THEORY OF WAVE CONVERSION AND SCATTERING ON PLASMA FLUCTUATIONS

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Wave scattering on plasma fluctuations can result in the conversion of one characteristic wave mode into another. We find the electromagnetic radiation and the intensity of longitudinal waves due to conversion in a nonisothermal plasma. The spectral correlation functions of various quantities characterizing the fluctuations are determined.

1. The scattering of electromagnetic waves on plasma oscillations has, in recent years, occupied the attention of a number of workers in connection with plasma diagnostics and the physics of the ionosphere and of the solar corona.[1-4]

Ginzburg and Zheleznyakov[5] and Sturrock[6] have pointed out the possibility of conversion of longitudinal waves into transverse waves in scattering on thermal fluctuations in a plasma as a possible explanation for type I and II bursts at frequencies Ω_e and $2\Omega_e$ in the spectrum of sporadic radio radiation from the Sun (Ω_e is the Langmuir frequency). However, a quantitative theory has been developed only for scattering of electromagnetic waves on plasma waves (cf. [1-4]). Furthermore, in general, electromagnetic waves can be radiated not only by conversion of longitudinal waves, but also by the conversion of any characteristic wave into a transverse wave in scattering on plasma fluctuations.

In the present work we have investigated the scattering and conversion of different kinds of waves due to the interaction with thermal fluctuations in a nonisothermal plasma. (We adopt the convention that scattering occurs if the output wave is a wave of the same kind as the incident wave and conversion occurs if the output wave is different from the incident wave.)

2. It is well known that four kinds of characteristic waves can propagate in a nonisothermal plasma when $T_e \gg T_i$ (T_e and T_i are respectively the electron and ion temperatures in energy units); these waves are described by the following dispersion relations:

$$\omega^2 = \Omega_e^2 + s_e^2 k^2, \quad \Omega_e^2 = 4\pi e^2 N/m,$$

$$s_e^2 = 3T_e/m \quad \text{longitudinal electron} \tag{1}$$

$$\omega^2 = \Omega_e^2 + c^2 k^2 \quad \text{transverse} \tag{2}$$

$$\omega^2 = \Omega_i^2 + s_i^2 k^2, \quad \Omega_i^2 = 4\pi e^2 N/M,$$

$$s_i^2 = 3T_i/M \quad \text{longitudinal ion} \tag{3}$$

$$\omega^2 = s^2 k^2, \quad s^2 = T_e/M \quad \text{acoustic} \tag{4}$$

(m and M are the electron ion masses and N is the density of the ions or electrons).

Each of these wave types can be scattered or converted into a wave of a different type as a result of a wave-wave interaction. There are limitations, imposed by energy and momentum conservation, on the frequency and wave vector of the scattered (converted) wave, however, so that not all of the indicated conversion processes can, in fact, occur. We give several examples.

Suppose that a wave characterized by frequency ω_0 and wave vector $\mathbf{k}_0$, obeying the dispersion relation $\omega_0 = \varphi_1(\mathbf{k}_0)$, is converted into a wave of frequency ω, wave vector $\mathbf{k}$, and dispersion relation $\omega = \varphi_3(\mathbf{k})$ in scattering on a wave with frequency ω' and wave vector $\mathbf{k}'$. Energy and momentum must be conserved so that

$$\omega' = \omega - \omega_0, \quad \mathbf{k}' = \mathbf{k} - \mathbf{k}_0. \tag{5}$$

The dispersion equation of the wave on which scattering occurs $\omega' = \varphi_2(\mathbf{k}')$, together with the dispersion relation for the scattered (converted) wave, determines uniquely the frequency and absolute magnitude of the wave vector of the scattered (converted) wave. Conversion occurs if these equations have real solutions and if frequency of the scattered wave is higher than the Langmuir frequency.

We now consider several particular wave interactions.

a) A transverse wave scattered on a longitudinal wave. The dispersion equation for the longitudinal wave on which scattering occurs is

$$(\omega - \omega_0)^2 = \Omega_e^2 + s_e^2 (\mathbf{k} - \mathbf{k}_0)^2. \tag{6}$$

Neglecting the second term in the right side of (6) and using (2) we find $\omega = \omega_0 \pm \Omega_e$ (combination scattering [1]). The doublet line arises if the frequency of the incident transverse wave is greater than twice the Langmuir frequency. If the neglected term is to be small $(s_e^2(\mathbf{k}-\mathbf{k}_0)^2 \ll \Omega_e^2)$ there must be an upper limit on the frequency of the incident wave

$$\omega_0 \ll \Omega_e/2 \frac{s_e}{c} \sin\frac{\theta}{2} \qquad \left(\cos\theta = \frac{\mathbf{k}\mathbf{k}_0}{kk_0}\right).$$

b) A longitudinal wave scattered on a longitudinal wave and converted into a transverse wave. Solving the dispersion equation for the longitudinal wave (6) and using (1) and (2) we find $\omega = 2\Omega_e$.

