$ if $\mathbf{p}\neq\mathbf{p}''$, and $\overline{F_{\mathbf{p}}^{2}}=\overline{F}_{\mathbf{p}}$.

$$\overline{\delta F_{\mathbf{p}}\,\delta F_{\mathbf{p}''}}=\begin{cases}0, & \mathbf{p}\neq\mathbf{p}''\\ \overline{F}_{\mathbf{p}}, & \mathbf{p}=\mathbf{p}''.\end{cases} \tag{8}$$

Here we shall everywhere neglect $\overline{F}_{\mathbf{p}}$ in comparison with unity. Ultimately Eq. (6) takes the form

$$-i\omega\gamma_{\mathbf{p}}^{k}+e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}\gamma_{\mathbf{p}}^{k}+\sum_{\mathbf{p}'}S_{\mathbf{pp}'}\gamma_{\mathbf{p}'}^{k}=\frac{ev_{\mathbf{p}}^{k}}{2\pi}\overline{F}_{\mathbf{p}}. \tag{9}$$

Furthermore,

$$(\delta J_{i}\delta J_{k})_{\omega}=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega\tau}\,\overline{\delta J_{i}(t+\tau)\,\delta J_{k}(t)}\,d\tau$$

$$=\frac{1}{2\pi}\int_{0}^{\infty}e^{i\omega\tau}\,\overline{\delta J_{i}(\tau)\delta J_{k}}\,d\tau+\frac{1}{2\pi}\int_{0}^{\infty}e^{-i\omega\tau}\,\overline{\delta J_{k}(\tau)\,\delta J_{i}}\,d\tau. \tag{10}$$

Finally, we then get

$$(\delta J_{i}\,\delta J_{k})_{\omega}=2e\sum_{\mathbf{p}}[v_{\mathbf{p}}^{i}\gamma_{\mathbf{p}}^{k}(\omega)+v_{\mathbf{p}}^{k}\gamma_{\mathbf{p}}^{i}(-\omega)] \tag{11}$$

(the factor 2 is connected with summation over the spins).

The following much simpler formula is valid for the diagonal elements of this tensor:

$$(\delta J_{i}^{2})_{\omega}=4e\,\mathrm{Re}\sum_{\mathbf{p}}v_{\mathbf{p}}^{i}\gamma_{\mathbf{p}}^{k}(\omega). \tag{11a}$$

It is interesting to compare Eq. (9) with that which determines the correction $g_{\mathbf{p}}^{k}(\omega)e^{-i\omega t}$ to the stationary distribution function in the action on the system of a variable electric field $\mathscr{E}^{k}=\mathscr{E}_{0}^{k}e^{-i\omega t}$. For unit amplitude of $\mathscr{E}_{0}^{k}$, it takes the form

$$-i\omega g_{\mathbf{p}}^{k}+e\left(E+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}g_{\mathbf{p}}^{k}=-\sum_{\mathbf{p}'}S_{\mathbf{pp}'}g_{\mathbf{p}'}^{k}-e\frac{\partial\overline{F}}{\partial p_{k}}. \tag{12}$$

The tensor Λ_{ik} which appears in the linear relations

$$j_{i}=\Lambda_{ik}\mathscr{E}_{k} \tag{12a}$$

(j is the alternating-current density), is

$$\Lambda_{ik}=2\sum_{\mathbf{p}}v_{\mathbf{p}}^{i}g_{\mathbf{p}}^{k}(\omega).$$

When the system of electrons in the absence of the variable field $\mathscr{E}$ is in equilibrium, $\overline{F}_{\mathbf{p}}$ is a Boltzmann distribution. Then

$$\frac{\partial\overline{F}}{\partial p_{k}}=v_{\mathbf{p}}^{k}\frac{\partial\overline{F}}{\partial\varepsilon}=-\frac{v_{\mathbf{p}}^{k}}{T}\overline{F}_{\mathbf{p}},$$

where T is the temperature (in energy units) and ϵ is the energy of the electron. In this (simplest) case we obtain the well-known relation

$$(\delta J_{i}\,\delta J_{k})_{\omega}=(T/2\pi)\,[\Lambda_{ik}(\omega)+\Lambda_{ki}^{*}(\omega)]. \tag{13}$$

In the general case of an arbitrary stationary non-equilibrium state, the inhomogeneities of Eqs. (9) and (12) do not differ at all by a constant factor and the general relations of the type (13) are not present.

