INELASTIC DIFFRACTION PROCESSES AT HIGH ENERGIES

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Submitted to JETP editor May 8, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1731-1742 (November, 1962)

It is shown that cross sections for inelastic processes determined by the diffraction of fast particles by nuclei can be calculated on the basis of the diagram technique. "Vertex" functions are obtained which describe elastic diffraction scattering and which should be inserted in the diagrams for inelastic diffraction processes. Using the model of a "black" spherical nucleus, the differential and total cross sections are calculated for the diffraction production of π mesons and strange particles as a result of the interaction of fast protons with nuclei, for the diffraction production of nucleon-antinucleon pairs in the collision of a fast π meson with a nucleus, and for the diffraction production of π-meson pairs by a high-energy γ quantum.

1. INTRODUCTION

IN a number of papers [1-3] a method has been developed for the calculation of inelastic diffraction processes at high energies (bremsstrahlung, production by γ quanta of π-meson and nucleon-antinucleon pairs, etc.) which are caused by the strong absorption of π mesons and nucleons in nuclei. The physical reason which makes it possible to calculate cross sections for processes involving strongly interacting particles consists of the fact that the conservation laws require smaller and smaller transfers of momentum to the external body (nucleus) as the energy of the incident particle is increased. If the momentum transferred to the nucleus in the direction of motion of the incident particle is

$$q_{\parallel} \lesssim 1/R \tag{1.1}$$

(R is the nuclear radius), then the inelastic processes occur at large distances from the nucleus, and the details of the strong interaction of π mesons and nucleons with the nucleus become unimportant. Under such conditions the strong interaction of nucleons, π mesons, and strange particles with the nucleus can be described phenomenologically by using the formulas describing diffraction by a black sphere [1]. In order for us to be able to consider a nucleus as a black sphere with a sharp boundary, it is necessary that the reciprocal thickness of the effective absorbing layer of the nucleus should be large in comparison to the total transferred momentum. This condition is fulfilled if [4]

$$q_{\perp} \lesssim \mu, \tag{1.2}$$

where μ is the π-meson mass, and $q_{\perp}$ is the momentum transferred to the nucleus in the perpendicular direction. Thus, under ultrarelativistic conditions it is sufficient to know the wave functions in the region of space $r \gtrsim R$ for an "exact" calculation of inelastic processes, if the particles produced emerge in a direction which differs little from the direction of the incident particle. The wave functions describing the particle diffracted by a black nucleus were obtained on the basis of Huygens' principle for particles having zero and non-zero spin [1,3] and have been generalized to the case of a semi-transparent nucleus [5], and also to the case of a nonspherical nucleus [6,7] (when the excitation of the low-lying rotational levels of the nucleus plays an essential role).

In Sec. 2 we shall obtain with the aid of such "exact" wave functions operators corresponding to elastic diffraction scattering, which must be inserted in diagrams describing inelastic diffraction processes involving small transfers of momentum.

In Secs. 3—6 explicit calculations are carried out of cross sections for processes of the following types

$$\gamma + A \rightarrow \pi^+ + \pi^- + A, \tag{1.3}$$

$$p + A \rightarrow n + \pi^+ + A, \tag{1.4}$$

$$p + A \rightarrow K^+ + \Lambda\,(\Sigma^0) + A, \tag{1.5}$$

$$\pi^- + A \rightarrow n + \bar{p} + A. \tag{1.6}$$

2. DIAGRAM TECHNIQUE

We consider the elastic scattering of spinless particles (π mesons) by a black spherical nucleus under ultrarelativistic conditions involving small momentum transfers $q_{||} \lesssim 1/R$ (diffraction scattering of π mesons by nuclei). We shall describe the process under discussion by the diagram of Fig. 1a (we omit the line corresponding to the nucleus, since for small momentum transfers the nucleus may be treated in the static approximation),

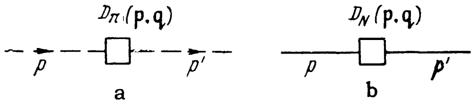

FIG. 1

where $D_\pi(\mathbf{p}, \mathbf{q})$ is the vertex function representing the matrix element for the diffraction scattering of a π meson by a nucleus without normalizing factors, $\mathbf{p}$ is the momentum of the incident particle, and $\mathbf{q}$ is the momentum transferred to the nucleus. The vertex function $D_\pi(\mathbf{p}, \mathbf{q})$ can be easily determined by comparing the scattering amplitude $f_\pi(\mathbf{p}, \mathbf{q})$, evaluated with the aid of the "exact" wave function of the diffracted particle, with the amplitude obtained from the diagram of Fig. 1a:

$$D_\pi(\mathbf{p}, \mathbf{q}) = 2|\mathbf{p}|\, J_\pi(\mathbf{q}, R_\pi),$$
$$J_\pi(\mathbf{q}, R_\pi) = \int e^{i\mathbf{q}\boldsymbol{\rho}} (1 - \Omega_\pi(\rho))\, d\boldsymbol{\rho}. \tag{2.1}$$

In the case of scattering by a "black" spherical nucleus

$$\Omega_\pi(\boldsymbol{\rho}) = \begin{cases} 1, & \rho > R_\pi \\ 0, & \rho < R_\pi \end{cases}, \quad \boldsymbol{\rho} \perp \mathbf{p},$$
$$J_\pi(\mathbf{q}, R_\pi) = \frac{2\pi R_\pi}{k} J_1(kR_\pi), \tag{2.2}$$

where k is the transverse component of the transferred momentum, R_π is the nuclear radius with respect to the absorption of π mesons, and $J_1(x)$ is a Bessel function of the first order.

We note that under the conditions under discussion the energy of the incident π meson is conserved at the diffraction vertex, since the energy transferred to the nucleus $\Delta E_n \approx q^2/2M_n$ turns out to be very small. In the case of diffraction scattering of a π meson by a "black" nucleus having the shape of an ellipsoid of revolution involving the excitation of a rotational level the function $J_1(x)$ in (2.2) is replaced by the function $J_{n0}(ka, z)$ introduced by Drozdov [6] (cf. also [7]). In the case of diffraction by a semi-transparent nucleus the function $\Omega_\pi(\rho)$ has the form

$$\Omega_\pi(\rho) = \begin{cases} \exp\left\{-\int\limits_\rho^{R'_\pi} \sqrt{r^2-\rho^2}\, B(r)\, dr\right\}, & \rho < R'_\pi, \\ 1, & \rho > R'_\pi \end{cases}$$

where $R'_\pi \gtrsim R_\pi$, $B(r)$ is the complex absorption coefficient for the nucleus (cf., for example [8]). In the most general case of the interaction of a π meson with a nucleus the function $D_\pi(\mathbf{p}, \mathbf{q})$ can be determined from experiment.