c) A wave characterized by the dispersion relation $\omega_0 = \varphi(\mathbf{k}_0)$ scattered on a wave of the same kind so that the dispersion relation for the output wave is of the same form $\omega = \varphi(\mathbf{k})$. According to (5) the dispersion relation for the wave on which scattering occurs is $\omega - \omega_0 = \varphi(|\mathbf{k}-\mathbf{k}_0|)$ or $\varphi(\mathbf{k}) - \varphi(\mathbf{k}_0) = \varphi(|\mathbf{k}-\mathbf{k}_0|)$. Setting k and k_0 equal to zero in turn, we easily verify that this condition is satisfied only when $\varphi(\mathbf{k}) = 0$. Thus, this scattering process cannot occur.

d) An ion longitudinal wave scattered on an ion longitudinal wave and converted into a transverse wave. The dispersion equation for the ion longitudinal wave on which the conversion occurs is of the form

$$(\omega - \omega_0)^2 = \Omega_i^2 + s_i^2 (\mathbf{k} - \mathbf{k}_0)^2 .$$

Solving this relation and using (2) and (3) we obtain the frequency of the transverse waves $\omega = 2\Omega_i$. A transverse wave with frequency lower than Ω_e, however, obviously cannot be propagated in the plasma.

3. The frequency of the scattered (converted) wave obtained from an analysis of the dispersion relations is obviously independent of the method of excitation of the interacting waves. To compute the intensity of the scattered (converted) wave one must make some assumptions as to the scattering mechanism. We consider the case in which the scattering occurs by virtue of thermal fluctuations in the plasma. In this and the next section we consider the interaction of high-frequency waves. Under these conditions the ion motion can be neglected. Scattering and conversion on low-frequency waves are treated in Sec. 5.

It will be shown below that conversion is due to spatial dispersion so that the problem is treated most conveniently in the kinetic approximation.

We write the distribution function in the form $f = f_0 + f_1 + f_2 + f_3$ where f_0 is the equilibrium distribution function while f_1, f_2, and f_3 are corrections due respectively to the field of the incident wave, the fluctuations, and their interaction. The field $\mathbf{E}$ is the sum of the field of the incident wave $\mathbf{E}_1$, the field due to fluctuations $\mathbf{E}_2$ and the field of the scattered (converted) wave $\mathbf{E}_3$. In order to use successive approximations we assume that $|f_3| \ll |f_1|$ and $|f_2|$ and $|\mathbf{E}_3| \ll |\mathbf{E}_1|$ and $|\mathbf{E}_2|$.

The starting equations for the distribution functions are

$$\frac{\partial f_1}{\partial t} + \mathbf{v}\nabla f_1 = -e\mathbf{E}_1 \frac{\partial f_0}{\partial \mathbf{p}} - \frac{f_1}{\tau}, \qquad (7)^*$$

$$\frac{\partial f_2}{\partial t} + \mathbf{v}\nabla f_2 = -e\mathbf{E}_2 \frac{\partial f_0}{\partial \mathbf{p}} - \frac{f_2}{\tau} + y(\mathbf{v};\mathbf{r},t), \qquad (8)$$

$$\frac{\partial f_3}{\partial t} + \mathbf{v}\nabla f_3 = -e\mathbf{E}_3 \frac{\partial f_0}{\partial p} - \frac{f_3}{\tau} - \left(e\mathbf{E}_1 + \frac{e}{c}[\mathbf{v}\mathbf{H}_1]\right)\frac{\partial f_2}{\partial \mathbf{p}} + \left(e\mathbf{E}_2 + \frac{e}{c}[\mathbf{v}\mathbf{H}_2]\right)\frac{\partial f_1}{\partial \mathbf{p}}. \qquad (9)$$

As is usually done in the general theory of fluctuations we have introduced random forces $y(\mathbf{v};\mathbf{r},t)$ in the right side of (8). Furthermore, in (7) and (8) we have neglected terms containing $\partial f_1/\partial \mathbf{p}$ and $\partial f_2/\partial \mathbf{p}$. These terms are of higher order than the other terms in (7) and (8) but, in general, are of the same order as the terms in (9). The first of the neglected terms characterizes the nonlinear effect of the interaction of the wave with its own field; it can be shown by calculations similar to those presented below that this term does not make a contribution to the intensity of the converted wave. The second term neglected in (7) is a small correction for the intensity of the thermal radiation and will not be considered.

The correction to the equilibrium distribution function f_3 can be written in the form $f_3 = f_3^0 + f_3'$, where f_3^0 is proportional to the field of the scattered wave while f_3' is determined by the cross term in (9). The quantity f_3^0 is responsible for the usual contribution to the dielectric constant of the plasma and is considered below in (10) and (11). The quantity f_3' determines the current that produces the scattered wave.

The vector and scalar potentials $\mathbf{A}$ and φ, that describe the field of the scattered wave ($\operatorname{div}\mathbf{A} = 0$) satisfy the equations

$$\Delta\mathbf{A} - \frac{\hat{\varepsilon}^{tr}}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} - \frac{\hat{\varepsilon}^{l}}{c}\frac{\partial}{\partial t}\operatorname{grad}\varphi = -\frac{4\pi}{c}\mathbf{j}, \qquad (10)$$

$$\hat{\varepsilon}^{l}\Delta\varphi = -4\pi\rho, \qquad (11)$$

where $\hat{\varepsilon}^{tr}$ and $\hat{\varepsilon}^{l}$ are the transverse and longitudinal dielectric constant operators; [9]

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*$[\mathbf{v}\mathbf{H}_1] = \mathbf{v} \times \mathbf{H}_1$.

$$\rho(\mathbf{r}, t) = e\int f_3' d\mathbf{v} \tag{12}$$

is the charge density and

$$\mathbf{j}(\mathbf{r}, t) = e\int f_3' \mathbf{v} d\mathbf{v} \tag{13}$$

is the current associated with the field of the scattered wave.

