ON THE THEORY OF THE ABSORPTION OF ULTRASOUND IN SUPERCONDUCTORS

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The peculiarities are investigated of ultrasonic absorption in superconductors, which are associated with two mechanisms of phonon scattering: absorption of the phonon by an electron and the decay of the phonon into a pair of quasi-particles. It is shown that the derivative of the absorption coefficient with respect to the temperature is finite at the critical point. In the isotropic model, the absorption coefficient undergoes a finite jump below the transition temperature; however, this jump is absent in an anisotropic superconductor. For high ultrasonic frequencies, the absorption coefficient increases with decrease in temperature.

THE absorption of ultrasound in superconductors was first investigated theoretically by Bardeen, Cooper, and Schrieffer.[1] In the case $\hbar\omega \ll kT$ (ω is the ultrasonic frequency, k is the Boltzmann's constant, T is the absolute temperature), and in the isotropic model the ratio of the absorption coefficient in the superconducting and normal states of the metal is simply related with the value of the energy gap Δ in the excitation spectrum:

$$\frac{\gamma_s}{\gamma_n} = 2\,[e^{\Delta/kT} + 1]^{-1}.$$

This dependence is obtained neglecting the decay of the phonons into a pair of quasi-particles, which make a contribution only of the order of $\hbar\omega/kT$ relative to the fundamental effect (absorption of the phonon by an electron) and differs from zero in a narrow range of temperatures close to the transmission temperature.[2]

Pokrovskiĭ[3] has discovered that if $T = 0$ the ultrasonic absorption is determined only by the decay of the phonon into two quasi-particles, which is possible only for frequencies above the threshold. The threshold frequency in the isotropic case is equal to $\omega_{th} = 2\Delta/\hbar$, where the absorption at the threshold point achieves its final value by a jump. In anisotropic superconductors at $T = 0$, the ultrasonic absorption close to threshold falls off to zero ($\gamma_s \sim \sqrt{\omega - \omega_{th}}$), while the threshold frequency is a function of the direction of propagation of the sound.[4]

It will be shown below that the threshold phenomena take place also at finite temperatures. In the isotropic model, the absorption coefficient changes discontinuously upon decrease in temperature at a value of T determined by $\hbar\omega = 2\Delta(T)$. In the anisotropic case, the $\gamma_s(T)$ and $\gamma_s(\omega)$ curves have vertical tangents on the right at the threshold point ($T > T_{th}$ and $\omega > \omega_{th}$). Experimental investigation of the threshold absorption for $T \neq 0$ can be used to establish the angular and temperature dependence of the energy gap.

At high frequencies ($\hbar\omega > 2\Delta(0)$), decay of the phonon into two quasi-particles is possible at any temperature, and plays a fundamental role in the ultrasonic absorption. In this case, the absorption coefficient increases upon decrease in temperature. In the limit $\hbar\omega \gg 2\Delta(0)$, the absorption coefficient does not depend on the temperature and is not changed in the superconducting transition ($\gamma_s/\gamma_n = 1$).

The derivative $d\gamma_s/dT$, computed by the formula of Bardeen, Cooper, and Schrieffer,[1] is infinite at the critical point. Other authors have come to the same conclusion.[2,5] Account of the anisotropy[6,7] does not change this result. The experimental investigation of ultrasonic absorption in superconductors[8] has apparently shown that $d\gamma_s/dT$ is finite at the critical point and that the absorption coefficient falls off in the immediate vicinity of the transition temperature more slowly than according to the Bardeen-Cooper-Schriefer formula. Bezuglyĭ, Galkin, and Korolyuk[8] have associated this with the existence of an anisotropy in the critical temperature, but this explanation contradicts the assumption of the theory of second-order phase transitions, and also the conclusions of the microscopic theory of anisotropic superconductors.[9]

We shall show that the finite value of the derivative $d\gamma_s/dT$ is a consequence of the general theory of superconductivity and is obtained upon correct account of the small contributions from the decay of the phonon into a pair of quasi-particles.

1. We proceed to the quantitative consideration of the problem. We write the Hamiltonian of the interaction of the electrons with the phonons in the Fröhlich form:

$$H_{int} = g \sum_{\mathbf{p},\sigma,\mathbf{q}} \left(\frac{\hbar\omega_{\mathbf{q}}}{2V}\right)^{1/2} (a^+_{\mathbf{p+q},\sigma} a_{\mathbf{p}\sigma} b_{\mathbf{q}} + a^+_{\mathbf{p-q},\sigma} a_{\mathbf{p}\sigma} b^+_{\mathbf{q}}). \quad (1)$$

$a^+_{p\sigma}$, $a_{p\sigma}$; b^+_q, b_q are the creation and annihilation operators of an electron with spin projection σ and momentum $\mathbf{p}$, and of a phonon with momentum $\mathbf{q}$, V is the volume of the crystal and g is the coupling constant.

