EFFECT OF INTERNAL THERMAL MOTION ON THE POLARIZABILITY OF PLASMOIDS

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A study is made of the resonance properties of restricted plasma entities; these properties are due to the oscillatory nature of the internal thermal motion of electrons. As an illustration, the interaction of an alternating field is considered with a plasma localized in a one-dimensional rectangular potential well.

In studying the possibilities of controlling the motion of plasmoids by means of alternating electromagnetic fields [1-5] it is assumed that the displacement of the particles in the plasmoid during one period of the high-frequency oscillation is small compared with both the length characterizing the nonuniformities in the field and with the dimensions of the plasmoid itself. By the same token, any influence of the internal thermal motion on the polarization properties of the plasma, apart from that associated with particle collisions, is effectively ignored. However, even in the absence of collisions in the plasma, this motion can be a very important factor which can influence the efficiency of interaction of a plasmoid with an external field.

The character of the thermal motion of electrons in a bounded plasma is chiefly determined by the structure of the force fields proposed for achieving stability of the given plasma configuration. The stabilizing action of these fields, as a rule, amounts to the creation of some kind of effective potential well for the charged particles. For example, such potential wells (at least for some directions of motion) are formed in so called magnetic bottles with constant or rotating fields and also in certain configurations of high frequency fields. In addition a potential well for electrons can be formed by the plasma Coulomb self field produced by perturbations of the quasi-neutrality at its boundary.

Since electrons localized in a potential well execute a finite oscillatory motion within it, they can be considered as a given aggregate of oscillators[1)] which is specified by a certain distribution of natural frequencies. If the most probable frequency of this distribution, Ω, is near the frequency of the external field, then it is reasonable to expect that the interaction of the field with the plasmoid will have a distinctly resonant character.

It is clear that the most strongly marked resonance phenomenon occurs in the case where the potential well is parabolic in form (in such circumstances all the oscillators are linear and have the same natural frequency). Since, however, most proposed methods of obtaining stable plasmoids depend on the use of stabilizing fields over only a comparatively narrow boundary region, it is of interest to consider potential wells which are approximately rectangular in shape.

The polarizability of a plasmoid localized in such a well is investigated below in the simplest case. We consider the interaction of an external quasi-uniform electric field, $\mathbf{E} = \mathbf{x}_0 E e^{i\omega t}$, with a plasma isolated between two parallel planes $x = 0$ and $x = L$ (the solution of this one dimensional problem can easily be generalized to the case of a three dimensional "potential well"). We assume that on the boundary planes the electron distribution function satisfies the boundary conditions of specular reflection

$$f(\dot{x}) = f(-\dot{x}). \tag{1}$$

The possibility of setting up such a boundary condition needs to be treated with reservation. Although specular reflection and the contrasting condition of diffuse reflection are often used in the study of the properties of a semi-bounded plasma [8,11], they are strictly speaking not applicable in actual typical cases, since they imply the production of sharp decreases in the plasma density over distances ΔL, which are much smaller than the Debye radius $D \approx v_m/\omega_0$ (v_m is the most prob-

[1)]It is interesting to note that such oscillations are completely analogous to those used in electronic devices similar to the Barkhausen-Kurtz generator. On this basis, the study of the oscillating properties of bounded plasma objects (see also[7]) is of definite interest from the point of view of possible applications in high-frequency electronics.

able thermal velocity of the electrons, and ω_0 is the plasma frequency). In actual fact, "instantaneous" reflection at the boundary requires that the inequality $\Delta L/v_m \ll 1/\omega$ be satisfied, which for $\omega_0 \lesssim \omega$ lead to $\Delta L \ll D$, while in the case when $\omega_0 \gg \omega$ and $\Delta L \approx D$, it may be necessary to take the currents and charges within the boundary layer itself into account. As a consequence, it should be borne in mind that results obtained by using the stated boundary condition are only illustrative in nature and give only a qualitative idea of the role of the boundary effects.