The change in the radiation energy of the scattered or converted wave per unit time is given by the expression[7]

$$\frac{\partial W}{\partial t} = \frac{1}{2}\operatorname{Re}\frac{1}{c}\int \mathbf{j}\,\frac{\partial \mathbf{A}^*}{\partial t}\,dV + \frac{1}{2}\operatorname{Re}\int\frac{\partial\rho}{\partial t}\varphi^* dV. \tag{14}$$

Here, the first term determines the intensity of the electromagnetic radiation while the second gives the strength of longitudinal plasma waves.

The expressions in (10) and (11) can be used to express the vector and scalar potentials in terms of the current j and charge density ρ. Fourier analyzing (9) and (10) we have

$$\mathbf{A}(\mathbf{k}, \omega) = \frac{4\pi}{c}\,\frac{\mathbf{j}(\mathbf{k}, \omega) - \mathbf{k}(\mathbf{k}\mathbf{j}(\mathbf{k}, \omega))k^{-2}}{k^2 - (\omega/c)^2\varepsilon^{tr}(\mathbf{k}, \omega)}, \tag{15}$$

$$\varphi(\mathbf{k}, \omega) = 4\pi\rho(\mathbf{k}, \omega)\,/\,k^2\varepsilon^l(k, \omega). \tag{16}$$

We introduce the quantity $I = -\overline{\partial W/\partial t}$, the radiation intensity of the scattered (converted) wave.

Taking the Fourier component from (13), substituting in (14) and carrying out the integration over k by means of the formula

$$\frac{1}{x + i0} = P\frac{1}{x} - i\pi\delta(x), \tag{17}$$

after statistical averaging we obtain the electromagnetic radiation per unit volume, unit solid angle, and unit frequency interval

$$\frac{1}{V}\frac{\partial I_{tr}}{\partial\omega\partial o} = \frac{(2\pi)^5}{4}\frac{\omega^2}{c^3}\langle j_i j_k^*\rangle_{\mathbf{k}\omega}\left(\delta_{ik} - \frac{k_i k_k}{k^2}\right)\sqrt{\varepsilon^{tr}(\omega, \mathbf{k})}, \tag{18}$$

where $\langle j_i j_k^*\rangle_{\mathbf{k}\omega}$ is the current spectral correlation function.

In similar fashion, using (13), (15), and (16) and the equation of continuity, we find the spectral intensity of the longitudinal waves:

$$\frac{1}{V}\frac{\partial I_l}{\partial\omega\,\partial o} = \frac{(2\pi)^5}{2}\sum_s \frac{\langle j_i j_k^*\rangle_{\mathbf{k}\omega}k_i^{(s)}k_k^{(s)}}{|\partial\varepsilon^l(k, \omega)/\partial k|_{k=k^{(s)}}}\,\frac{1}{\omega}, \tag{19}$$

where $k^{(s)} = k^{(s)}(\omega)$ is a root of the dispersion equation for the longitudinal waves $\epsilon^l(\omega, k) = 0$ and the summation is carried out over all roots of this equation.

4. In order to find the intensities (18) and (19) in explicit form we must determine the Fourier components of the current correlations.

Solving (9) in the Fourier representation we find the correction to the equilibrium distribution function due to the wave interaction

$$f_3'(\mathbf{k}, \omega) = -i\,\frac{\mathbf{F}_1(\omega_0, \mathbf{k}_0)\,\partial f_2(\omega', \mathbf{k}')/\partial\mathbf{p} + \mathbf{F}_2(\omega', \mathbf{k}')\,\partial f_1(\omega_0, \mathbf{k}_0)/\partial\mathbf{p}}{\omega - \mathbf{k}\mathbf{v} + i/\tau}, \tag{20}$$

where $\mathbf{F} = e\mathbf{E} + e\omega^{-1}\mathbf{v}\times(\mathbf{k}\times\mathbf{E})$.

Using (13) and (20) we express the current spectral correlation function in terms of the correlation functions for the distributions and field fluctuations:

$$\begin{aligned}\langle j_i j_k^*\rangle_{\mathbf{k}\omega} = \left(\frac{e}{m}\right)^2\iint \frac{\partial}{\partial v_l}\,\frac{v_i}{\omega - \mathbf{k}\mathbf{v} + i/\tau}\,\frac{\partial}{\partial v_m'}\,\frac{v_k'}{\omega - \mathbf{k}\mathbf{v}' - i/\tau}\\ \times\{F_{1l}F_{1m}^*\langle f_2(\mathbf{v})\,f_2^*(\mathbf{v}')\rangle_{\mathbf{k}'\omega'} + f_1(\mathbf{v})\,f_1^*(\mathbf{v}')\langle F_{2l}F_{2m}^*\rangle_{\mathbf{k}'\omega'}\\ + F_{1l}f_1^*(\mathbf{v}')\langle f_2(\mathbf{v})\,F_{2m}^*\rangle_{\mathbf{k}'\omega'}\\ + F_{1m}^*f_1(\mathbf{v})\langle f_2^*(\mathbf{v}')\,F_{2l}\rangle_{\mathbf{k}'\omega'}\}\,d\mathbf{v}d\mathbf{v}'.\end{aligned} \tag{21}$$