We introduce the creation and annihilation operators of the excitations by the canonical transformation of Bogolyubov:[10]

$$a_{p+} = u_p \alpha_{p0} + v_p \alpha^+_{p1}, \quad a_{-p,-} = u_p \alpha_{p1} - v_p \alpha^+_{p0};$$
$$u^2_p = \tfrac{1}{2}(1 + \xi/\varepsilon), \quad v^2_p = \tfrac{1}{2}(1 - \xi/\varepsilon) \quad (2)$$

(ξ is the energy of the electron in the normal state, calculated from the Fermi energy, and $\epsilon = \sqrt{\Delta^2 + \xi^2}$ is the energy of interaction).

To determine the probability of absorption of a sound quantum, we calculate the squares of the matrix elements for the transitions $\alpha^+\alpha b$ (absorption of a phonon by the electron) and $\alpha^+\alpha^+ b$ (decay of the phonon into a pair of quasi-particles), and also for the reverse transitions. The ultrasonic absorption coefficient calculated by this method is given by

$$\gamma_s = \frac{\pi g^2\omega}{(2\pi\hbar)^3} \int d\mathbf{p} \left\{ \left(1 + \frac{\xi\xi' - \Delta\Delta'}{\varepsilon\varepsilon'}\right)(f - f')\,\delta(\varepsilon' - \varepsilon - \hbar\omega) + \frac{1}{2}\left(1 - \frac{\xi\xi' - \Delta\Delta'}{\varepsilon\varepsilon'}\right)(1 - f - f')\,\delta(\varepsilon + \varepsilon' - \hbar\omega)\right\}. \quad (3)$$

Here the unprimed quantities have the argument $\mathbf{p}$ while the primed quantities have the argument $\mathbf{p+q}$; $f = (e^{\epsilon/kT} + 1)^{-1}$. The delta-function in the integrand of the expression corresponds to the laws of conservation of energy $\epsilon' - \epsilon = \hbar\omega$ (the transition $\alpha^+\alpha b$) and $\epsilon' + \epsilon = \hbar\omega$ (for the transition $\alpha^+\alpha^+ b$).

We transform to the variables ξ, ξ' and φ (φ is an angle in the plane $\mathbf{p}\cdot\mathbf{q} = 0$):

$$d\mathbf{p} = m^2 q^{-1} d\xi\, d\xi'\, d\varphi; \quad (4)$$

m is the mass of the "normal" electron. Integrating over φ, we get

$$\gamma_s = a \iint d\xi\, d\xi' \left\{ \left(1 + \frac{\xi\xi' - \Delta\Delta'}{\varepsilon\varepsilon'}\right)(f - f')\,\delta(\varepsilon' - \varepsilon - \hbar\omega) + \frac{1}{2}\left(1 + \frac{\Delta\Delta'}{\varepsilon\varepsilon'}\right)(1 - f - f')\,\delta(\varepsilon + \varepsilon' - \hbar\omega)\right\}; \quad (5)$$

$$a = 2\pi^2 g^2 \omega m^2 / q\,(2\pi\hbar)^3. \quad (6)$$

The region of integration in the plane $\xi\xi'$ is limited by the curves

$$\xi' = \xi + q^2/2m \pm p(\xi)\, q/m. \quad (7)$$

The line $\epsilon + \epsilon' - \hbar\omega = 0$ for any value of Δ is located wholly in this region and is symmetric relative to the axes ξ and ξ'. The line $\epsilon' - \epsilon = \hbar\omega$ does not possess symmetry in the limits of the region of integration. Therefore, the odd terms, which are proportional to $\xi\xi'$, are omitted in the second component but remain in the first [compare (3) and (5)].

Integrating with account of the δ-functions, we get

$$\gamma_s = 4a \left\{ \int_\Delta^\infty d\varepsilon \frac{\varepsilon(\hbar\omega + \varepsilon) - \Delta^2}{\sqrt{\varepsilon^2 - \Delta^2}\sqrt{(\hbar\omega + \varepsilon)^2 - \Delta^2}} [f(\varepsilon) - f(\hbar\omega + \varepsilon)] + \frac{1}{2}\int_b^\infty d\varepsilon \left[1 - \frac{\varepsilon(\hbar\omega + \varepsilon) - \Delta^2}{\sqrt{\varepsilon^2 - \Delta^2}\sqrt{(\hbar\omega + \varepsilon)^2 - \Delta^2}}\right] \times [f(\varepsilon) - f(\hbar\omega + \varepsilon)] + \int_\Delta^{\hbar\omega/2} d\varepsilon \frac{\varepsilon(\hbar\omega - \varepsilon) + \Delta^2}{\sqrt{\varepsilon^2 - \Delta^2}\sqrt{(\hbar\omega - \varepsilon)^2 - \Delta^2}} \times [1 - f(\varepsilon) - f(\hbar\omega - \varepsilon)]\right\}. \quad (8)$$

Here

$$b \approx qv_0/2 + \Delta^2/qv_0, \quad (9)$$

v_0 is the velocity of the electron on the Fermi surface. The expression for b is written in the approximation $\Delta/qv_0 \ll 1$. In the opposing case, the explicit form of b is not needed. The decay of the phonon into two quasi-particles corresponds to the third component on the right hand side of Eq. (8).

