SOVIET PHYSICS JETP VOLUME 16, NUMBER 4 APRIL, 1963

ON THE MASS OF A COMPLEX VECTOR FIELD

I. B. KHRIPLOVICH

Institute of Nuclear Physics, Siberian Division, Academy of Sciences, U.S.S.R.

Submitted to JETP editor May 4, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1493-1495 (October, 1962)

It is shown that a massless complex vector field interacting with an electromagnetic field is not invariant under gauge transformations of the second kind. Consequently a charged vector meson acquires electromagnetic mass even in the absence of bare mass. In this case, its mass is determined by the cut-off parameter and the coupling constant.

As is well-known, the Lagrangian for a free massless charged vector field,

$$L_0 = -\tfrac{1}{2}\,\psi^+_{\mu\nu}\psi_{\mu\nu}, \qquad \psi_{\mu\nu} \equiv \partial_\mu\psi_\nu - \partial_\nu\psi_\mu \tag{1}$$

depends on only the transverse (in a four-dimensional sense) part of the vector field. Therefore, it is invariant with respect to a gauge transformation of the second kind

$$\psi_\mu \to \psi_\mu + \partial_\mu\varphi, \tag{2}$$

which obviously changes only the longitudinal part. The presence of a meson mass would lead to the appearance in the Lagrangian of a term $m^2\psi^+_\mu\psi_\mu$, which violates this invariance.

We now introduce the interaction of the massless charged field with an electromagnetic field by the usual method, i.e., we carry out the following substitution in L_0:

$$\partial_\mu \to \partial_\mu - ieA_\mu \equiv D_\mu. \tag{3}$$

Considering the interaction Lagrangian obtained in this connection,

$$L_i = -ieA_\mu(\psi^+_\nu\psi_{\mu\nu} - \psi^+_{\mu\nu}\psi_\nu) - e^2(A_\mu A_\mu\psi^+_\nu\psi_\nu - A_\mu A_\nu\psi^+_\nu\psi_\mu), \tag{4}$$

or the equation of motion directly,

$$D_\mu(D_\mu\psi_\nu - D_\nu\psi_\mu) = 0, \tag{5}$$

it is not difficult to verify that a massless complex vector field loses gauge invariance of the second kind while interacting with an electromagnetic field. In this respect it differs markedly from the electromagnetic field, which retains this invariance for arbitrary interactions, and therefore cannot acquire mass. Thus, there is no reason to discard the constant part of the self-energy of a charged vector meson without bare mass, and there is no reason to assume that the electromagnetic mass of such a particle equals zero.

We shall attempt to relate the sought-for electromagnetic mass with the coupling constant $\alpha = e^2/4\pi$ and the cut-off parameter Λ. We rewrite $L_0 + L_i$ in the form

$$L = L_0 + m^2\psi^+_\mu\psi_\mu + L_i - m^2\psi^+_\mu\psi_\mu. \tag{6}$$

The first two terms will be regarded as the Lagrangian of a free vector field with renormalized mass m^2. The corresponding Green's function is

$$G_{\mu\nu}(k) = \frac{g_{\mu\nu} - k_\mu k_\nu/m^2}{i(k^2 - m^2)}. \tag{7}$$

We denote the sum of all compact meson self-energy diagrams by $\Sigma^*(k^2, m^2, \Lambda^2)$. Then m^2 is determined from the equation

$$m^2 = \Sigma^*(m^2, m^2, \Lambda^2). \tag{8}$$

It is clear that the mass is proportional to the cut-off parameter, where the coefficient of proportionality depends on only α.

From now on we shall take into account only the most divergent terms in the compact self-energy diagrams. To this end we first calculate the maximal degree of divergence D of an arbitrary diagram containing n_1 single-photon vertices, n_2 two-photon vertices, p_i internal photon lines, p_e external photon lines, v_i internal vector meson lines, and v_e external vector meson lines. We note that the pairing

$$\overline{\psi^+_{\mu\nu}\psi_\alpha} = \frac{k_\nu g_{\mu\alpha} - k_\mu g_{\nu\alpha}}{k^2 - m^2}$$

makes a smaller contribution to D than $G_{\mu\nu}$ does, and the pairing

$$\overline{\psi^+_{\mu\nu}\psi_{\alpha\beta}} = \frac{k_\mu k_\alpha g_{\nu\beta} + k_\nu k_\beta g_{\mu\alpha} - k_\mu k_\beta g_{\nu\alpha} - k_\nu k_\alpha g_{\mu\beta}}{i(k^2 - m^2)}$$