Thus, to determine the current correlation function we need the correlation functions of the field fluctuations and distribution functions. Using (7) we write the Fourier component of the distribution function f_2 in terms of the Fourier component of the random forces

$$f_2(\mathbf{v}) = -ie\frac{(\mathbf{E}_2\mathbf{v})\,\partial f_0/\partial\varepsilon}{\omega - \mathbf{k}\mathbf{v} + i/\tau} + i\,\frac{y(\mathbf{v};\,\omega,\,\mathbf{k})}{\omega - \mathbf{k}\mathbf{v} + i/\tau}. \tag{22}$$

Using the results of [8] we find the Fourier component of the mean values of the products of the random forces:

$$(2\pi)^4\langle y(\mathbf{v})\,y^*(\mathbf{v}')\rangle_{\mathbf{k}\omega} = \frac{2}{\tau}f_0(v)\,\delta(\mathbf{v} - \mathbf{v}'). \tag{23}$$

Averaging the product of intensities of the fluctuating fields and using (23) we find[8]

$$\begin{aligned}(2\pi)^4\langle E_i E_k^*\rangle_{\mathbf{k}\omega} = \frac{8\pi}{\omega}\Bigg[\frac{k_i k_k}{k^2}\,\frac{T^e\operatorname{Im}\varepsilon_e^l + T^i\operatorname{Im}\varepsilon_i^l}{|\varepsilon^l|^2}\\ + \left(\delta_{ik} - \frac{k_i k_k}{k^2}\right)\frac{\omega^4}{c^4}\,\frac{T^e\operatorname{Im}\varepsilon_e^{tr} + T^i\operatorname{Im}\varepsilon_i^{tr}}{|k^2 - (\omega/c)^2\varepsilon^{tr}|^2}\Bigg].\end{aligned} \tag{24}$$

Using (22)—(24) and letting τ approach infinity, we find the correlations of the distribution functions

$$\begin{aligned}(2\pi)^4\langle f_2^a(\mathbf{v})\,f_2^{b*}(\mathbf{v}')\rangle_{\mathbf{k}\omega} = 2\pi f_0(v)\,\delta_{ab}\delta(\mathbf{v} - \mathbf{v}')\,\delta(\omega - \mathbf{k}\mathbf{v})\\ \pm\frac{8\pi\omega e^2}{c^2}f_0^a(v)\,f_0^b(v')\Bigg[\frac{4\omega^2}{c^2}\,\frac{1}{T^aT^b}\,\frac{v_i v_k'\psi_{ik}}{(\omega - \mathbf{k}\mathbf{v} + i0)(\omega - \mathbf{k}\mathbf{v}' - i0)}\\ + \frac{1}{T^a}\,\frac{v_i v_k' L_{ik}}{\omega - \mathbf{k}\mathbf{v} + i0}\,\delta(\omega - \mathbf{k}\mathbf{v}')\\ + \frac{1}{T^b}\,\frac{v_i' v_k L_{ik}^*}{\omega - \mathbf{k}\mathbf{v}' - i0}\,\delta(\omega - \mathbf{k}\mathbf{v})\Bigg],\end{aligned} \tag{25}$$

where for convenience we have used the notation $(2\pi)^4\langle E_i E_k^*\rangle = (4\pi\omega/c^2)^2(\omega/2\pi)\psi_{ik}$ while $L_{ik}(\mathbf{k}, \omega)$ is the Fourier transform of the Green's tensor in the Maxwell equation:[9]

$$L_{ik}(\mathbf{k}, \omega) = \frac{k_i k_k}{k^2}\,\frac{1}{(\omega/c)^2\varepsilon^l(k, \omega)} - \frac{\delta_{ik} - k_i k_k/k^2}{k^2 - (\omega/c)^2\varepsilon^{tr}(k, \omega)}. \tag{26}$$

In (25) the indices a and b denote the particle species and the upper (lower) sign is taken in the case of similar (different) particles. Equation (25) goes over to the corresponding formula from [8] for the case of a longitudinal field.