Evidently one can consider fluctuations of other physical quantities in this way. For example, the contribution of fluctuations of the energy flux density of purely electronic origin to the spectral density is

$$(\delta Q_{i}\,\delta Q_{k})_{\omega}=2e\sum_{\mathbf{p}}\varepsilon_{\mathbf{p}}\,[v_{\mathbf{p}}^{i}\psi_{\mathbf{p}}^{k}(\omega)+v_{\mathbf{p}}^{k}\psi_{\mathbf{p}}^{i}(-\omega)],$$

where $\psi_{\mathbf{p}}^{k}(\omega)$ is determined from the equation

$$-i\omega\psi_{\mathbf{p}}^{k}(\omega)+e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vH}]\right)\frac{\partial}{\partial\mathbf{p}}\psi_{\mathbf{p}}^{k}(\omega)+\sum_{\mathbf{p}'}S_{\mathbf{pp}'}\psi_{\mathbf{p}'}^{k}=\varepsilon_{p}v_{\mathbf{p}}^{k}\overline{F}_{\mathbf{p}}/2\pi.$$

In this connection let us make one remark. It would have been possible from the very beginning to consider not the correlators $\overline{\delta F_{\mathbf{p}}(\tau)\delta J_{\mathbf{k}}}$ or $\overline{\delta F_{\mathbf{p}}(\tau)\delta Q_{\mathbf{k}}}$ but the correlator $\overline{\delta F_{\mathbf{p}}(\tau)\delta F_{\mathbf{p}'}}$ as was done in [4]. Obviously $\overline{\delta F_{\mathbf{p}}(\tau)\delta F_{\mathbf{p}'}}$ is a more universal function, since one can consider not only the current but also a series of other physical quantities in terms of it. Nevertheless, in the majority of cases one finds it more convenient to deal with functions of the form $\overline{\delta F_{\mathbf{p}}(\tau)\delta J_{\mathbf{k}}}$ for the following reason. All these correlators satisfy integro-differential kinetic equations of the type (9), the left sides of which are identical while the right sides (which contain the inhomogeneity) differ. Solution of a kinetic equation of the type (9) is as a rule accomplished in analytical form only under the simplest assumptions both relative

[5)] The normalization volume which actually appears in this case and in a number of succeeding expressions and which is eliminated on going from summation over p to integration will be set equal to unity for simplicity.

to the collision operator and relative to the form of the right-hand side containing the inhomogeneity. Thus, in a number of cases, it is possible to solve the kinetic equation with the right-hand side having the form of a product of the velocity of the electron by a function depending only on its energy. However, in most of these cases, it is impossible to find the Green's function of the operator on the right side of the equation, i.e., it is impossible to find a solution of an equation of the type (9) the right side of which is proportional to $\delta^3(\mathbf{p}-\mathbf{p}_0)$ ($\mathbf{p}_0$ takes on all values).

A similar situation is encountered in the calculation of the conductivity in a weak electric field $\mathscr{E}^k$. If the Green's function of the operator on the left side of an equation of the type (9) were known, then one could compute by means of it not only the differential conductivity but any linear kinetic coefficient generally. However, the usual problem in such a general arrangement is insoluble; for the calculation of the conductivity one must solve an equation of the type (9) with a right-hand side of special form.

We shall pause on the question of the limits of applicability of the use of the approach used here. On the one hand, they must include the usual criteria of applicability of the kinetic equation $\hbar/t_e\bar{\epsilon} \ll 1$, where τ_e is the relaxation time and ϵ the mean energy of the electron in the field **E**. On the other hand, Eq. (4), being classical, is suitable for a description of the behavior of the system only after time intervals $\tau > \hbar/\bar{\epsilon}$. Therefore, Eq. (11) is valid in the frequency range $\omega \ll \bar{\epsilon}/\hbar$.