In a similar manner we can find the vertex function $D_N(\mathbf{p}, \mathbf{q})$ for the diffraction scattering of nucleons by nuclei (Fig. 1b). With the aid of the "exact" wave function for spinor particles the scattering amplitude can be written in the form ($\bar{u}$ and u are bispinors)

$$f_N(\mathbf{p}, \mathbf{q}) = -\frac{E}{2\pi}(\bar{u}_{p'}(\gamma\mathbf{n})\, u_{\mathbf{p}})\, J_N(\mathbf{q}, R_N);$$
$$E = \sqrt{\mathbf{p}^2 + m^2}, \qquad u_p^+ u_p = 1.$$

Comparing this with the amplitude evaluated in accordance with the diagram of Fig. 1b we obtain

$$D_N(\mathbf{p}, \mathbf{q}) = -i(\gamma\mathbf{n})\, J_N(\mathbf{q}, R_N), \quad \mathbf{n} = \mathbf{p}/p,$$
$$J_N(\mathbf{q}, R_N) = \int e^{i\mathbf{q}\boldsymbol{\rho}} (1 - \Omega_N(\boldsymbol{\rho}))\, d\boldsymbol{\rho}, \quad \boldsymbol{\rho} \perp \mathbf{p}. \tag{2.3}$$

For a "black" spherical nucleus we have

$$\Omega_N(\rho) = \begin{cases} 1, & \rho > R_N \\ 0, & \rho < R_N \end{cases}, \quad J_N(\mathbf{q}, R_N) = \frac{2\pi R_N}{k} J_1(kR_N), \tag{2.4}$$

where R_N is the nuclear radius with respect to nucleon absorption, γ are the Dirac matrices ($\gamma = -i\beta\alpha$). The vertex function $D_N(\mathbf{p}, \mathbf{q})$ in the case of diffraction by a semi-transparent nucleus, a nonspherical nucleus, etc., can be obtained in the same way as in the case of diffraction of a π meson by a nucleus.

With the aid of the vertex functions $D_\pi(\mathbf{p}, \mathbf{q})$ and $D_N(\mathbf{p}, \mathbf{q})$ the matrix elements for the inelastic diffraction processes of the type of bremsstrahlung, or of pair production in the field of the nucleus, can be calculated on the basis of the usual diagram technique. In this case the momentum $\mathbf{p}$ should be interpreted as the momentum of a real particle. We also note that the Coulomb interaction of charged particles with a nucleus can be taken into account in the method proposed here by means of the usual diagram technique [5].

3. DIFFRACTION PRODUCTION OF PION PAIRS BY A GAMMA QUANTUM

In connection with the possibility of obtaining narrowly collimated π-meson beams with the aid of electron accelerators it is of particular interest to investigate the process of diffraction production of π-meson pairs as a result of the interaction of a high-energy γ quantum with a nucleus (1.3). From the conservation laws it follows that condition (1.3) is satisfied for a γ-quantum energy of $\omega \gtrsim 2\mu A^{1/3}$ (for A = 125, $\omega \gtrsim 1.4$ BeV), where A is the atomic mass number. Diagrams correspond-

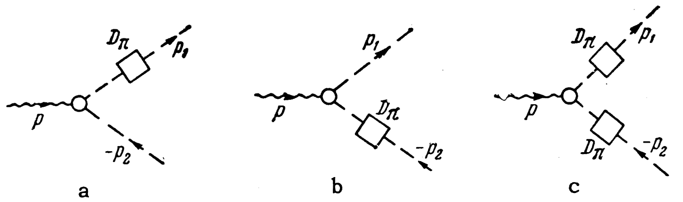

FIG. 2

ing to the production of π-meson pairs by a γ quantum are shown in Fig. 2.

On the basis of these diagrams the matrix element for the production of a π-meson pair by a black spherical nucleus can be written in the form [1)]

$$T = \frac{8\sqrt{\alpha}\,\pi^{3/2}}{\sqrt{2E_1E_2\omega}}\left\{\frac{ip_2R_\pi J_1(gR_\pi)(\mathbf{j}\mathbf{p}_1)}{g[(\mathbf{p}_1-\mathbf{p})^2-\mathbf{p}_2^2]} - \frac{ip_1R_\pi J_1(gR_\pi)(\mathbf{j}\mathbf{p}_2)}{g[(\mathbf{p}-\mathbf{p}_2)^2-\mathbf{p}_1^2]} + \frac{p_1p_2}{8\pi^4}\int e^{-i\mathbf{f}\rho_1}e^{i(\mathbf{f}+\mathbf{p})\rho_2}\frac{(\mathbf{j}\mathbf{f})\,d^3f\,d\rho_1 d\rho_2}{[\mathbf{f}^2-\mathbf{p}_1^2-i\varepsilon][(\mathbf{f}+\mathbf{p})^2-\mathbf{p}_2^2-i\varepsilon]}\right\}. \tag{3.1}$$

Here $\epsilon \to 0$; $\boldsymbol{\rho}_1 \perp \mathbf{p}_1$; $\boldsymbol{\rho}_2 \perp \mathbf{p}_2$; $\rho_1, \rho_2 < R_\pi$; ω is the energy of the γ quantum; $\mathbf{p}$ is the momentum of the γ quantum; $\mathbf{p}_1, \mathbf{p}_2$ are the momenta of the π mesons; $\mathbf{g}$ is the momentum transferred to the nucleus in the direction perpendicular to $\mathbf{p}$; E_1, E_2 are the energies of the π mesons.

In (3.1) we have set for the electric form factors $F = 1$. In the case of heavy nuclei ($\mu R_\pi \gg 1$) the differential cross section for the process averaged over the polarizations has the form [2)]

$$d\sigma(g, b, E_1) = \frac{\alpha}{2\pi^2}\frac{R_\pi^2}{g^2}\frac{J_1^2(gR_\pi)}{\omega^3}\frac{E_1(\omega-E_1)}{(\mu^2+b^2)^2}\,b^2 d\mathbf{b}\,dg, \tag{3.2}$$

where $\mathbf{b} = E_2\theta_2$ (θ_2 is the angle of emission of one of the π mesons, $\theta_2^2 \ll \mu^2/E_2^2$). In the derivation of (3.2) it was taken into account that $E_1, E_2, \omega \gg \mu$ and $g_{\mathrm{eff}} \sim 1/R \ll \mu$. On integrating the cross section (3.2) with respect to g between the limits $0 \le g \le \infty$ (because of good convergence) we obtain Drell's result [9] apart from the notation (Drell evaluated the cross section for the production of a π meson in the collision of a γ quantum with a nucleus on the basis of the diagram of Fig. 3), if we assume that at the vertex Γ elastic diffraction scattering of a π meson by a nucleus occurs with total cross section πR_π^2. It should be noted that the result (3.2) was obtained, in contrast to Drell's calculations, on the basis of the three diagrams of Fig. 2. Of particular interest is the fact that the diagram of Fig. 2c, where both newly created π mesons are diffracted by the nucleus, gives a contribution of the same order of magnitude as the diagrams of Fig. 2a and Fig. 2b where only one of the π mesons is diffracted.

