SCATTERING OF GAMMA QUANTA AND POLARIZABILITY OF NUCLEI AND NUCLEONS

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A definition of the polarizability of atomic nuclei and nucleons, based on the dispersion relations and the spin structure of the γ-ray scattering amplitude, is discussed. An estimate of the magnetic polarizability of the proton is obtained. The applications of crossing symmetry requirements are indicated for the polarizability of nuclei with arbitrary spins.

MIGDAL[1] was apparently the first to use the notion of polarizability of nuclear matter in the analysis of the scattering of low-energy gamma quanta by atomic nuclei. Later on Levinger[2], Ramsey, Malenka, and Kruse[3], and recently Baldin[4,5] analyzed various aspects of the notion of polarizability of nuclei. Migdal[1] gave general formulas for the polarizability of complex nuclei in the spin-free case. Levinger and Rustgi[6] examined in 1957 the then available experimental data on polarizability, and Levinger[7] made a detailed comparison of Migdal's formulas with the experimental data. Most recently Baldin[4,5] advanced an idea that the polarizability of atomic nuclei has an anisotropic character.

On the other hand, the concept of nucleon polarizability was introduced in connection with experiments on the scattering of slow neutrons by nuclei, and also in connection with the analysis of photoproduction of pions by nucleons and the scattering of gamma quanta by nucleons (see [8-12]).

In most papers the introduction of the electric polarizability is related with the classical analogy. Then, using the connection between the polarizability α_e and the mean value of the dipole moment, one arrives at the formula

$$\alpha_e = \frac{\hbar c}{2\pi^2}\int \frac{\sigma_{E1}(\nu)}{\nu^2}\, d\nu \tag{1}$$

and the analogy with Rayleigh scattering is made.

In the analysis of photon scattering one can start directly from the spin structure of the Compton-effect amplitude (see for example[13-14] for particles with spin ½ and[15] for scattering of gamma quanta by spin-1 particles) and the polarizability can be regarded, along with the charge and the magnetic and higher moments, as a quantity characterizing the limiting value of the amplitude of two-photon particle interaction. The use of dispersion relations permits an estimate of the polarizability of nucleons and nuclei when the partial cross sections for gamma absorption are known over a sufficiently wide range of energies.

A group at the Physics Institute of the Academy of Sciences[16] recently obtained, in the course of an experimental investigation of the proton Compton effect, experimental data on the electric and magnetic polarizabilities of the proton. From the way the polarizability is estimated in these experiments, it can be likewise formally defined in the general case as the coefficient of the square of the radiation frequency in the amplitude R_1 (α_e—electric polarizability) or in the expansion of R_2 (α_m—magnetic polarizability). These definitions are suggested by the character of the states in which the photon absorption predominates for each of the aforementioned amplitudes. In the absorption of the dipole gamma quanta the only states contributing to R_1 are of the electric type, while those contributing to R_2 are magnetic. In spite of the fact that in most states, as can be seen from the unitarity relations [see (7) in [14] and (22) in [15]], for spin values ½ and 1, magnetic states also contribute to the amplitude R_1, as do electric states to R_2, we retain for α_e and α_m the designations electric and magnetic polarizability. Although we do not make use here of the classical analogy, it can be noted that these definitions are convenient for an analysis of scattering and for symmetry considerations (see the following remarks on crossing symmetry and the polarizabilities).

The dispersion relations for the amplitude $R_1 + R_2$ yield for the sum of the electric and magnetic polarizabilities $\alpha_e + \alpha_m$

$$\alpha_e + \alpha_m = \lim_{\nu_0^2\to 0} \frac{d}{d\nu_0^2}(R_1 + R_2) = \frac{\hbar c}{2\pi^2}\int_{\nu_t}^{\infty} \frac{\sigma_t(\nu)}{\nu^2}\, d\nu, \tag{2}$$

where ν_t is the threshold of the inelastic processes (photonuclear processes for atomic nuclei and pion photoproduction for nucleons). This expression differs from the one customarily given in that the integral in the total absorption cross section contains contributions from all states, and not only from dipole absorption. For nuclei where the dipole absorption is the principal one, the definition introduced above for the polarizability does not contradict in practice the definition connected with the mean value of the dipole moment. In the case of nucleons, the term quadratic in the frequency in $R_1 + R_2$ is determined by the contribution of all the states to the pion photoproduction cross section, although the production of pions in the s state, connected with the electric dipole absorption, plays an important role. We note the interesting fact that definition (2) does not depend on the number of subtractions in the dispersion relations.

The magnetic polarizability of the nuclei, if we define it as above, is much smaller than the electric polarizability. For the deuteron this follows from the known formulas for the cross sections of the dipole electric and magnetic deuteron splitting at low energies. For the electric polarizability of the deuteron we have

$$\alpha_e = \frac{1}{32}\frac{e^2\hbar^2}{M\varepsilon^2}(1-\gamma r_0)^{-1} = 0.64\cdot 10^{-39}\ \text{cm}^3 \quad (3)$$

The magnetic polarizability of the deuteron is

$$\alpha_m = \frac{e^2}{Mc^2}\left(\frac{\hbar c}{\varepsilon}\right)^2 \frac{1}{12}\left(1+\sqrt{\frac{\varepsilon'}{\varepsilon}}\right)\frac{\varepsilon}{M_p c^2}(\mu_p-\mu_n)^2 \quad (4)$$

(the notation is the same as in [15]). A comparison of (3) and (4) shows that $\alpha_m \ll \alpha_e$. The magnetic polarizability of heavier nuclei is smaller because of the relatively small role of magnetic transitions in photonuclear processes.