For $\omega \gtrsim \bar{\epsilon}/\hbar$ a quantum mechanical analysis is necessary. In this case, it is natural in the calculation of the correlator $\overline{\delta J_i(\tau)\delta J_k}$ to use a technique similar to that applied by Callen and Welton[3] in the analysis of fluctuations close to an equilibrium state. To be precise, it is necessary to average the operator

$$\tfrac{1}{2}\left[\delta\hat{J}_i(\tau)\,\delta\hat{J}_k + \delta\hat{J}_k\delta\hat{J}_i(\tau)\right]$$

(where $\delta\hat{J}_i = \hat{J}_i - \bar{J}_i$, $\hat{\mathbf{J}}$ is the current density operator) with the help of the density matrix of the system found in a stationary state. It can be shown that for low frequencies ($\omega \ll \bar{\epsilon}/\hbar$) such a procedure gives the same result as a solution of Eq. (9). For high frequencies, an expression is obtained for the quantum fluctuations. However, consideration of the latter problem goes beyond the framework of the present paper.

Finally, we emphasize that here and below the collision operator $\hat{S}$ is regarded as independent of the electric field **E** and the magnetic field **H**. So far as the dependence on E is concerned, there is a well-known series of cases on the one hand where the departure from Ohm's law is large while the electric field has no effect on the collision operator.[8,9,12] On the other hand the extremely interesting problem of the investigation of the stationary state has apparently not been solved to date even in any single case where $\mathbf{H} = 0$ and where such an effect is substantial. Therefore, there is no sense in investigating fluctuations in similar cases while the stationary state itself is unknown. So far as the dependence on **H** is concerned, it, as is well known,[13,14] begins to play a role for $\hbar\Omega/\bar{\epsilon} \sim 1$, where $\Omega = eH/mc$, m is the effective mass of the conduction electron. In this case, the present analysis is inapplicable.

2. CURRENT FLUCTUATIONS IN ATOMIC SEMICONDUCTORS

Let us consider by this method one concrete example of fluctuations close to a stationary state —current fluctuations in an atomic semiconductor in a strong electric field for $\mathbf{H} = 0$. The stationary state of such a system has been studied in the work of Davydov.[12] Following [12], we assume that the conduction electrons are scattered by acoustic phonons which are in equilibrium, and that the electron and phonon dispersion laws are isotropic and have respectively the following forms:

$$\varepsilon_p = p^2/2m, \qquad \omega_q = wq,$$

where m is the effective mass of the electrons, w is the velocity of the longitudinal sound vibrations, q is their wave vector.

For this case, Eq. (9) takes on the following form:

$$\begin{aligned} &-i\omega\gamma_p^k + e\mathbf{E}\frac{\partial}{\partial \mathbf{p}}\gamma_p^k + \frac{2\pi}{\hbar}\sum_q |c_q|^2 \\ &\times\{\gamma_p^k(N_q+1)\,\delta(\varepsilon_{p-\hbar q} - \varepsilon_p + \hbar\omega_q) \\ &+ \gamma_p^k N_q \delta(\varepsilon_{p+\hbar q} - \varepsilon_p - \hbar\omega_q) \\ &- \gamma_{p-\hbar q}^k N_q\,\delta(\varepsilon_{p-\hbar q} - \varepsilon_p + \hbar\omega_q) \\ &- \gamma_{p+\hbar q}^k(N_q+1)\,\delta(\varepsilon_{p+\hbar q} - \varepsilon_p - \hbar\omega_q)\} = ev_p^k\bar{F}_p/2\pi. \end{aligned} \tag{14}$$

Here

$$|c_q|^2 = E_0^2\hbar q/2V_0\rho w, \tag{15}$$

where E_0 is a constant of the deformation potential, ρ is the density of the crystal, and V_0 is the normalized volume (we set $V_0 = 1$).

In the work of Davydov[12] on the solution of Eq. (14), the following circumstance was used. Collisions of electrons with phonons are almost elastic. As a result, relaxation in the momenta

takes place far more rapidly than relaxation in the energy. As a consequence of this, the symmetric (relative to an exchange of $\mathbf{p}$ for $-\mathbf{p}$) part of the distribution function, even in the presence of an electric field, depends only on the energy of the electron ϵ_p. For $\epsilon_E \equiv T^{1/2} eEl \times (6mw^2)^{-1/2} \gg T$ (which is also assumed in the following) it depends essentially on the electric field, and is equal to

$$F_0(\varepsilon_p) = C \exp\{-\varepsilon_p^2/2\varepsilon_E^2\}, \tag{16}$$

$$C = \frac{(2\pi\hbar)^3}{2} \frac{\Gamma(5/4)\, n}{2^{5/2}\pi^2 (m\varepsilon_E)^{3/2}}, \tag{17}$$

n is the concentration of conduction electrons. The antisymmetric part f(p) (which generates a constant current $\bar{J}$) is equal to

$$f(\mathbf{p}) = -l \frac{e\mathbf{E}\mathbf{p}}{p} \frac{\partial F_0}{\partial \varepsilon_p}, \tag{18}$$

where

$$l = \frac{\pi\hbar^4 \rho w^2}{E_0^2 m^2 T} \tag{19}$$

is the mean free path of the conduction electron. Obviously, $f(\mathbf{p}) \ll F_0(\epsilon_p)$ (naturally, it is assumed that $T \gg mw^2$).