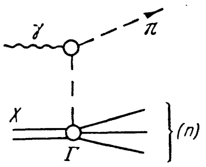

FIG. 3

[1)]The metric is $ab = \mathbf{a}\cdot\mathbf{b} - a_0b_0$.

[2)]The result (3.2) was first obtained by Pomeranchuk.[2]

4. DIFFRACTION PRODUCTION OF PIONS UPON SCATTERING OF A FAST NUCLEON BY A NUCLEUS

We consider the process of the diffraction production of a π meson in the collision of a fast nucleon with a nucleus (1.4) [4,10]. The energy threshold for this process when the inequality (1.1) is satisfied can be obtained from the condition

$$-(p_1+p_2)^2 = -(p-q)^2 \geqslant (m+\mu)^2, \tag{4.1}$$

where $p = (E, \mathbf{p})$ is the four-momentum of the incident nucleon, $p_1 = (E_1, \mathbf{p}_1)$ is the four-momentum of the nucleon after the interaction, $p_2 = (E_2, \mathbf{p}_2)$ is the four-momentum of the created π meson, q is the four-momentum transferred to the nucleus, and m is the nucleon mass. Going over on the left hand side of the inequality (4.1) to the laboratory system of coordinates (l.s.), we have

$$m^2 - q^2 - 2Eq_0 + 2pq_\parallel \geqslant m^2 + 2m\mu + \mu^2. \tag{4.2}$$

Since $q_0 \approx q^2/2M_n$ (M_n is the nuclear mass) and the recoil energy of the nucleus can be neglected under the conditions (1.1) and (1.2), then from expression (4.2) we can easily determine the energy threshold for the reaction (1.4):

$$E_{\mathrm{thr}} \gtrsim (2m\mu + \mu^2)/2q_{\parallel\,max} \approx m\mu R. \tag{4.3}$$

For a nucleus containing $A = 125$ nucleons, condition (1.5) is satisfied already at energies $E_{\mathrm{thr}} \ge 5$ BeV, which are quite easily attainable with modern accelerators.

We now proceed to the direct calculation of the cross section for the diffraction production of π mesons on the basis of the simplest diagrams of Fig. 4 where the interaction of π mesons and nucleons with the nucleus is taken into account exactly in the sense indicated above. We treat the πN vertex as an exact one by means of introducing the pion-nucleon form factor $g\gamma_5 F(-g_1^2, -g_2^2, -g_3^2)$ (g_1 is the four-momentum of the meson, g_2, g_3 are the four-momenta of the nucleon before and after the interaction), which is equal to $g\gamma_5$ on the mass shell when all the particles are physical. We neg-

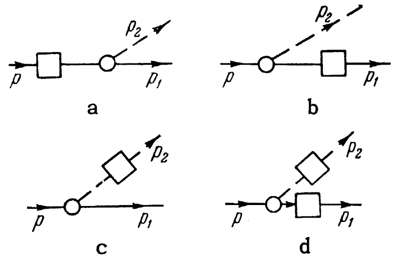

FIG. 4

lect the contribution from diagrams of the type shown in Fig. 5 since they contain weakly interfering diffraction vertices of the incident and the created particles [5]. The approximation which we are utilizing takes into account the large class of diagrams of the type of Fig. 6. However, we cannot estimate the contribution from diagrams containing the "enveloping" π-meson lines (cf., Fig. 7). and their role remains unclear. In view of the aforementioned difficulties which are generally characteristic of modern theory, the estimates for the cross sections for the diffraction production of particles at high energies should be taken as correct only in order of magnitude.

Thus, in the approximations indicated earlier the matrix element for the reaction (1.4) can be written in the form

$$T = \frac{4\pi^{3/2}g}{\sqrt{E_2}}\bar{u}_{p_1}\left\{\frac{\gamma_5[i(\mathbf{p}-\mathbf{q})\gamma-\gamma_4E-m](\boldsymbol{\gamma}\mathbf{n})}{(\mathbf{p}_1+\mathbf{p}_2)^2-\mathbf{p}^2}\frac{R_NJ_1(kR_N)}{k}\right.$$

$$\times F(\mu^2, m^2, -(p_1+p_2)^2)$$

$$+\frac{(\boldsymbol{\gamma}\mathbf{n}_1)[i(\mathbf{p}_1+\mathbf{q})\boldsymbol{\gamma}-\gamma_4E_1-m]\gamma_5}{(\mathbf{p}-\mathbf{p}_2)^2-\mathbf{p}_1^2}\frac{R_NJ_1(kR_N)}{k}$$

$$\times F(\mu^2, -(p-p_2)^2, m^2)$$

$$+\frac{2i\gamma_5p_2R_\pi J_1(kR_\pi)}{k[(\mathbf{p}-\mathbf{p}_1)^2-\mathbf{p}_2^2]}F(-(p-p_1)^2, m^2, m^2)$$

$$+\frac{ip_2}{8\pi^4}\int(\gamma\mathbf{n}_1)\frac{[i(\mathbf{p}+\mathbf{f})\gamma-\gamma_4E_1-m]\gamma_5}{(\mathbf{p}+\mathbf{f})^2-\mathbf{p}_1^2-i\varepsilon}\frac{F(-f^2, -(p+f)^2, m^2)}{\mathbf{f}^2-\mathbf{p}_2^2-i\varepsilon}$$

$$\left.\times e^{i(\mathbf{p}+\mathbf{f})\boldsymbol{\rho}_1}e^{-i\mathbf{f}\boldsymbol{\rho}_2}d\boldsymbol{\rho}_1\,d\boldsymbol{\rho}_2\,d^3f\right\}u_p;$$

$$\varepsilon\to 0,\quad \mathbf{n}_1=\mathbf{p}_1/p_1,\quad \mathbf{n}=\mathbf{p}/p,\quad \boldsymbol{\rho}_1^2\leqslant R_N,$$

$$\boldsymbol{\rho}_2^2\leqslant R_\pi^2,\quad \boldsymbol{\rho}_1\perp\mathbf{p}_1,\quad \boldsymbol{\rho}_2\perp\mathbf{p}_2. \qquad (4.4)$$

