ANALYTIC PROPERTIES OF PARTIAL AMPLITUDES

Yu. M. MALYUTA

Physics Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor April 23, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1397-1399 (October, 1962)

The diagram technique is employed to determine the nearest singular points of partial wave amplitudes in $\pi\pi$ scattering.

1. INTRODUCTION

ACCORDING to Polkinghorne and Screaton[1] and Taylor and Warburton[2] the singular points of the partial wave amplitudes

$$A_l(k^2) = \int_{-1}^{1} d\cos\theta A(k^2, \cos\theta) P_l(\cos\theta)$$

(where k is the magnitude of the three-dimensional momentum of the pion in the barycentric frame and θ is the scattering angle in the same frame) are determined in perturbation theory by the following system of equations:

$$\Sigma \pm p_j + \Sigma \pm q_i = 0, \quad \text{(1a)}$$

$$q_i^2 = m_i^2, \quad \text{(1b)}$$

$$\sum_C \pm \alpha_i q_i = 0, \quad \text{(1c)}$$

$$\Sigma \alpha_i = 1, \qquad \alpha_i > 0, \quad \text{(1d)}$$

$$\cos\theta = \pm 1, \quad \text{(1e)}$$

where p_j is an external momentum, q_i is the momentum of the i-th internal line, m_i is the mass of the i-th internal line, and α_i is a Feynman parameter.

In this work we will show for the reaction of pions scattering on pions that the condition (1e) makes possible the application of the majorization method of Symanzik[3-5] for the determination of the nearest singular points of the partial wave amplitudes $A_l(k^2)$, since under that condition the set of external momenta p_j becomes Euclidean.

2. DETERMINATION OF PRIMITIVE DIAGRAMS

Let us restrict the discussion to the scalar version of the pion theory with vertices of the four-point type. We consider the diagram for $\pi\pi$ scattering shown in Fig. 1. Let p_1, p_2 be the initial, and $-p_3$, $-p_4$ the final, momenta. Then the invariants on which the amplitude A depends may be written in the barycentric frame in the following form

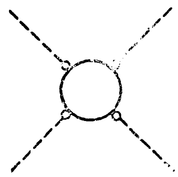

FIG. 1

$$p_1^2 = p_2^2 = p_3^2 = p_4^2 = \mu^2,$$

$$s = 4(k^2 + \mu^2), \qquad t = -2k^2(1 - \cos\theta),$$
$$u = -2k^2(1 + \cos\theta), \quad \text{(2)}$$

where μ is the pion mass.

After constructing Gram determinants out of the vectors p_1, p_2, p_3, and p_4 and using Eqs. (1e) and (2) one sees easily that the set of vectors p_1, p_2, p_3, and p_4 is Euclidean[1)] if

$$-\mu^2 \leqslant k^2 \leqslant 0. \quad \text{(3)}$$

Let us pass to the determination of primitive diagrams. The majorization rules necessary for what follows are shown in Fig. 2.[3] It follows from the strong connectivity of the diagram, Fig. 1, from the four-point structure of the vertices, and from the continuity of the pionic lines, that the diagram of Fig. 1 may be majorized (according to Fig. 2a) by the primitive diagrams shown in Fig. 3.

Since the diagrams of Fig. 3 are still too complicated for direct study we majorize them (according to Fig. 2b) by the "box" diagram (Fig. 4) with artificial vertices of the three-point type.

3. DETERMINATION OF THE NEAREST SINGULAR POINTS

To study the diagram of Fig. 4 we make use of the Taylor-landau[6,7] method of dual diagrams. In agreement with Eqs. (1) and (2) we construct

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1)A set of vectors is Euclidean if the Gram determinants constructed from these vectors are nonnegative.

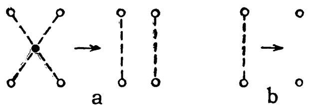

FIG. 2

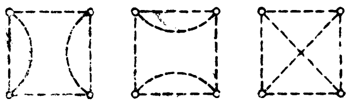

FIG. 3

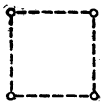

FIG. 4

for the diagram of Fig. 4 the dual diagram shown in Fig. 5. The dual diagram, Fig. 5, constructed to scale determines no singular points in the Euclidean region (3), since at its internal vertex O the stability condition (1c) is not satisfied. We therefore pass to the study of the reduced diagrams, obtained from Fig. 4 by collapsing to a point corresponding lines.

The reduced diagrams of the "triangle" type (Fig. 6) also determine no singular points in the Euclidean region (3), since at the internal vertices O of the corresponding dual diagrams (Fig. 7) the stability condition (1c) is not satisfied. We consider next reduced diagrams of the "loop" type (Fig. 8). Solving for these diagrams Eq. (1) with Eq. (2) taken into account we find for the location of the nearest singular points of the partial wave amplitudes $A_l(k^2)$:

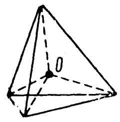

FIG. 5

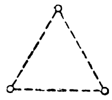

FIG. 6

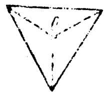

FIG. 7

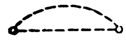

FIG. 8

$$k_-^2 = -\mu^2, \qquad k_+^2 = 0.$$

The author is grateful to N. N. Bogolyubov for valuable discussion of this work.

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Translated by A. M. Bincer
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