For simplicity it is assumed further that 1) the particle number density in the plasma is low ($\omega_0 \ll \omega$), so that the external field is almost undisturbed by it; 2) the electron collision frequency $\nu \ll \omega$; 3) the amplitude of the field E is small enough so that the problem can be solved in a linear approximation.

The kinetic equation for the non-equilibrium term $f_1 e^{i\omega t}$ which is added to the stationary electron distribution function f_0 is

$$i\omega f_1 + \dot{x}\frac{\partial f_1}{\partial x} + \frac{e}{m}E\frac{\partial f_0}{\partial \dot{x}} = -\nu f_1$$

(e is the charge and m the mass of the electron), and has as a solution satisfying boundary condition (1) the function

$$f_1 = \frac{eE}{im\tilde{\omega}}\frac{\partial f_0}{\partial \dot{x}}\left\{\frac{1}{\sin(\tilde{\omega}L/\dot{x})}\exp\left(-\frac{i\tilde{\omega}x}{\dot{x}}\right)\left[\exp\left(\frac{i\tilde{\omega}L}{\dot{x}}\right) - 1\right] - 1\right\}$$

$$\tilde{\omega} = \omega - i\nu.$$

From this result, it is not difficult to find the current density averaged over the range $0 \le x \le L$*

$$j_{av} = \tilde{\sigma}E, \qquad \tilde{\sigma} = \frac{e^2}{im\omega}\left(N + \int\limits_{-\infty}^{+\infty}\frac{2\dot{x}^2}{\omega L}\frac{\partial f_0}{\partial \dot{x}}\,\mathrm{tg}\,\frac{\tilde{\omega}L}{2\dot{x}}\,d\dot{x}\right),$$

where N is the equilibrium electron number density. Taking into account the fact that the function under the integral sign has singularities near the real axis ($\nu \ll \omega$) and assuming that the distribution function f_0 is Maxwellian, we obtain the following expressions for the equivalent dielectric constant

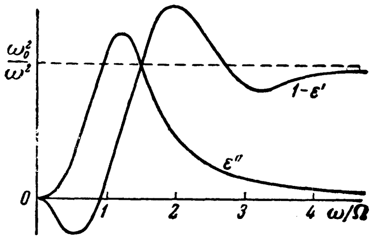
$$\varepsilon = 1 + 4\pi\tilde{\sigma}/i\omega = \varepsilon' - i\varepsilon'';$$

$$\varepsilon' = 1 + \frac{16}{\pi^2}\frac{\omega_0^2}{\Omega^2}\sum_{n=0}^{\infty}(2n+1)^{-4}G_{2n+1}\left(\frac{\omega}{\Omega}\right),$$

$$\varepsilon'' = \frac{16}{\pi^{3/2}}\frac{\omega_0^2\omega}{\Omega^3}\sum_{n=0}^{\infty}(2n+1)^{-5}\exp\left(-\frac{\omega^2}{\Omega^2(2n+1)^2}\right);$$

$$G_n(\xi) = 1 - 2\frac{\xi}{n}e^{-(\xi/n)^2}\int\limits_0^{\xi/n}e^{z^2}dz, \qquad \Omega = \pi\frac{v_m}{L}.$$

As is evident from the plots of the dependence of ϵ' and ϵ'' on ω/Ω, for a fixed ratio ω_0/ω,



$\epsilon' - 1$ changes sign near the point $\omega/\Omega = 1$, and ϵ'' reaches its maximum value of $1.2\,\omega_0^2/\omega^2$. The example considered shows that even in the case where the natural oscillations of the electrons in the plasmoid are markedly different from harmonic and have a frequency spectrum extending over a wide range, the resonance phenomena associated with these oscillations are fairly pronounced and can be used in particular for controlling the phase difference between the current in a plasmoid and the external field. As a result, additional possibilities for increasing the efficiency of so-called radiation methods of plasma acceleration become apparent.

Furthermore, the resonance effects considered can be of importance in the scattering of radio waves from various kinds of ionization disturbances in the atmosphere. The possibility of using the results obtained for studying the dispersion properties of certain long-wave transmission lines filled with plasma should also be noted.

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*tg = tan.

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