Here E and p are the energy and the momentum of the incident nucleon, E_1 and p_1 are the energy

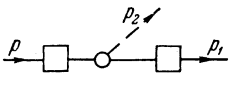

FIG. 5

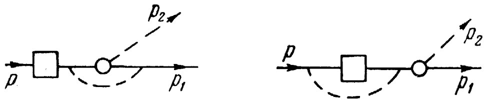

FIG. 6

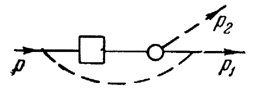

FIG. 7

and the momentum of the nucleon after the reaction; E_2 and p_2 are the energy and the momentum of the created π-meson; q is the momentum transferred to the nucleus; $\mathbf{k} = -\mathbf{q}_\perp$; R_N and R_π are the nuclear radii with respect to the absorption of a nucleon or a π-meson; $g^2 = 14.5$ is the coupling constant for the πN interaction; $\bar{u}_{p_1}$ and u_p are bispinors describing respectively the final and the initial nucleon states, γ, γ_4, γ_5, are Dirac matrices. In the case of the creation of a neutral meson the matrix element (4.4) is smaller by a factor $2^{1/2}$. The diffraction mechanism for the production of π mesons is realized at high energies $E, E_1 \gg m$, $E_2 \gg \mu$, for small angles of emission of nucleons and of π mesons. Under ultrarelativistic conditions expression (4.4) is significantly simplified. The integral in the curly brackets can be calculated in the case of heavy nuclei $\mu R \gg 1$ in the same manner as was done in Pomeranchuk's [2] paper. In carrying out the integration we assumed that the form factor has no singularities in the upper complex half-plane of the variable $\mathbf{f}_{\|} = (\mathbf{fp})\mathbf{p}/p^2$. On introducing the notation

$$\mathbf{p}_1 = p_1\mathbf{n}(1-k_1^2/2p_1^2)+\mathbf{k}_1,\qquad \mathbf{k}_1\perp\mathbf{p},\quad k_1\ll p_1,$$

$$\mathbf{p}_2 = p_2\mathbf{n}(1-k_2^2/2p_2^2)+\mathbf{k}_2,\qquad \mathbf{k}_2\perp\mathbf{p},\quad k_2\ll p_2, \qquad (4.5)$$

and on neglecting terms of order m^2/E_1^2 and μ^2/E_2^2 compared to the main terms, and on taking $R_N = R_\pi = R$ we obtain

$$T = \frac{8i\pi^{3/2}gRJ_1(kR)E_1E_2}{kE\sqrt{E_2}}\bar{u}_{p_1}\left\{\frac{\left[1-\frac{1}{2E_1}(\boldsymbol{\gamma}\mathbf{n}_1)(\boldsymbol{\gamma}\mathbf{k})\right]\gamma_5F(\mu^2, -g_2^2, m^2)}{\frac{m^2E_2^2}{E^2}+\frac{\mu^2E_1}{E}+k_2^2}\right.$$

$$-\frac{\gamma_5\left[1+\frac{1}{2E}(\gamma\mathbf{k})(\gamma\mathbf{n})\right]F(\mu^2, m^2, -g_3^2)}{\frac{m^2E_2^2}{E^2}+\frac{\mu^2E_1}{E}+\frac{E_1^2E_2^2}{E^2}\left(\frac{\mathbf{k}_1}{E_1}-\frac{\mathbf{k}_2}{E_2}\right)^2}+\frac{\gamma_5F(-g_1^2, m^2, m^2)}{\frac{m^2E_2^2}{E^2}+\frac{\mu^2E_1}{E}+k_1^2}$$

$$\left.-\frac{\left(1-\frac{1}{4E_1}(\boldsymbol{\gamma}\mathbf{n}_1)(\boldsymbol{\gamma}\mathbf{k}_1)\right)\gamma_5F(-g_1'^2, m^2, m^2)}{\frac{m^2E_2^2}{E^2}+\frac{\mu^2E_1}{E}+\frac{1}{4}(\mathbf{k}_1-\mathbf{k}_2)^2}\right\}u_p; \qquad (4.6)$$

$$-g_1^2 = -(m^2E_2^2-k_1^2E^2)/EE_1,$$

$$-g_1'^2 = \left[-E_2^2m^2-\frac{1}{4}E^2(\mathbf{k}_1-\mathbf{k}_2)^2\right]/EE_1,$$

$$-g_2^2 = (E_1E_2m^2 - E^2k_2^2)/EE_2,$$
$$-g_3^2 = [m^2EE_2 + \mu^2EE_1 + (\mathbf{k}_1E_2 - \mathbf{k}_2E_1)^2]/E_1E_2.$$

In deriving formula (4.6) we took into account the fact that $q_{\|} \ll q_{\perp} = k$ in accordance with conditions (1.1) and (1.2). It follows from (4.6) that the effective quantities are $k_1 \sim k_2 \sim mE_2/E$, if $E_2 \gtrsim \mu E/m$, or $k_1 \sim k_2 \sim \mu(E_1/E)^{1/2} \sim \mu$, if $E_2 \lesssim \mu E/m$. The quantities k_1 and k_2 are simply related to the angles of emission of π mesons and nucleons: $k_1 = E_1\theta_1$, $k_2 = E_2\theta_2$, and for $E_2 \gg \mu E/m$ the π mesons are emitted at angles $\theta_2 < m/E$, while nucleons are emitted at angles $\theta_1 \le mE_2/EE_1$. The form factors $F(-g_1^2, -g_2^2, -g_3^2)$ in front of the various terms in the curly brackets of expression (4.6) differ in magnitude since they depend on different variables. We now evaluate the amounts by which the variables of the form factors differ from their values on the mass shell (for $E_2 \gg \mu E/m$):

$$\begin{aligned}
-g_2^2 - m^2 &\approx -E_2m^2/E, \quad \theta_2^2 \ll m^2/E^2,\\
-g_3^2 - m^2 &\approx m^2E_2/E_1, \quad (\theta_2 - \theta_1)^2 \ll m^2/E_1^2,\\
-g_1^2 - \mu^2 &\approx -m^2E_2^2/EE_1, \quad \theta_1^2 \ll m^2E_2^2/E^2E_1^2,\\
-g_1'^2 - \mu^2 &\approx -m^2E_2^2/EE_1, \quad \tfrac{1}{4}(\theta_2 - \theta_1E_1/E_2)^2 \ll m^2/E^2.
\end{aligned} \tag{4.7}$$

