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A FORM FACTOR SATISFYING THE UNITARITY CONDITION

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A simple relation between the scattering amplitude and particle form factor is established on the basis of the unitarity condition by applying the R-matrix formalism.

A form factor satisfying the unitarity and analyticity conditions is usually obtained in terms of the scattering phase shift as a solution of a singular integral equation [1,2]. It is somewhat easier to obtain the solution by using the R-matrix formalism, where under simple assumptions concerning the character of the scattering there is no need to solve the integral equations.

The two-particle unitarity condition for the partial amplitude T(s)

$$T^* - T = 2i\rho T^* T,$$

$$\rho = s^{-1}\{[s-(m_1+m_2)^2][s-(m_1-m_2)^2]\}^{1/2}$$

(the factor ρ corresponds to the statistical weight of the two-particle state) is satisfied identically if we put

$$T = R(1-i\rho R)^{-1} \equiv (\rho'/\rho) R' (1-i\rho' R')^{-1},$$
$$\rho' = \sqrt{s-(m_1+m_2)^2}, \qquad (1)$$

where R′ does not have a right-hand cut but has all other singularities of T. Such a simple solution is applicable because the two-particle cut corresponds to an ambiguity which is guaranteed by the fact that ρ' is in the form of a square root.

This fact uncovers the possibility of a "semiphenomenological" description of scattering in a certain limited region of values of s by suitable choice of R′ [3,4]. Putting R′ = const, we obtain an approximation of the scattering length. If R′ is a polynomial in s, this corresponds to expansion in powers of the "effective radius." The pole R′ on the real axis describes a Breit-Wigner type of resonance.

The unitarity condition for the form factor F(s) (here s is square of the momentum transfer) is, if only one partial wave is taken into account,

$$F^* - F = 2i\rho F^* T. \qquad (2)$$

Substituting (1) in (2) we verify that $F = f(1-i\rho' R')^{-1}$ satisfies (2) identically for any f(s) which has no right-hand cut. It is necessary to choose f such as to obtain the correct analytic properties of F(s).

$1 - i\rho' R'$ can have a left-hand cut corresponding to the cut of T(s). The function f should then contain a left-hand cut, and its discontinuity is chosen such as to make the form factor F an analytic function in the left-plane. We thus obtain for f the Muskhelishvili-Omnes equation

$$\frac{\operatorname{Im} f}{f} = -\frac{|\rho'| \operatorname{Im} R'}{1+|\rho'| R'}, \qquad (3)$$

which, as is well known, is solved with accuracy to an arbitrary polynomial, the degree of which determines the behavior of f when $s \to \infty$. The function f also has poles coinciding with the poles of R′. The requirement that F have no poles means that f have no zeros and have all the poles of R′. Then f is determined uniquely as the fundamental solution of (3) if R′ is known, i.e., if the scattering amplitude is known.

In a small region of variation of s the left-hand cut can be approximated by a finite number of constants, and R′ can be assumed to have no cuts, i.e., to be a rational function of s. Then f is also a rational function of s, which has no other poles except those possessed by R′. This function is determined completely by the zeros of F. If we do not specify the zeros of F, then to reduce the arbitrariness in the choice of f we can use the requirement, reasonable under these conditions, that |F| not increase when $s \to \infty$, a requirement which determines the maximum degree of f.

Let us consider by way of an example the behavior of F near the values $s = s_0$ at which resonance takes place in the scattering:

$$R' = \frac{a}{s-s_0}, \qquad F = \frac{f(s-s_0)}{s-s_0-i\rho' a}.$$

The requirements that |F| be non-increasing and F(0) = 1 yield

$$f \sim \frac{s-s_1}{s-s_0}, \qquad F = \frac{s_0-(m_1+m_2)a}{s_1} \frac{s-s_1}{s-s_0-i\rho' a}.$$

If $F \neq 0$ in the region of interest to us, i.e., the zero of F (the point s_1) is situated far away from s_0, then we can assume that F has a zero at infinity. Then, obviously,

$$F = \frac{-s_0 + (m_1 + m_2)\,a}{s - s_0 - i\rho\, a}.$$

Grashin and Mel'nikov[2,5] have pointed out that the zero s_1 of the form factor can be determined uniquely from the condition that the inelastic processes have a small influence, with s_1 situated in the resonant region. However, without insisting on the point of view that analyticity and unitarity must determine uniquely the scattering-matrix elements, such a choice of the solution turns out to be arbitrary. The requirement that an infinitesimally narrow resonance in the scattering make no contribution to the form factor, i.e., that $s_1 \to s_0$ as $a \to 0$, does not contradict the example presented, in which $|s_1 - s_0|/ma \gg 1$ for finite a.

Thus, in a small region of momentum-transfer values it is possible to obtain in very simple fashion a form factor satisfying the unitarity condition in the two-particle approximation. At the same time, the ambiguity typical of problems of this kind, first pointed out by Castillejo, Dalitz, and Dyson, arises in this case.

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