We also give the expression for the correlation $\langle f_2 F_2^* \rangle_{k\omega}$ required for computing the current correlation function by means of (21):

$$\langle f_2^a(\mathbf{v}') F_{2m}^* \rangle_{\mathbf{k}\omega} = e\,[\delta_{km} + \omega^{-1}(k_m v_k - \delta_{km}\mathbf{k}\mathbf{v}')]\,\langle f_2^a(\mathbf{v}') E_{2k}^* \rangle_{\mathbf{k}\omega},$$

$$\langle f_2^a(\mathbf{v}) E_{2k}^* \rangle_{\mathbf{k}\omega} = \frac{4\pi e\omega}{c^2}\left[i\,\frac{\omega^2}{T^a}\,\frac{v_l \psi_{kl}}{\omega - \mathbf{k}\mathbf{v} + i0} + 2\pi L_{kl}^* v_l f_0^a(v)\,\delta(\omega - \mathbf{k}\mathbf{v})\right]. \quad (27)$$

In general, the formulas obtained above allow us to compute the intensity for both relativistic and nonrelativistic cases. Inasmuch as the condition $v_\varphi \gg v_T$ is satisfied (v_φ is the phase velocity and v_T is the thermal velocity) for all waves considered below, which are nonrelativistic, in using (25) we can neglect terms containing $\delta(\omega - \mathbf{k}\cdot\mathbf{v})$: after integration over the equilibrium distribution function these terms make a contribution $\exp[-v_\varphi^2/v_T^2]$ times smaller than that of the other terms in the expression for the current correlations. Direct calculation shows that this procedure corresponds to neglecting the random forces in (22).

Thus, in this case it is convenient to compute the current correlation functions in the following way. The solution of (7) is of the form

$$f_1(\omega, \mathbf{k}) = -ie\frac{(\mathbf{E}_1\mathbf{v})\,\partial f_0/\partial\varepsilon}{\omega_0 - \mathbf{k}_0\mathbf{v} + i0}. \quad (28)$$

Neglecting the term with random forces in (22), substituting (22) and (27) in (20), and using (13) we write the current expression in the form

$$j_i = e^3 Q_{ikl} E_{1k} E_{2l}, \quad (29)$$

where

$$Q_{ikl} = \int \frac{v_i}{\omega - \mathbf{k}\mathbf{v} + i/\tau}\left\{\delta_{km} + \frac{1}{\omega_0}[k_{0m}v_k - \delta_{km}(\mathbf{k}_0\mathbf{v})]\right\} \times \frac{\partial}{\partial p_m}\frac{v_l f_0'(\varepsilon)}{\omega' - \mathbf{k}'\mathbf{v} + i/\tau}\,d\mathbf{v} \int \frac{v_i}{\omega - \mathbf{k}\mathbf{v} + i/\tau} \times \left\{\delta_{lm} + \frac{1}{\omega'}[k_m' v_l - \delta_{lm}(\mathbf{k}'\mathbf{v})]\right\}\frac{\partial}{\partial p_m}\frac{v_k f_0'(\varepsilon)}{\omega_0 - \mathbf{k}_0\mathbf{v} + i/\tau}\,d\mathbf{v}. \quad (30)$$

We note that (29) vanishes if spatial dispersion is neglected.

Lettint τ approach infinity and expanding the integrand in powers of $\mathbf{k}\cdot\mathbf{v}/\omega \sim v_T/v_\varphi \ll 1$, after integration we find

$$Q_{ikl} = \frac{4\pi}{3}\frac{A}{m}\Gamma_{ikl}, \quad A = \int_0^\infty v^4 f_0'(\varepsilon)\,dv,$$

$$\Gamma_{ikl} = \left\{\frac{1}{\omega\omega'}\left[\left(\frac{k_l'}{\omega'} + \frac{k_l}{\omega}\right)\delta_{ik} + \frac{k_l^0\delta_{lk} - k_i^0\delta_{ik}}{\omega^0} + \frac{k_k}{\omega}\delta_{il}\right] + \frac{1}{\omega\omega_0}\left[\left(\frac{k_k^0}{\omega^0} + \frac{k_k}{\omega}\right)\delta_{il} + \frac{k_i'\delta_{kl} - k_k'\delta_{il}}{\omega'} + \frac{k_l}{\omega}\delta_{ik}\right]\right\}. \quad (31)$$

In the case of a quadratic dispersion relation we have $A = -3N/4\pi m$.

Averaging the product of the currents (29) and using (24) we find the current spectral correlation function for a Maxwellian distribution ($r_0 = e^2/mc^2$):

$$(2\pi)^4 \langle j_i j_k^* \rangle_{\mathbf{k}\omega} = \frac{1}{2\pi}\frac{r_0}{mc^2}\Omega_e^4 \omega'^3 E_{1l} E_{1m}^* \psi_{pr}(\mathbf{k}', \omega')\,\Gamma_{ilp}\Gamma_{kmr}. \quad (32)$$

It is evident from (24) and (32) that the scattering exhibits a resonance. Using the fact that the imaginary part of the dielectric constant is small it is easy to show that the corresponding terms in (24) are proportional to $\gamma/[(\omega - \omega_s)^2 + \gamma^2]$ where γ is the damping due to the imaginary part of $\epsilon(k, \omega)$ and ω_s is the frequency of the scattered (converted) wave. For scattering on longitudinal waves we have

$$\gamma = \mathrm{Im}\,\varepsilon^l / |\partial\varepsilon^l/\partial\omega|_{\omega=\omega_s},$$

while for transverse waves

$$\gamma = \frac{\omega^2}{c^2}\frac{\mathrm{Im}\,\varepsilon^{tr}}{|\partial\varphi/\partial\omega|_{\omega=\omega_s}}, \qquad \varphi(k, \omega) = k^2 - \frac{\omega^2}{c^2}\varepsilon^{tr}(k, \omega).$$

5. Substituting (32) in (18) and making use of the fact that the attenuation is small we obtain a final expression for the spectral intensity of the electromagnetic waves produced by scattering of longitudinal waves on transverse waves:

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = \frac{\pi r_0}{12}|\mathbf{E}_1|^2\Omega_e^4\frac{s_e^2}{c^3}(\omega - \Omega_e)\,\omega^2\left|\frac{(\mathbf{k}'\mathbf{n})\,\omega}{c\sqrt{\omega^2 - \Omega_e^2}} - \frac{\omega - \Omega_e}{c^2}\right|^{-1} \times \sum_{s=1}^{2}\frac{\gamma}{(\omega - \omega_s)^2 + \gamma^2}(\delta_{ik} - n_i n_k)\,n_{0l}n_{0m}(\delta_{pr} - n_p' n_r')\,\Gamma_{ilp}\Gamma_{kmr}, \quad (33)$$

where

$$\mathbf{n}_0 = \frac{\mathbf{k}_0}{k_0}, \qquad \mathbf{n} = \frac{\mathbf{k}}{k}, \qquad \mathbf{n}' = \frac{\mathbf{k}'}{k'};$$

$$\omega_{1,2} = \frac{(\omega_0^2 - \alpha^2)\,\omega_0 \pm [\omega_0^2(\omega_0^2 - \alpha^2)^2 - 4(\omega_0^2 - \alpha^2\cos^2\theta)(^1/_4(\omega_0^2 - \alpha^2)^2 + \alpha^2\Omega_e^2\cos^2\theta)]^{1/2}}{2(\omega_0^2 - \alpha^2\cos^2\theta)}; \qquad \alpha^2 = c^2/s_e^2\,(\omega_0^2 - \Omega_e^2).$$

The analysis is rather complicated for arbitrary θ. We limit ourselves to the cases $\theta = \pi/2$ and $\theta = 0$. In the $\theta = \pi/2$ case the scattered wave is at a frequency $\omega = \Omega_e/2 - \alpha^2/2\Omega_e < \Omega_e$ so that conversion is forbidden. When $\theta = 0$, the frequency $\omega = \frac{1}{2}[\Omega_e + \alpha^2/\sqrt{\alpha^2 - \Omega_e^2}]$ and conversion is allowed if $\omega > \Omega_e$. This leads to the condition $\Omega_e/s_e \gg k_0 \gg \Omega_e/c$.

The electromagnetic waves due to scattering of longitudinal waves on longitudinal waves are characterized by an intensity

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = \frac{\pi}{\sqrt{3}} r_0 \Omega_e^6 \frac{s_e^2}{c} |\mathbf{E}_1|^2 \frac{\gamma}{(\omega - 2\Omega_e)^2 + \gamma^2}$$
$$\times n_l^0 n_m^0 n_p' n_r' (\delta_{ik} - n_i n_k) \Gamma_{ilp} \Gamma_{kmr}. \tag{34}$$

In similar fashion, using (13) and (32) we find the intensity of longitudinal waves due to scattering of transverse waves and longitudinal waves on transverse waves

$$\frac{1}{V}\frac{\partial I}{\partial\omega\partial o} = \frac{r_0 \Omega_e^6}{18\, s_e} \sum_{s=1}^{2} \sqrt{\varepsilon^l(\omega_s)} \left| \frac{\Omega_e}{k(\omega_s)} \frac{(\mathbf{k}'\mathbf{n})}{s_e^2} - \frac{\omega'}{c^2} \right|^{-1} \frac{\gamma}{(\omega - \omega_s)^2 + \gamma^2}$$
$$\times E_{1l} E_{1m}^* n_i n_k (\delta_{pr} - n_p' n_r') \Gamma_{ilp} \Gamma_{kmr}. \tag{35}$$

When the incident wave is a transverse wave $\omega_{1,2} = \Omega_e + \delta\omega_{1,2}$ where

$$\delta\omega_{1,2} = \left\{ \frac{s_e}{c} \frac{1}{\sqrt{2\Omega_e}} (\sqrt{\omega_0^2 - \Omega_e^2} \cos\theta \pm [\Omega_e^2 \sin^2\theta + \omega_0(\omega_0 \cos^2\theta - 2\Omega_e)^{1/2}) \right\}^2.$$

We note that conversion is impossible when $\theta = \pi/2$. When the incident wave is a longitudinal wave $\mathbf{E}_1 = E_1\mathbf{n}_0$ and the frequency shift is $\omega - \Omega_e = \delta\omega_{1,2}$ where

$$\delta\omega_{1,2} = \left(\sqrt{\omega_0 - \Omega_e} \pm \left[\frac{\Omega_e}{2}\frac{s_e}{c} - (\omega_0 - \Omega_e)\left(1 - \frac{1}{4}\cos^2\theta\right)\right]^{1/2}\right)^2.$$

The intensity of longitudinal waves due to scattering of transverse waves on longitudinal waves is

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = \frac{r_0 c^2}{18\, s_e} \Omega_e^6 \frac{\gamma}{(\omega - \Omega_e)^2 + \gamma^2}$$
$$\times \sqrt{\varepsilon^l(\omega)}\, E_{1l} E_{1m}^* \Gamma_{ilp} \Gamma_{kmr} n_p' n_r' n_i n_k. \tag{36}$$

As we have indicated, scattering of transverse waves on longitudinal waves has been treated in [8]. The corresponding expression for the intensity of the electromagnetic radiation can be obtained from (18), (24), and (32).