It can be seen from (4.7) that the arguments of the form factors differ considerably from the squares of the masses and, consequently, that the form factors themselves must differ appreciably from unity. However, in order to obtain at least a rough estimate of the magnitude of the effect, we shall assume that the numerical values of the form factors appearing in (4.6) are the same and of order of magnitude unity. For $E_2 \gtrsim \mu E/m$ the ratio $k/k_2 \ll 1$ ($k_{\text{eff}} \sim 1/R$). Therefore, in (4.6) we carry out an expansion in powers of k/k_2 keeping only the principal terms:

$$T = \frac{8\pi^{3/2}igRJ_1(kR)E_1E_2}{kE\sqrt{E_2}}\bar{u}_{p_1}\gamma_5u_p\frac{(\mathbf{k}\mathbf{k}_2)(1-2E_2/E)}{(E_2^2m^2/E^2+k_2^2)^2}. \tag{4.8}$$

It follows from expression (4.8) that in the first approximation of perturbation theory in terms of the strong interaction a strong cancellation occurs between contributions of diagrams of Fig. 4. The matrix element (4.8) turns out to be of order μ/m in comparison with the matrix elements obtained on the basis of each of the diagrams of Fig. 4. The existence of this cancellation indicates the important role played by the polar single nucleon diagrams of Fig. 4a, b and by the diagram of Fig. 4d. Such a result contradicts Drell's conclusions [11] that polar single nucleon diagrams are unimportant for the diffraction mechanism for the production of π mesons. We should note that Drell's conclusions refer to the diffraction production of π mesons by nuclei which can no longer be regarded as stationary. Nevertheless, the existence of this cancellation is a very important fact pointing to the lack of justification for neglecting the polar single nucleon diagrams. Taking into account the departure from spherical shape and the semi-transparency of the nucleus does not eliminate the cancellation. However, the cancellation must be partially destroyed by taking the πN vertex into account exactly by means of form factors, and by taking into account the fact that the cross sections for the interaction of π mesons and of nucleons with nuclei are different ($R_N = R_\pi$). In view of what we have just said formula (4.8) and all subsequent estimates can be considered valid only in order of magnitude. The cross section for the production of π mesons is related to the matrix element by the formula

$$d\sigma = \int |T|^2\, 2\pi\delta(E - E_1 - E_2)\frac{E}{p}\frac{d^3p_1d^3p_2}{(2\pi)^6}. \tag{4.9}$$

On carrying out the integration with respect to E_1 in (4.9) and on replacing the variables $\theta_1 = k_1/E_1$, $\theta_2 = k_2/E_2$, $\mathbf{k}_1 = \mathbf{k} - \mathbf{k}_2$, we obtain with the aid of (4.8)

$$d\sigma = \frac{2g^2R^2J_1^2(kR)}{\pi^2k^2}|\bar{u}_{p_1}\gamma_5u_p|^2\frac{(\mathbf{k}\mathbf{k}_2)^2(1-2E_2/E)^2}{(E_2^2m^2/E^2+k_2^2)^4}\frac{E_1^2E_2}{E^2}dE_2d^3k_2d^3k. \tag{4.10}$$

On averaging over the initial spin states of the nucleon and summing over the final ones in (4.10) we obtain

$$d\sigma = \frac{g^2R^2J_1^2(kR)}{2\pi^2k^2}\varepsilon(1-2\varepsilon)^2(\mathbf{k}\mathbf{k}_2)^2\frac{d\varepsilon d\mathbf{k}_2d\mathbf{k}}{(m^2\varepsilon^2+k_2^2)^2}, \tag{4.11}$$

where $\epsilon = E_2/E$. On integrating (4.11) over the angles and over k between the limits $0 \le k \le \mu$ [cf. (1.2)], we obtain

$$d\sigma = \frac{g^2R\mu}{\pi}\frac{\varepsilon(1-2\varepsilon)^2d\varepsilon\, k_2^3dk_2}{(m^2\varepsilon^2+k_2^2)^3}. \tag{4.12}$$

In view of the rapid convergence the integration over k_2 can be carried out between the limits $0 < k_2 < \infty$. As a result of this the energy spectrum of fast π mesons ($\epsilon > \mu/m$) can be expressed by the formula

$$d\sigma = \frac{g^2\mu^2}{4\pi m^2}\frac{R}{\mu}\frac{(1-2\varepsilon)^2}{\varepsilon}d\varepsilon. \tag{4.13}$$

On carrying out the integration over ϵ between the limits $\mu/m \le \epsilon \le 1$ we obtain an estimate for the total cross section for the production of fast π mesons at small angles $\theta_2 \le m/E$:

$$\sigma \approx (0.7g^2R/4\pi\mu)(\mu/m)^2 \approx 1.5 \text{ mb for } A = 125. \tag{4.14}$$

It can be seen from (4.14) that the cross section for the diffraction production of π mesons at high energies is proportional to the nuclear radius and does not depend on the energy. The result (4.14) agrees with the qualitative estimates made by Pomeranchuk and Feĭnberg [4]. Thus, even in the presence of strong cancellation in the diagrams the cross section for the process (1.4) attains an appreciable value. Experimental measurement of the differential cross section could in principle yield information on the value of the pion-nucleon form factor. However, as can be seen from (4.6), this is a very difficult problem since the differential cross section depends on the values of the form factor at three different points with respect to different variables.

5. THE DIFFRACTION PRODUCTION OF STRANGE PARTICLES UPON SCATTERING OF A FAST NUCLEON BY A NUCLEUS

We now consider briefly the processes of the diffraction production of strange particles in the interaction of a fast nucleon with a nucleus. The energy threshold for such processes when condition (1.1) is fulfilled is evaluated in the same manner as in Sec. 4.