By analogy with [12], we seek a solution of (14) in the form

$$\gamma_p^k = x^k(\varepsilon_p) + y^k(\mathbf{p}), \tag{20}$$

where $x^k(\epsilon_p)$ is the symmetric part of the function γ_p^k which can be regarded as depending only on the energy, while $y^k(\mathbf{p}) \ll x^k(\epsilon_p)$ is the antisymmetric (current) part. We shall asume (and below shall check on the validity of such an assumption) that $y^k(\mathbf{p})$ has the form

$$y^k(\mathbf{p}) = \mathbf{p}y^k/p, \tag{21}$$

where the vector $\mathbf{y}^k$ depends only on the energy ϵ_p.

The course of the subsequent calculations is essentially the same as in [12]. Therefore, we only express the fundamental idea in a few words. We substitute (20) in (14). The action of the operator $\hat{S}$ in (14) on a function of the form (21) reduces, as is well known,[8] simply to multiplication by $1/\tau_e = v/l$. As a result, by separating the antisymmetric part, we obtain the equation

$$-i\omega y^k + \frac{v}{l} y^k = \frac{ev^k}{2\pi} F_0(\varepsilon_p) - eEv_p^z \frac{\partial x^k}{\partial \varepsilon_p} \tag{22}$$

(the electric field E is directed along the z axis).

Inasmuch as x^k depends only on ϵ, it is most convenient to obtain a second equation for this function by multiplying (14) by $\delta(\epsilon - \epsilon_p)$ and by integrating over d^3p, as was done by Kazarinov and Skobov.[15] This gives

$$-i\omega x^k(\varepsilon) \int d^3p\,\delta(\varepsilon - \varepsilon_p) + e\mathbf{E} \int d^3p\,\delta(\varepsilon - \varepsilon_p) \frac{\partial}{\partial \mathbf{p}} \gamma_p^k = \int d^3p\,\delta(\varepsilon - \varepsilon_p) \hat{S}x^k(\varepsilon_p) + \frac{e}{2\pi} \int d^3p\,\delta(\varepsilon - \varepsilon_p) v_p^k f(\mathbf{p}). \tag{23}$$

Here

$$\int d^3p\,\delta(\varepsilon - \varepsilon_p) = 4\pi\sqrt{2}\, m^{3/2}\varepsilon^{1/2}, \tag{24}$$

$$\frac{e}{2\pi} \int d^3p\,\delta(\varepsilon - \varepsilon_p)\, v_p^k f(\mathbf{p}) = -\frac{4e^2 E l m \varepsilon}{3} F_0'(\varepsilon). \tag{25}$$

In the calculation of the first term on the right side of (23), the function $\hat{S}x^k(\epsilon_p)$ can be expanded in a series in a small parameter proportional to $\hbar\omega_q$. As a result, we get

$$\int d^3p\,\delta(\varepsilon - \varepsilon_p) \hat{S}x^k(\varepsilon_p) = \frac{\partial}{\partial \varepsilon} \left\{ a(\varepsilon) \left[x^k(\varepsilon) + T \frac{\partial}{\partial \varepsilon} x^k(\varepsilon) \right] \right\}, \tag{26}$$

where

$$a(\varepsilon) = \frac{1}{(2\pi)^3} \int d^3p \int d^3q\, |c_q|^2\, \delta(\varepsilon - \varepsilon_p)\, \delta(\varepsilon_p - \varepsilon_{p+\hbar q}) = \frac{16\pi m^2 w^2 \varepsilon^2}{Tl}. \tag{27}$$

Finally, the second term on the left side of (23) easily reduces to the form

$$eE \frac{\partial}{\partial \varepsilon} \int d^3p\, v^z y^k(\mathbf{p})\, \delta(\varepsilon - \varepsilon_p). \tag{28}$$