makes the same contribution as $G_{\mu\nu}$. Thus, in the present case, the presence of derivatives in the interaction Lagrangian does not increase D. Taking this into account, it is not difficult to find by the usual method that

$$D = 4 - 2v_e - p_e + n_1 + 2n_2. \quad (9)$$

We note that each matrix element is proportional to $(e/4\pi)^{n_1+2n_2}$. Furthermore, for any given process n_1 is either always even or always odd. In the case of self-energy diagrams $p_e = 0$, $v_e = 2$, $D = n_1 + 2n_2$, n_1 is even and the leading terms constitute a series in $\alpha\Lambda^2/4\pi m^2$. Equation (8) acquires the form

$$m^2 = \frac{\alpha\Lambda^2}{4\pi}\sum_{n=0}^{\infty} a_n \left(\frac{\alpha\Lambda^2}{4\pi m^2}\right)^n$$

(direct calculation gives $a_0 = 2$) or

$$1 = \sum_{n=0}^{\infty} a_n (\alpha\Lambda^2/4\pi m^2)^{n+1}. \quad (10)$$

It is now clear that, with acceptable precision,

$$m^2 = b\alpha\Lambda^2/4\pi, \quad (11)$$

where b is a numerical coefficient. The method fails to find the general form of a_n and, by the same token, to determine b.

Using (11), it is possible to eliminate Λ^2 from the matrix elements. But at the same time, the powers of α are correspondingly lowered in terms in which this substitution is made. Moreover, a logarithmic term in α will also appear. After such a substitution each diagram will, generally speaking, contain terms which depend on the coupling constant in different ways. Therefore, in principle it may be necessary to investigate an infinite set of diagrams in order to calculate some process to any given order in α. It is possible to partially eliminate terms containing lower powers of α by carrying out charge renormalization, which will, of course, be finite.

If D did not depend on n_1 and n_2, then the calculation of m^2 would not be difficult. For example, in the case of scalar electrodynamics, the maximum degree of divergence (independent of whether the bare mass vanishes or not) is

$$D_0 = 4 - p_e - s_e,$$

where s_e is the number of external scalar lines. Here all self-energy diagrams are quadratically divergent. Therefore, confining ourselves to the simplest diagrams, we easily find $m^2 = 3\alpha\Lambda^2/4\pi$, correct to order α^2.

Recently the hypothesis concerning the existence of a charged vector meson field which transmits the weak interactions has been widely discussed. However, the question of the origin of the mass of this meson, which must at any rate exceed the mass of the K meson, remains open. It seems quite possible that this mass is purely electromagnetic. Then the idea [1] of the existence of a triplet of vector particles, consisting of two charged mesons which transmit the weak interactions and a neutral photon which transmits the electromagnetic interactions, becomes rather natural. Actually the charged mesons acquire mass while interacting with the photon, whereas the photon, retaining gauge invariance of the second kind, remains massless. But from the proposed point of view, it would be difficult to explain the mass of the neutral intermediate bosons. [2]

Attempts have been made in several articles (a review of these articles is given in [3], and a criticism in [4]) to introduce hypothetical vector fields which transmit the strong interactions in the following way. The requirement of invariance with respect to a certain gauge transformation of the first kind, with the phase depending on the coordinates, is imposed on the baryon field. Then, in order to compensate the additional gradient terms which appear at the same time in the Lagrangian, charged and neutral vector fields are introduced which are invariant under gauge transformations of the second kind. Since it is possible to introduce the interaction of a complex vector meson field with the electromagnetic field by using just such a principle, but at the same time upsetting the invariance of the vector field to gauge transformations of the second kind, it becomes clear that one field cannot simultaneously be both compensating and also compensable. Thus, the electromagnetic interactions not only violate the conservation laws (for example, isotopic spin) which the appropriate gauge transformations of the first kind give rise to, but they also take away the invariance of the charged carriers of the interaction with respect to gauge transformations of the second kind.

In conclusion, I express my deep gratitude to V. N. Baĭer, V. V. Sokolov, and S. A. Kheĭfets for their constant interest in the work, valuable comments, and criticism.

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Translated by H. H. Nickle
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