For the process (1.5) we obtain

$$E_{\text{thr}} \gtrsim [(m_\Lambda + m_K)^2 - m^2] / 2q_{\parallel max}$$

$$\gtrsim \tfrac{1}{2} [(m_\Lambda + m_K)^2 - m^2] R. \tag{5.1}$$

In the case of a nucleus of mass number A = 125 we have $E_{thr} \gtrsim 30$ BeV. The matrix element for the process (1.5) can be easily obtained on the basis of the diagrams of Fig. 4 with small changes associated with the nature of the KNΛ vertex and with the masses of the particles:

$$T = \frac{4\pi^{3/2} g_{K\Lambda N}}{\sqrt{2E_2}} \bar{u}_{\mathbf{p}_1} \left\{ \frac{\Gamma [i(\mathbf{p}-\mathbf{q})\gamma - \gamma_4 E - m](\gamma\mathbf{n})}{(\mathbf{p}_1+\mathbf{p}_2)^2 - \mathbf{p}^2} \frac{R_N J_1(kR_N)}{k} \right.$$

$$\times \Phi(m_K^2, m_\Lambda^2, -(p_1+p_2)^2)$$

$$+ \frac{(\gamma\mathbf{n}_1)[i(\mathbf{p}_1+\mathbf{q})\gamma - \gamma_4 E_1 - m_\Lambda]\Gamma}{(\mathbf{p}-\mathbf{p}_2)^2 - \mathbf{p}_1^2} \frac{R_\Lambda J_1(kR_\Lambda)}{k}$$

$$\times \Phi(m_K^2, -(p-p_2)^2, m^2) + \frac{2i\Gamma p_2}{(\mathbf{p}-\mathbf{p}_1)^2 - \mathbf{p}_2^2} \frac{R_K J_1(kR)}{k}$$

$$\times \Phi(-(p-p_1)^2, m_\Lambda^2, m^2) + \frac{ip_2}{8\pi^4} \int (\gamma\mathbf{n}_1)$$

$$\times \frac{[i(\mathbf{p}+\mathbf{g})\gamma - \gamma_4 E_1 - m_\Lambda]\Gamma}{(\mathbf{p}+\mathbf{g})^2 - \mathbf{p}_1^2 - i\varepsilon} \frac{\Phi(-g^2, -(p+g)^2, m^2)}{g^2 - p_2^2 - i\varepsilon}$$

$$\left. \times e^{i(\mathbf{p}+\mathbf{g})\rho_1} e^{-ig\rho_2} d\rho_1 \, d\rho_2 d^3 g \right\} u_{\mathbf{p}}; \tag{5.2}$$

$$\varepsilon \to 0, \quad \rho_1 \perp \mathbf{p}_1, \quad \rho_2 \perp \mathbf{p}_2, \quad \mathbf{n}_1 = \mathbf{p}_1/p_1, \; \mathbf{n} = \mathbf{p}/p,$$

$$\rho_1^2 \leqslant R_\Lambda^2, \; \rho_2^2 \leqslant R_K^2, \; \mathbf{k} = -\mathbf{q}_\perp.$$

Here E and $\mathbf{p}$ are the energy and the momentum of the incident nucleon, E_1 and $\mathbf{p}_1$ are the energy and the momentum of the Λ hyperon, E_2 and $\mathbf{p}_2$ are the energy and the momenum of the K meson; R_N, R_Λ, R_K are the nuclear radii with respect to the absorption respectively of nucleons, Λ hyperons, and K mesons, $g_{K\Lambda N}$ is the coupling constant for the KΛN coupling; Γ = 1 if the KΛN coupling is scalar, and $\Gamma = \gamma_5$ if the KΛN coupling is pseudoscalar; $\Phi(-g_1^2, -g_2^2, -g_3^2)$ is the vertex function for the KΛN-vertex. If $R_N = R_\Lambda = R_K = R$ and all the form factors $\Phi \approx 1$, then the expression (5.2) is very much simplified. The integral contained in the curly brackets of formula (5.2) can be evaluated in the case of a heavy nucleus [2]. On introducing the notation

$$\mathbf{p}_1 = p_1\mathbf{n}(1 - k_1^2/2p_1^2) + \mathbf{k}_1, \quad k_1 \ll p_1, \quad \mathbf{k}_1 \perp \mathbf{p},$$
$$\mathbf{p}_2 = p_2\mathbf{n}(1 - k_2^2/2p_2^2) + \mathbf{k}_2, \quad k_2 \ll p_2, \quad \mathbf{k}_2 \perp \mathbf{p} \tag{5.3}$$

and on going to the limit $E \gg m$, $E_1 \gg m_\Lambda$, $E_2 \gg m_K$ (m_Λ is the mass of the Λ hyperon, m_K is the mass of the K meson), we obtain

$$T = \frac{8\pi^{3/2} i g_{K\Lambda N} R J_1(kR) E_1 E_2}{kE\sqrt{2E_2}}$$

$$\times \bar{u}_{\mathbf{p}_1} \left\{ - \frac{\Gamma\left[1 + \frac{1}{2E}(\gamma\mathbf{k})(\gamma\mathbf{n})\right]}{\frac{E_2}{E} m_\Lambda^2 - \frac{E_1E_2}{E^2} m^2 + \frac{E_1}{E} m_K^2 + \frac{(\mathbf{k}_1 E_2 - \mathbf{k}_2 E_1)^2}{E^2}} \right.$$

$$- \frac{\left[1 - \frac{1}{2E_1}(\gamma\mathbf{n}_1)(\gamma\mathbf{k})\right]\Gamma}{\frac{E_1}{E} m_K^2 + \frac{E_2}{E} m_\Lambda^2 - \frac{E_1E_2}{E^2} m^2 + k_2^2}$$

$$+ \frac{\Gamma}{\frac{E_2}{E} m_\Lambda^2 + \frac{E_1}{E} m_K^2 - \frac{E_1E_2}{E_2} m^2 + k_1^2}$$

$$\left. - \frac{\left[1 - \frac{1}{4E_1}(\gamma\mathbf{n}_1)(\gamma\mathbf{k})\right]\Gamma}{\frac{E_2}{E} m_\Lambda^2 + \frac{E_1}{E} m_K^2 - \frac{E_1E_2}{E^2} m^2 + \frac{1}{4}(\mathbf{k}_1 - \mathbf{k}_2)^2} \right\} u_{\mathbf{p}};$$

$$\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2. \tag{5.4}$$

On taking into account the fact that effectively $k_1 \sim k_2 \sim m$ and $\mathbf{k}_1 \sim -\mathbf{k}_2$ for $E_1 \sim E_2$, while $k_{eff} \sim 1/R$, we can carry out an expansion in formula (5.4) in powers of k/k_2 keeping only the term linear in k. After some straightforward calculations the matrix element assumes the comparatively simple form:

$$T = \frac{8\pi^{3/2} i g_{K\Lambda N} R J_1(kR) E_1 E_2}{kE\sqrt{2E_2}} \bar{u}_{\mathbf{p}_1} \Gamma u_{\mathbf{p}}$$

$$\times \frac{(\mathbf{k}\mathbf{k}_2)(1 - 2E_2/E)}{[E_2 m_\Lambda^2/E - E_1E_2 m^2/E^2 + E_1 m_K^2/E + k^2]^2}. \tag{5.5}$$