Let us first consider the case when $\omega l/\bar{v} \ll 1$ (where $\bar{v}$ is the mean electron velocity, of the order of $\sqrt{\epsilon_E/m}$). One can then neglect the first component on the left side of (22). Then by finding $y^k(\mathbf{p})$ from this equation, we get

$$\int v^z y^k(\mathbf{p})\, \delta(\varepsilon - \varepsilon_p)\, d^3p = \frac{4mel}{3} \varepsilon F_0(\varepsilon)\, \delta_{kz} - \frac{8\pi e E l m}{3} \varepsilon \frac{\partial x^k}{\partial \varepsilon}, \tag{29}$$

where δ_{kz} is the Kronecker symbol. As a result, Eq. (23) takes the following form:

$$-i\omega\tau_s \left(\frac{\varepsilon}{\varepsilon_E}\right)^{1/2} x^k(\varepsilon) + \varepsilon_E \frac{d}{d\varepsilon} \left(\varepsilon \frac{dx^k}{d\varepsilon} \right) + \frac{1}{\varepsilon_E} \frac{d}{d\varepsilon} \left[\varepsilon^2 \left(x^k + T \frac{dx^k}{d\varepsilon} \right) \right] = \varepsilon_E \frac{\delta_{kz}}{2\pi E} F_0(\varepsilon), \tag{30}$$

where

$$\tau_s = (T/2\sqrt{2}\, mw^2)\, l\, (m/\varepsilon_E)^{1/2}.$$

We note two important circumstances. First, the inhomogeneity of this equation is proportional to δ_{kz}. This means that $x^k = 0$ in the current fluctuations in a direction perpendicular to $\mathbf{E}$, and it suffices for its consideration to solve only Eq. (22) for the function $y^k(\mathbf{p})$. In the second case, in consideration of the fluctuations in the direction of $\mathbf{E}$ a new parameter with the dimensions of time appears in the theory. This parameter τ_s characterizes the relaxation time of the symmetric

part of the distribution. This time is very large and its presence indicates significant dispersion (dependence on ω) of the function $(\delta J_z^2)_\omega$ even in the radio frequency region. We note that for $\omega \sim 1/\tau_s$, dispersion of the same origin should be observed even for the quantity $\Lambda_{zz}(\omega)$, determined with the aid of relation (12a).[6)]

Furthermore, once again we delineate two limiting cases: 1) $\omega\tau_s \gg 1$ and 2) $\omega\tau_s \ll 1$. In the first case, we can generally neglect the second component on the right side of (22) in comparison with the first. Then

$$y^k(p) = (lC/2\pi)\, n_k \exp\{-\varepsilon^2/2\varepsilon_E^2\}, \tag{31}$$

where $n_k = p_k/p$. We then find

$$(\delta J_i \delta J_k)_\omega = \frac{4\Gamma(5/4)}{3^{5/4}\pi^2} \frac{ne^2 l^{3/2}(eE)^{1/2}T^{1/4}}{m^{3/4}\omega^{1/2}} \delta_{ik}, \tag{32}$$

i.e., the fluctuations are proportional to $E^{1/2}$.

Let us consider the second case, $\omega\tau_s \gg 1$. Then the first component on the right side of (30) can be discarded. The remaining equation can be integrated once with respect to ϵ, keeping it in mind that the second component on the right side of (23) is proportional to the current fluctuation of the electrons with given energy, and therefore should tend to zero as $\epsilon \to \infty$ [compare with [12]]. This gives

$$\frac{dx^k}{d\varepsilon}\left(1 + \frac{T\varepsilon}{\varepsilon_E^2}\right) + \frac{\varepsilon}{\varepsilon_E^2} x^k = -\frac{\delta_{kz}}{2\pi E}\frac{\varepsilon_E^2}{\varepsilon}\int_\varepsilon^\infty F_0(\eta)\, d\eta. \tag{33}$$