6. The interaction of low-frequency waves, the ion longitudinal wave and the acoustic wave, can be treated using the same technique as that used above for the high-frequency waves. After substitution of the appropriate quantities the interaction with ion longitudinal waves is described by the same formulas obtained above for the electron longitudinal waves.

In the case of low-frequency waves we need not consider spatial dispersion in (7) and (9) since the frequency shift is small (as indicated above we cannot neglect spatial dispersion completely). This corresponds to the condition $\mathbf{k}\cdot\mathbf{v}/\omega \ll \mathbf{k}'\cdot\mathbf{v}/\omega'$ i.e., $v_\varphi \gg v_\varphi'$ which is satisfied for low-frequency oscillations. Neglecting $\mathbf{k}\cdot\mathbf{v}$ compared with ω in the denominator of (20) and using (13) we write the current expression in the form

$$\mathbf{j}(\mathbf{k}, \omega) = -ie^2\mu^{-1}\omega^{-1}[\rho_2(\omega', \mathbf{k}')\,\mathbf{E}_1(\omega_0, \mathbf{k}_0)$$
$$+ \rho_1(\omega_0, \mathbf{k}_0)\,\mathbf{E}_2(\omega', \mathbf{k}')], \tag{37}$$

where

$$\rho_{1,2}(\omega', \mathbf{k}') = \mu(M\rho_{1,2}^e + m\rho_{1,2}^i)/(m + M),$$
$$\mu = mM/(m + M)$$

(the indices 1 and 2 refer to the incident and scattered wave respectively).

Consequently, the mean of the product of the currents is expressed in terms of the spectral correlations of the field fluctuations and the fluctuations in particle density. Using Poisson's equation for the field fluctuations and the expression for the spectral correlation function for the particle density given in [8] we find the current correlation

$$\langle j_i j_k^* \rangle_{\mathbf{k}\omega} = \frac{2}{\pi^2}\frac{e^2}{\mu^2} \sum_s \frac{|\mathbf{k}(\omega_s) - \mathbf{k}_0|^2}{\omega_s^2(\omega_s - \omega_0)} \frac{|\varkappa_e^l|^2}{|\partial\varepsilon/\partial\omega|_{\omega=\omega_s}} \frac{\gamma}{(\omega - \omega_s)^2 + \gamma^2}$$
$$\times \frac{T_e \operatorname{Im}\varkappa_e^l + T_i \operatorname{Im}\varkappa_i^l}{\varepsilon''} S_{ik}, \tag{38}$$

where

$$S_{ik} = A_{1i}A_{1k}^* + A_{2i}A_{2k}^* + A_{1i}A_{2k}^* + A_{1k}A_{2i}^*,$$
$$\mathbf{A}_1 = \frac{4\pi i e\mathbf{k}'}{k'^2}\frac{\rho_1}{\mu} - \frac{\mathbf{E}_1}{m}, \qquad \mathbf{A}_2 = -\frac{4\pi i e\mathbf{k}'}{k'^2}\frac{\rho_1}{\mu} + \frac{\mathbf{E}_1}{M}.$$

(Expressions for the susceptibility $\kappa_{e,i}$ are given in [7].)

Substituting (38) in (18) and (19) we determine the intensities of transverse and longitudinal waves due to conversion and scattering on low-frequency oscillations.

Scattering of electromagnetic waves on sound waves has also been studied in [8]. We note that the scattered wave spectrum contains an additional peak at the ion Langmuir frequency. Furthermore, radiation of transverse waves can occur by virtue of conversion of longitudinal waves in scattering on low-frequency oscillations. The radiation intensity for conversion of longitudinal electron waves into transverse waves in scattering on ion longitudinal waves is found to be

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = (2\pi)^4 \frac{c^2}{c^3} k'^2 \frac{\Omega_e}{\Omega_i} |\varkappa_e^l|^2 \frac{T_e \operatorname{Im}\varkappa_e^l + T_i \operatorname{Im}\varkappa_i^l}{\varepsilon''}$$

$$\times \frac{\gamma}{[\omega - (\Omega_e + \Omega_i)]^2 + \gamma^2} S_{ik} (\delta_{ik} - n_i n_k). \qquad (39)$$

Electromagnetic waves can also be radiated by scattering of low-frequency waves on high-frequency waves. For example, in scattering of the ion longitudinal wave on the electron longitudinal wave the radiation intensity is given by (34) where the resonance factor is of the form $\gamma/\{[\omega - (\Omega_e + \Omega_i)]^2 + \gamma^2\}$.