The cross section for the process is related to the matrix element by means of formula (4.9). On substituting (5.5) into (4.9) and on carrying out the integration with respect to the energy of the Λ hyperon E_1 we obtain

$$d\sigma = \frac{g^2_{K\Lambda N} R^2 J_1^2(kR) E_1^2 E_2}{\pi^2 k^2 E^2} |\bar{u}_{p_1}\Gamma u_p|^2 \times \frac{(\mathbf{k}\mathbf{k}_2)^2 (1-2E_2/E)^2 dE_2 d\mathbf{k} d\mathbf{k}_2}{[E_2 m_\Lambda^2/E - E_1E_2m^2/E^2 + E_1 m_K^2/E + k_2^2]^4}. \quad (5.6)$$

On averaging over the initial spin states of the nucleon, and on summing over the final spin states of the Λ hyperon we obtain from (5.6)

$$d\sigma_{P,S} = \frac{g^2_{P,S} R^2 J_1^2(kR) E_2}{4\pi^2 k^2 E^2}\left[k_2^2 + \left(m_\Lambda \mp m \pm \frac{E_2 m}{E}\right)^2\right] \times \frac{(\mathbf{k}\mathbf{k}_2)^2 (1-2E_2/E)^2 dE_2 d\mathbf{k} d\mathbf{k}_2}{[m_\Lambda^2 E_2/E - m^2E_1E_2/E^2 + m_K^2 E_1/E + k_2^2]^4}, \quad (5.7)$$

where the index P (upper sign) corresponds to pseudoscalar KΛN coupling, while the index S (lower sign) corresponds to scalar coupling. On carrying out the integration over the azimuthal angles of the vectors **k** and $\mathbf{k}_2$ and over the absolute value k between the limits from 0 to μ [cf. (1.2)], we obtain the energy and the angular spectra of the K mesons:

$$d\sigma_{P,S} = \frac{g^2_{P,S} R\mu}{2\pi}[k_2^2 + (m_\Lambda \mp (1-\varepsilon) m)^2] \times \frac{\varepsilon(1-2\varepsilon)^2 d\varepsilon\, k_2^3 dk_2}{[m_\Lambda^2\varepsilon - m^2\varepsilon(1-\varepsilon) + m_K^2(1-\varepsilon) + k_2^2]^4}. \quad (5.8)$$

Here $\epsilon = E_2/E$, while k_2 is simply related to the angle of emission of the K-meson: $\theta_2 = k_2/E_2$.

The integration over k_2 can be carried out between the limits from 0 to ∞ in view of the good convergence. The energy spectrum of the K mesons then has the form

$$d\sigma_{P,S} = \frac{g^2_{P,S} R\mu}{12\pi}\left[1 + \frac{(m_\Lambda \mp (1-\varepsilon) m)^2}{2(m_K^2 + \varepsilon(m_\Lambda^2 - m^2 - m_K^2) + \varepsilon^2 m^2)}\right] \times \frac{\varepsilon(1-2\varepsilon)^2 d\varepsilon}{[m_K^2 + \varepsilon(m_\Lambda^2 - m^2 - m_K^2) + \varepsilon^2 m^2]}. \quad (5.9)$$

We note that in comparing formulas (5.8) and (5.9) with experimental data we can, in principle, obtain information on the relative KΛ parity and on the coupling constant for KΛN coupling. Naturally, an important role in this can be played by the form factors introduced in (5.2) which we have set equal to unity. If the form factors differ appreciably from unity in the range of energies and angles considered by us, then the analysis with respect to parity and the derivation of the form factors themselves from experimental data becomes in practice a very complicated problem. If we assume that $g_P^2 \approx 10$, then the total cross section for the production of K-mesons is $\sigma_P \approx 0.5$ mb.

6. THE DIFFRACTION PRODUCTION OF NUCLEON-ANTINUCLEON PAIRS BY A FAST PION

We consider the production of nucleon-antinucleon pairs accompanying the diffraction of fast π mesons by nuclei. The energy threshold for pair production under the condition (1.1) is determined in the same way as in section 4:

$$E_{\text{thr}} \gtrsim 2m^2R. \quad (6.1)$$

For a nucleus of mass number A = 125 we have $E_{\text{thr}} \gtrsim 60$ BeV. The matrix element for the process (1.6) can be calculated on the basis of the diagrams of Fig. 8 and has the form

$$T = \frac{8i\pi^{3/2} RE_1E_2 g}{kE^{3/2}} J_1(kR)\bar{u}_{p_1} \times \Big\{\frac{[1-(\gamma\mathbf{n})(\gamma\mathbf{k})/2E_1]\gamma_5}{k_2^2+m^2}F(\mu^2, -(p-p_2)^2, m^2) + \frac{[1-(\gamma\mathbf{k})(\gamma\mathbf{n}_2)/2E_2]\gamma_5}{k_1^2+m^2}F\left(\mu^2, m^2, -(p-p_1)^2\right) - \frac{\gamma_5 F(-(p_1+p_2)^2, m^2, m^2)}{m^2+E^{-2}(\mathbf{k}_1E_2-\mathbf{k}_2E_1)^2} - \frac{[1-(\gamma\mathbf{n}_1)(\gamma\mathbf{k})/4E_1 - (\gamma\mathbf{k})(\gamma\mathbf{n}_2)/4E_2]\gamma_5}{m^2+(\mathbf{k}_1-\mathbf{k}_2)^2/4} \times F(\mu^2, -g_1^2, m^2)\Big\} v_{-p_2};$$

$$\mathbf{n}_1 = \mathbf{p}_1/p_1,\quad \mathbf{n}_2 = \mathbf{p}_2/p_2,\quad \mathbf{k} = \mathbf{k}_1+\mathbf{k}_2,\quad \mathbf{n} = \mathbf{p}/p,$$
$$\mathbf{p}_1 = p_1\mathbf{n}(1-k_1^2/2p_1^2)+\mathbf{k}_1,\quad k_1 \ll p_1,\quad \mathbf{k}_1 \perp \mathbf{p},$$
$$\mathbf{p}_2 = p_2\mathbf{n}(1-k_2^2/2p_2^2)+\mathbf{k}_2,\quad k_2 \ll p_2,\quad \mathbf{k}_2 \perp \mathbf{p}, \quad (6.2)$$

where E, **p** is the energy and the momentum of the π meson, $E_1, \mathbf{p}_1$ and $E_2, \mathbf{p}_2$ are the energy and the momentum of the nucleon and the antinucleon, $\bar{u}_{p_1}$ and v_{-p_2} are bispinors describing the spin states of the nucleon and the antinucleon, $g^2 = 14.5$, and m is the nucleon mass. In the derivation of formula (6.2) we assume that $E, E_1, E_2 \gg m$, $R_\pi = R_N = R_{\bar{N}} = R$. The quantities k_1 and k_2 are simply related to the angles of emission of the nucleon and the antinucleon: $\mathbf{k}_1 = E_1\boldsymbol{\theta}_1$, $\mathbf{k}_2 = E_2\boldsymbol{\theta}_2$.