If we neglect $T\epsilon/\epsilon_E^2$ in comparison with unity, then the solution of this equation has the form

$$x^k(\varepsilon) = -\frac{\delta_{kz}C}{2\pi E} e^{-\varepsilon^2/2\varepsilon_E^2} \int_{\varepsilon_0}^{\varepsilon} \frac{d\eta}{\eta} e^{\eta^2/2\varepsilon_E^2} \int_\eta^\infty e^{-\zeta^2/2\varepsilon_E^2} d\zeta, \tag{34}$$

where the integral over η must be taken in the sense of the principal value. The constant of integration ϵ_0 must be determined from the condition of constancy of the electron concentration in spatially homogeneous fluctuations. It has the form

$$\int_0^\infty d\varepsilon\, \varepsilon^{1/2} e^{-\varepsilon^2/2\varepsilon_E^2} \int_{\varepsilon_0}^{\varepsilon} \frac{d\eta}{\eta} e^{\eta^2/2\varepsilon_E^2} \int_\eta^\infty d\zeta e^{-\zeta^2/2\varepsilon_E^2} d\zeta = 0. \tag{35}$$

We then easily find that $\epsilon_0 = \epsilon_E\sqrt{2x_0}$, where x_0 is a number of the order of unity, satisfying the equation

$$\int_{x_0}^\infty e^{x^2}\frac{dx}{x}\int_x^\infty e^{-y^2}dy = \frac{4}{\pi}\Gamma\left(\frac{5}{4}\right)\int_0^\infty dx\, x^{1/2}e^{-x^2}\int_x^\infty \frac{dy}{y}e^{y^2}\int_y^\infty e^{-z^2}. \tag{36}$$

[6)]Obviously, dispersion of such an origin in the range of comparatively low frequencies should take place not only in the case considered here, but also generally for "hot" electrons (in this case also for $H \neq 0$), if they undergo nearly elastic collisions.

Substituting (34) in (22) and integrating, we get, finally,

$$(\delta J_z^2)_\omega = (\delta J_x^2)_\omega (1 - D), \tag{37}$$

where

$$D = 2\int_0^\infty e^{-x^2} dx \int_{x_0}^x \frac{dy}{y} e^{y^2} \int_y^\infty e^{-z^2} dz. \tag{38}$$

Thus, in the given case, $(\delta J_Z^2)_\omega$ and $(\delta J_k^2)_\omega$ are also proportional to $E^{1/2}$; however, the constants of proportionality are different for the two quantities. This result was obtained in a different way in the paper of Price.[7]

We now proceed to the case $\omega l/\bar{v} \gg 1$. With the help of a contribution similar to those made above, we find that in this range of frequencies

$$(\delta J_i \delta J_k)_\omega = \frac{1}{\pi\sqrt{2}} \frac{ne^2\varepsilon_E^{3/2}}{m^{3/2}\omega^2 l}\delta_{ik}. \tag{39}$$

It is interesting to compare the quantity $(\delta J_i \delta J_k)_\omega$ with the tensor Λ_{ik} which figures in Eq. (13). For example, if $\omega\tau_s \ll 1$, then $\Lambda_{ik} = \partial\bar{J}_i/\partial E_k$. But, in accord with Davydov,[12]

$$\bar{J}_i = \frac{4\Gamma(5/4)}{3^{3/4}\pi^{1/2}} ne\left(\frac{e^2E^2l^2\omega^2}{mT}\right)^{1/4}\frac{E_i}{E}. \tag{40}$$

Hence

$$\Lambda_{ik} = \frac{4\Gamma(5/4)}{3^{3/4}\pi^{1/2}} ne\left(\frac{e^2l^2\omega^2}{mTE^2}\right)^{1/4}\left(\delta_{ik} - \frac{E_iE_k}{2E^2}\right). \tag{41}$$

The components of the tensors $(\delta J_i \delta J_k)_\omega$ and Λ_{ik} are connected by the relations

$$(\delta J_x^2)_\omega = \frac{1}{\pi}\left(\frac{2}{\pi}\right)^{1/2}\varepsilon_E\Lambda_{xx}, \qquad (\delta J_z^2)_\omega = \left(\frac{2}{\pi}\right)^{3/2}(1-D)\,\varepsilon_E\Lambda_{zz}. \tag{42}$$

These indicate that for hot electrons a relation of the type (13) remains in force for the order of magnitude, if only we replace T in it by the mean energy of the electron in a strong electric field. However, it is important to note that the tensors $(\delta J_i \delta J_k)$ and Λ_{ik} are not proportional to each other: the ratios of their various components have in general different values.