To determine the numerical values of the polarizabilities it is very desirable to set up experiments on the determination of the total absorption cross sections over a wide range of energies, and to analyze the absorption data so as to obtain information on the partial cross sections. For protons, Gol'danskiĭ et al [16] reached the conclusion that the electric polarizability of the protons is

$$\alpha_e = (9 \pm 2)\cdot 10^{-43}\ \text{cm}^3$$

and the magnetic polarizability amounts to

$$\alpha_m = (2 \pm 2)\cdot 10^{-43}\ \text{cm}^3$$

On the basis of the dispersion relations, Gol'danskiĭ et al [16] and Baldin obtained for the sum of the electric and magnetic polarizabilities of the proton the estimate

$$\alpha_e + \alpha_m = 11\cdot 10^{-43}\ \text{cm}^3 \quad (5)$$

There are no known estimates for the magnetic polarizability of protons, similar to that given above. An analysis of the amplitudes of γp forward scattering, made in [14], yields the magnetic polarizability of the protons without any additional manipulations. For this purpose it is sufficient to find the term quadratic in the frequency in the expansion of the real part of the amplitude R_2.

Recognizing that by its very nature the nucleon polarizability is determined by the principal term of an expansion in the form $(\nu/\nu_t)^2$, and not $(\nu/M)^2$, which is connected with recoil effects, and considering the dispersion relations for both $R_1 + R_2$ and $R_1 - R_2$, we obtain

$$\alpha_m = \frac{2}{\pi}\int_{\nu_t}^{\infty}\frac{d\nu}{\nu^2} \times \left\{|M_1|^2 + 2|M_3|^2 + \frac{1}{3}|M_2|^2 - \frac{1}{6}|E_2|^2\right\} \quad (6)$$

for the magnetic polarizability, and

$$\alpha_e = \frac{2}{\pi}\int_{\nu_t}^{\infty}\frac{d\nu}{\nu^2}\left\{|E_1|^2 + 2|E_3|^2 + \frac{1}{3}|E_2|^2 - \frac{1}{6}|M_2|^2\right\} \quad (7)$$

for the electric one. The photoproduction amplitudes have been determined in [14].

From (6) and (7) we see once more that $\alpha_m(\alpha_e)$ contains along with magnetic (electric) amplitudes also those of the electric (magnetic) absorption. Although in general only the sum $\alpha_e + \alpha_m$ is a positive quantity for the photoproduction of pseudoscalar particles on a fermion of spin $1/2$, for a proton the presence of the small contributions $|E_2|^2$ and $|M_2|^2$ cannot really alter the positive nature of α_e and α_m individually.

If we consider pion photoproduction in the resonant p state only, we arrive at the estimate

$$\alpha_m \approx 2\cdot 10^{-43}\ \text{cm}^3,$$

with the aid of which we obtain from (5)

$$\alpha_e \approx 9\cdot 10^{-43}\ \text{cm}^3. \quad (9)$$

Both estimates are close to the experimental data. To improve the reliability of the agreement of (8) and (9) with the experimental data, it may be necessary to use a more thorough statistical reduction.

The use of the data assumed in [14] for the photoproduction of pions in the s state leads to the following estimate for the electric polarizability

$$\alpha_e > 4.2\cdot 10^{-43}\ \text{cm}^3$$

which coincides with the results of Baldin and Foldy [11].

In estimates of the lowest terms quadratic in the frequency of the amplitudes analogous to R_1, and in the case of arbitrary values of the spins, it is interesting to use the crossing symmetry requirements. As is well known, one introduces along with the c.m.s. amplitudes R_i also invariant amplitudes (designated T_i in [17,18]), which by virtue of the crossing symmetry have simple properties when the frequency ν is replaced by $-\nu$. From the relations between the amplitudes R_i and T_i [formula (1) in [18]] it follows, for example, that the expansion of the real part of the quantity $R_3 - R_4$ for spin-$1/2$ particles should contain, apart from recoil effects, only odd powers of ν/ν_t.

For spin-1 particles, neglecting recoil effects, the amplitudes A, B, C, and D, (determined, for example, in [15]) also have a definite parity relative to the substitution $\nu \rightarrow -\nu$, so that the crossing symmetry requirements lead, for example, to a vanishing of the vector polarizability, if one attempts to define the latter as the lowest coefficient of ν^2 in the amplitude B. Analogous considerations will be useful in the development of a scattering theory for the scattering of gamma quanta by atomic nuclei with arbitrary spins.

It is clear that the polarizability can characterize elastic scattering of gamma quanta by nucleons and nuclei only at energies much lower than the threshold energies of the inelastic processes. The data obtained in the energy region above the threshold of the inelastic processes should be analyzed in greater detail with the aid of the dispersion relations. They do not reduce to polarizability alone. From this point of view there is hardly justification for the analysis, in terms of polarizability, of the data on the scattering of gamma quanta by nuclei in the recently published paper [19].

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Note added in proof (15 September 1962). After this article went to press, Fedyanin published a paper containing an estimate of the magnetic polarizability of the proton[20].

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234