The intensity of longitudinal oscillations for scattering on low-frequency waves is given by

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = 4\pi e^2 \sum_s \frac{|\mathbf{k}(\omega_s) - \mathbf{k}(\omega_0)|^2}{\omega_s^2(\omega_s - \omega_0)} \frac{1}{|\partial\varepsilon'(k', \omega')/\partial\omega|_{\omega=\omega_s}} |4\pi\varkappa_e^l|^2$$

$$\times \frac{k^2(\omega_s)}{|\partial\varepsilon^l(k, \omega)/\partial k|} \cdot \frac{T_e \operatorname{Im}\varkappa_e^l + T_i \operatorname{Im}\varkappa_i^l}{\varepsilon''} \frac{\gamma}{(\omega - \omega_s)^2 + \gamma^2} S_{ik} n_i n_k. \qquad (40)$$

7. Conversion and scattering on free particles can be treated starting from the expression for the distribution function correlations. In this case we retain only the first term in (25). Using (21) and (25) and neglecting terms due to spatial dispersion in the denominator of the integrand we find

$$\langle j_i j_k^* \rangle_{\mathbf{k}\omega} = \left(\frac{2\pi}{3}\right)^{1/2} \frac{e^4}{m^2} \frac{N}{s_e} \frac{1}{k'\omega^2} E_{1i} E_{1k}^* \exp\left[-\frac{m}{2T}\left(\frac{\omega'}{k'}\right)^2\right], \qquad (41)$$

whence, using (18) and (19) we obtain an expression for the intensity of the electromagnetic radiation

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = \frac{\pi}{2}\left(\frac{2\pi}{3}\right)^{1/2} r_0^2 N \frac{c}{s_e} \frac{\sqrt{\varepsilon^{tr}(\omega)}}{k'}$$

$$\times \exp\left\{-\frac{m}{2T}\left(\frac{\omega'}{k'}\right)^2\right\} (|\mathbf{E}_1|^2 - |\mathbf{n}\mathbf{E}_1|^2), \qquad (42)$$

and for the intensity of longitudinal waves

$$\frac{1}{V}\frac{\partial I}{\partial\omega\,\partial o} = 2\pi\left(\frac{2\pi}{3}\right)^{1/2} \frac{e^4}{m^2} \frac{N}{s_e}$$

$$\times \sum_s \frac{\exp\left\{-\frac{m}{2T}\left(\frac{\omega - \omega_0}{k - k_0}\right)^2\right\} |\mathbf{E}_1 \mathbf{k}^{(s)}|^2}{|k_s^{(s)} - k_0| \, |\partial\varepsilon^l/\partial k|_{k=k^{(s)}}} \frac{1}{\omega^3(k^{(s)})}. \qquad (43)$$

8. Another mechanism for scattering and conversion, in addition to thermal fluctuations, is plasma turbulence.

We consider conversion of high-frequency waves on turbulent fluctuations. Since the frequencies of the turbulent oscillations are small compared with the frequency of the incident wave we can neglect the frequency change in conversion. Starting with the linearized hydrodynamic equations and taking account of the situation indicated above it can be shown that because the velocity of sound is small compared with the phase velocity of the incident wave the current expression becomes

$$\mathbf{v}_3 = e\rho_2 \mathbf{v}_1, \qquad \mathbf{v}_1 = -i\frac{e}{m\omega_0} \mathbf{E}_1 e^{i\omega_0 t}. \qquad (44)$$

Averaging the product of the currents and using the equation of motion we have

$$\langle j_i j_k^* \rangle_{\mathbf{k}\omega} = \frac{e^4}{m^2\omega^2} E_{1i} E_{1k}^* \langle |\rho_2^2| \rangle_{\mathbf{k}\omega}. \qquad (45)$$

If the turbulence is uniform and isotropic the mean square density given by Villars and Weisskopf [10] can be used; in this case (45) becomes

$$\langle j_i j_k^* \rangle_{\mathbf{k}\omega} = \frac{e^4}{m^2 s_0^4} \frac{\rho_0^2}{\omega^2} \left(\frac{S_0}{\rho_0}\right)^{4/3} \frac{1}{|\mathbf{k} - \mathbf{k}_0|^{4/3}} E_{1i} E_{1k}^*, \qquad (46)$$

where $S_0 = \rho_0 v_0^3/L_0$ is the energy flux corresponding to the largest-scale fluctuations L_0, $s_0 = \sqrt{T/M}$.

Equation (46) applies in the equilibrium region. The scattering of electromagnetic waves on turbulent fluctuations has been studied in [10]. The corresponding result can be obtained from (18) and (14).

The radiation intensity of electromagnetic waves produced by conversion of longitudinal waves is found to be

$$\frac{1}{V}\frac{\partial I}{\partial\omega\partial o} = \frac{(2\pi)^5}{4} \frac{e^4}{m^2 s_0^4} \frac{\rho_0^2}{c^3} \left(\frac{S_0}{\rho_0}\right)^{4/3} \frac{\sqrt{\varepsilon^{tr}}}{|\mathbf{k} - \mathbf{k}_0|^{4/3}} |E_1^2| \sin^2\theta. \qquad (47)$$

Similarly, the intensity of the longitudinal waves is given by

$$\frac{1}{V}\frac{\partial I}{\partial\omega\partial o} = \frac{(2\pi)^5}{4} \frac{e^4}{m^2 s_0^6} \rho_0^2 \left(\frac{S_0}{\rho_0}\right)^{4/3} \frac{\sqrt{(\omega^2 - \Omega_e^2)m}}{T} \frac{|\mathbf{E}_1 \cdot \mathbf{n}|^2}{|\mathbf{k} - \mathbf{k}_0|^{4/3}}. \qquad (48)$$

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