It is interesting to compare the results of these calculations with the results of the work of Bunkin.[5] In one case, one can carry out a direct comparison, inasmuch as the case of fluctuations in a weakly ionized plasma in a strong electric field was also considered in [5]; one can extend the results of Davydov [12] which apply to semiconductors, and consequently our results, to this case by means of a simple change in notation. The results of the two cases, pertaining to transverse fluctuations, are identical, while those pertaining to longitudinal fluctuations differ. In particular, a conclusion on the isotropy of the ratio $(\delta J_i \delta J_k)_\omega/\Lambda_{ik}$ is made

in [5], whereas it is found to be anisotropic from (42).

It appears to us that the reason for the above-mentioned divergence lies in the following. The effect of velocity fluctuations on the current only was considered in [5]; that is, in practice, the fluctuations of the antisymmetric part of the distribution function. In the case of thermodynamic equilibrium, fluctuations of the symmetric part of the distribution function do not generate current fluctuations. However, in the stationary state which arises under the action of a constant electric field, a constant current flows which depends on the form of the symmetric part of the distribution function; therefore fluctuations of the latter can create additional fluctuations in the longitudinal current. In other words, when the electron current depends on the electron temperature, the fluctuations of the longitudinal current are determined not only by the fluctuations of the longitudinal electron velocity, but also by the fluctuations of the electron temperature.[7)]

In conclusion, we shall analyze the important problem as to the measure in which the theory just set forth is applicable to a classical atomic semiconductor of the type of germanium and silicon. The electron and phonon spectra in such semiconductors are much more complicated than the very simple model which served as the basis of the Davydov theory.[12] Moreover, electrons in these semiconductors can interact not only with acoustic, but also with optical phonons, and the effect of this interaction on the electrical conductivity of the semiconductor in a strong electric field has been observed experimentally.[14] However, for sufficiently low temperatures and not very strong electric fields, where one can assume that the optical phonons are practically unexcited, the qualitative conclusions of the Davydov theory, and consequently of ours, must remain in force. In particular, even in this case, one can represent the electron distribution function $\overline{F}_p$ (owing to the small inelasticity of the electron-phonon collisions) in the form of a sum of a symmetric part $F_0(\epsilon_p)$, which depends only on the energy of the electron, and an antisymmetric part $f(\mathbf{p})$.

A similar statement applies to $\gamma_p^k = x^k(\epsilon_p) + y^k(\mathbf{p})$. Then formulas of the type (32) have the sense of estimates, yielding the correct order of magnitude of the current fluctuations and their dependence on $\mathbf{E}$. The conclusion as to the presence of dispersion associated with the time τ_s also remains in force. However, the numerical coefficients entering into the various formulas (among which is the quantity x_0) apparently must be changed.

Quite recently, there appeared the first experimental work of Erlbach and Gunn[17] in which the fluctuations of the transverse current in an electric field in electronic germanium were studied. It was discovered that the ratio $(\delta J_x^2)_\omega / \Lambda_{xx}$ is proportional to $\mathbf{E}$ in the low frequency region. This result is in agreement with the conclusions of the given theory, which takes into account only scattering of electrons by long wavelength acoustical phonons in the limits of a single energy minimum. It would be extremely interesting in further experiments to choose the geometry of the experiment and the range of measured frequencies in such a fashion as to observe dispersion associated with the presence of the time τ_s.

However, it must be noted that, as was emphasized in [17], the law $\overline{J} \sim E^{1/2}$ is rather inaccurately obeyed in strong fields. It is possible that this is connected with the effect of other scattering mechanisms (optical phonons, interlinear transitions), although the authors[17] assume their role to be negligibly small. And perhaps the following circumstance plays a role here.

As has recently become known,[18] in a sufficiently strong electric field, where $\overline{J}/ne > w$, the state with the distribution function (16)—(18) can be shown to be unstable relative to the generation of sound vibrations. Then, strictly speaking, the actual stationary state must be determined with account of the given effect of generation. For this purpose, it is necessary to substitute in the kinetic equation (1) as a phonon distribution function N_q not the equilibrium Planck distribution, but a non-equilibrium function determined from a solution of its own kinetic equation. Such a problem has not been solved to date, although the numerical determination of $\overline{J}(E)$ with account of the given fact is of great interest.

In conclusion, I express my gratitude to A. L. Éfros for very interesting discussions.

[7)] Obviously, the concept of electron temperature is not precise in the given case and it is used only for a qualitative explanation of the additional reason for the fluctuations. As has been made known to the author, the problem of the effect of fluctuations of the electron temperature (for the case in which such a concept can have strict meaning) on current fluctuations in semiconductors has also been considered by Sh. M. Kogan.