We shall calculate the amount by which the variables of the form factors in formula (6.2) differ from their values on the mass shell:

$$-(p-p_2)^2 - m^2 \approx -Em^2/E_2,\quad \theta_2^2 \ll m^2/E_2^2,$$
$$-(p-p_1)^2 - m^2 \approx -Em^2/E_1,\quad \theta_1^2 \ll m^2/E_1^2,$$
$$-(p_1+p_2)^2 - \mu^2 \approx m^2E^2/E_1E_2,\quad (\theta_1-\theta_2)^2 \ll m^2E^2/E_1^2E_2^2,$$
$$-g_1^2 - m^2 \approx -Em^2/E_2,\quad \tfrac{1}{4}(\theta_2 - E_1\theta_1/E_2)^2 \ll m^2/E_2^2. \quad (6.3)$$

From this it can be seen that the arguments of the form factors differ appreciably from the squares of the masses and, consequently, the values of the

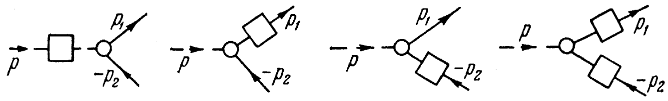

FIG. 8

form factors can differ appreciably from unity. Thus, on comparing the differential cross section calculated with the aid of formula (6.2) with the experimental value we can obtain, in principle, information on the pion-nucleon form factor. In order to obtain at least a rough estimate of the magnitude of the effect we shall assume that the form factors differ but little from their value on the mass shell, i.e., all $F \approx 1$.

It follows from (6.2) that on integrating the differential cross section over the angles the principal role is played by $\mathbf{k}_1 \approx -\mathbf{k}_2$ and $k_1 \sim k_2 \sim m$. Under such conditions the expression (6.2) for the matrix element can be expanded in powers of k/k_2 ($k_{\text{eff}} \sim 1/R$). Keeping only the term linear in k we obtain

$$T = i\,\frac{8\pi^{3/2} g R E_1 E_2 J_1(kR)}{kE^{3/2}}\,\bar{u}_{p_1}\gamma_5 v_{-p_2}\,\frac{(\mathbf{k}\mathbf{k}_2)(1-2E_2/E)}{(m^2+k_2^2)^2}. \qquad (6.4)$$

In this expression we also have the diagram cancellation noted in Sec. 4. The differential cross section for the diffraction production of nucleon-antinucleon pairs calculated with the aid of the matrix element (6.4) has the form

$$d\sigma = \frac{2g^2R^2E_1^2E_2^2J_1^2(kR)}{\pi^2E^3k^2}\,\frac{(\mathbf{k}\mathbf{k}_2)^2(1-2E_2/E)^2}{(m^2+k_2^2)^4}\,|\bar{u}_{p_1}\gamma_5 v_{-p_2}|^2\,dk\,d\mathbf{k}_2 dE_2. \qquad (6.5)$$

On summing over the spin states of the nucleon and the antinucleon, and on integrating over the angles we obtain

$$d\sigma = 2g^2R^2J_1^2(kR)\,k_2^3\,(k_2^2+m^2)^{-3}\,(1-2\varepsilon)^2\,kdk\,dk_2 d\varepsilon, \qquad (6.6)$$

where $\epsilon = E_2/E$. The integration over k should be carried out between the limits from 0 to μ [cf. (1.2)], as a result of which we obtain the energy and the angular distributions of the antinucleons:

$$d\sigma = (2g^2 R\mu/\pi)\,k_2^3\,dk_2\,(k_2^2+m^2)^{-3}\,(1-2\varepsilon)^2\,d\varepsilon. \qquad (6.7)$$

From this it follows that the effective angles of emission of the antinucleon $\theta_{2\,\text{eff}} \le m/E_2$. In view of the good convergence the integration over k_2 can be carried out from 0 to ∞. Then the energy spectrum of the antinucleons has the form

$$d\sigma = (g^2R\mu/2\pi m^2)\,(1-2\varepsilon)^2\,d\varepsilon. \qquad (6.8)$$

On carrying out the integration over ϵ from 0 to 1 we obtain an estimate for the total cross section for the diffraction production of nucleon-antinucleon pairs by a π meson:

$$\sigma = (g^2/6\pi)\,(\mu/m)^2\,R/\mu. \qquad (6.9)$$

For a nucleus of mass number A = 125 we have $\sigma \approx 1.7$ mb. Formula (6.9) has the linear dependence on the nuclear radius R characteristic of inelastic diffraction processes, and does not depend on the energy.

7. CONCLUSION

The estimates of the differential and total cross sections for the diffraction production of particles in the interaction between nucleons, γ quanta and π mesons with nuclei given above show that the diffraction mechanism can play a significant role at high energies. The product particles carry away practically the whole energy of the primary particle, and are emitted within a narrow angular range. This fact is very important for obtaining narrow beams of π mesons and of strange particles of high energy utilizing proton and electron accelerators.

From the theoretical point of view of greatest interest is the effect of cancellation in first-order perturbation theory of pole diagrams containing nucleon and π-meson virtual lines. This result, obtained for the case of diffraction by nuclei, does not agree with Drell's conclusions [11] that polar single nucleon diagrams are unimportant for diffraction processes in nucleon-nucleon interactions. The possibility is not excluded that the cancellation noted by us is of a general nature within the framework of perturbation theory. A more consistent method of taking into account strong interactions between π mesons and nucleons by means of introducing form factors must partially destroy this cancellation, since the arguments of the form factors can differ appreciably from their values on the mass shell.

The authors express their gratitude to I. Ya. Pomeranchuk for suggesting the problem and for many fruitful discussions, and also to the participants in the theoretical seminar of the Moscow Engineering-Physics Institute for a number of critical remarks.

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Translated by G. Volkoff
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