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305

ERRATA

Vol	No	Author	page	Correction
15	6	Turov	1098	The article contains an erroneous statement that weak ferromagnetism cannot exist in any cubic crystal (with collinear or weakly noncollinear antiferromagnetic structure. This was found to be true only for crystal classes T and T_h, and for others weak ferromagnetism will appear in antiferromagnets with magnetic structure type $3^+ 4^-$, and only due to invariants of third and higher orders in the antiferromagnetism vector L. Consequently a line (14) should be added to the table on p. 1100: $$14 \mid 207\text{-}230 \mid \text{Cubic} \mid 3^+, 4^- \mid M_x L_x (L_y^2 - L_z^2) + M_y L_y (L_z^2 - L_x^2) + M_z L_z (L_x^2 - L_y^2) \mid \text{VI}$$ The Cartesian axes are directed here parallel to the fourfold symmetry axes. The tensors $g^{(1)}$ and $g^{(2)}$ for this (sixth) group of weakly ferromagnetic structures will be identically equal and isotropic: $$g^{(1)}_{\alpha\beta} = g^{(2)}_{\alpha\beta} = g\delta_{\alpha\beta}$$
16	1	Valuev	172	At the end of the article there are incorrect expressions pertaining to $K\mu_3$ decay. The correct formula can be easily obtained from the main formula of the article by putting $g_S = g_T = 0$. The tangent of the angle between the $\mid m \mid^2$ curve and the cos θ axis will be $\approx \beta_e$ if $g_{V2}/g_{V1} = -0.5$ and ≈ 0 if $g_{V2}/g_{V1} = 4.5$ and $\beta_e \sim 1$, so that in fact the difference in the angle correlations between these cases is even somewhat stronger than indicated in the article.
16	1	Zhdanov et al	246	The horizontal parts of curves 2 and 3 in Fig. 2 should be drawn with solid lines (they correspond to the asymptotic calculated values of the ionization losses, i.e., to the region in which the theory describes the relation between g/g_0 and the particle energy exactly).
16	1	Deutsch	478 & 481	When account is taken of thermoelectric processes it is necessary to add in the first curly bracket of (24) the term $$A = 3v_0^2 H_y c (\alpha_{xz} - \alpha_{zx})/2$$ and in Eq. (31) the term A/9.
16	1	Nguyen	920 Eqs. (4), (6), (7), & (8)	The combinations $V^1 \pm V^2$, $A^1 \pm A^2$, and $I^1 \pm I^2$ should be divided by $\sqrt{2}$.
16	1	Gershtein et al	1097 Eq. (1)	Reads $G/\sqrt{2}$, should read G/2
16	5	Gurevich		An error has crept into Eq. (30). The right half of this formula is actually equal to $$\varepsilon_E \frac{\delta_{kz}}{2\pi E}\left[F_0(\varepsilon) + 2\varepsilon \frac{d}{d\varepsilon} F_0(\varepsilon)\right].$$

16 5 Gurevich (cont.)

As a result, the corrected equation (33) assumes the form

$$\left(1+\frac{\varepsilon T}{\varepsilon_E^2}\right)\frac{dx^k}{d\varepsilon}\frac{\varepsilon}{\varepsilon_E^2}x^k=-\frac{\delta_{kz}}{2\pi E\varepsilon}\int_{\varepsilon}^{\infty}\left[F_0(\eta)+2\eta\frac{d}{d\eta}F_0(\eta)\right]d\eta. \qquad (33)$$

Suitable corrections must be made in (34) and (35) by replacing $\exp(-\zeta^2/2\varepsilon_E^2)$ by $(1-2\zeta^2/\varepsilon_E^2)\exp(-\zeta^2/2\varepsilon_E^2)$. As a result, relations (36) and (38), which determine the numerical constants x_0 and D, assume the form

$$\int_0^{\infty}dxx^{1/2}e^{-x^2}\int_{x_0}^{x}\frac{dy}{y}\int_y^{\infty}dz\,(1-4z^2)\,e^{-z^2}=0, \qquad (36)$$

$$D=2\int_0^{\infty}e^{-x^2}\int_{x_0}^{x}\frac{dy}{y}\,e^{y^2}\int_y^{\infty}dze^{-z^2}(1-4z^2). \qquad (38)$$