For $\Delta = 0$ we get the following expression for the absorption coefficient in the normal state:

$$\gamma_n = 2a\hbar\omega. \quad (10)$$

2. We first show that the derivative $d\gamma_s/dT$ is finite for $T = T_c$. Considering the temperature dependence of the energy gap to be

$$\Delta = c\sqrt{T_c - T} \quad (11)$$

(T_c is the critical temperature, and the constant c is equal to $3.2k\sqrt{T_c}$ in the isotropic model), it is not difficult to see that

$$\frac{d\gamma_s(T_c)}{dT} = -\frac{1}{2}c^2 \lim_{\Delta\to 0} \frac{1}{\Delta}\frac{\partial\gamma_s}{\partial\Delta}. \quad (12)$$

In the calculation of the derivative $d\gamma_s(T_c)/dT$, it is convenient to reduce Eq. (8) for the absorption coefficient to the form

$$\gamma_s = F(\Delta) + \varphi(\Delta) + \psi(\Delta); \quad (13)$$

$$F(\Delta)=4a\left\{\int_{\Delta}^{\infty}d\varepsilon\frac{\varepsilon(\hbar\omega+\varepsilon)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega+\varepsilon)^2-\Delta^2}}\right.$$
$$\times[f(\varepsilon)-f(\hbar\omega+\varepsilon)]+\int_{\Delta}^{\hbar\omega/2}d\varepsilon\frac{\varepsilon(\hbar\omega-\varepsilon)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega-\varepsilon)^2-\Delta^2}}$$
$$\left.\times[1-f(\varepsilon)-f(\hbar\omega-\varepsilon)]\right\}, \qquad (14)$$

$$\varphi(\Delta)=4a\Delta^2\left\{\int_{\Delta}^{\hbar\omega/2}\frac{1-f(\varepsilon)-f(\hbar\omega-\varepsilon)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega-\varepsilon)^2-\Delta^2}}d\varepsilon\right.$$
$$\left.-\int_{\Delta}^{\infty}\frac{f(\varepsilon)-f(\hbar\omega+\varepsilon)}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega+\varepsilon)^2-\Delta^2}}d\varepsilon\right\}, \qquad (15)$$

and $\psi(\Delta)$ coincides with the second integral on the right hand side of Eq. (8). The limit of $d\psi/d\Delta^2$ is known to be finite as $\Delta \to 0$; therefore, we shall not consider the function $\psi(\Delta)$ at present.

It is not possible to differentiate the integrals (14) and (15) with respect to the lower limit Δ, since the integrand functions have singularities at $\epsilon = \Delta$. These singularities vanish after integration by parts. By integrating, we get from Eq. (14)

$$\frac{1}{4a}\lim_{\Delta\to 0}\frac{1}{\Delta}\frac{\partial F}{\partial\Delta}=\lim_{\Delta\to 0}\left\{\int_{\Delta}^{\infty}\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}[f'(\varepsilon)-f'(\hbar\omega+\varepsilon)]\right.$$
$$-\int_{\Delta}^{\hbar\omega/2}\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}[f'(\varepsilon)-f'(\hbar\omega-\varepsilon)]$$
$$-\int_{0}^{\infty}\varepsilon\frac{\partial}{\partial\varepsilon}\left[\frac{f(\varepsilon)-f(\hbar\omega+\varepsilon)}{(\hbar\omega+\varepsilon)^2}\right]d\varepsilon$$
$$\left.-\int_{0}^{\hbar\omega/2}\varepsilon\frac{\partial}{\partial\varepsilon}\left[\frac{1-f(\varepsilon)-f(\hbar\omega-\varepsilon)}{(\hbar\omega-\varepsilon)^2}\right]d\varepsilon\right\}. \qquad (16)$$

Here the prime, in contrast with the expressions (3)—(7), denotes differentiation with respect to the argument.

Each of the first two terms is logarithmically divergent as $\Delta \to 0$. We note that these terms correspond to different processes ($\alpha^+\alpha^+b$ and $\alpha^+\alpha b$) and enter into (14) with different signs. The difference of the diverging integrals as $\Delta \to 0$ is equal to

$$\int_{\Delta}^{\hbar\omega/2}\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}[f'(\hbar\omega-\varepsilon)-f'(\hbar\omega+\varepsilon)]$$
$$+\int_{\hbar\omega/2}^{\infty}\frac{d\varepsilon}{\sqrt{\varepsilon^2-\Delta^2}}[f'(\varepsilon)-f'(\hbar\omega+\varepsilon)$$

and does not have a singularity as $\Delta \to 0$.

The derivatives of the individual components in $\varphi(\Delta)$, which correspond to different processes of phonon absorption, also diverge logarithmically as $\Delta \to 0$. However, the sum of these derivatives has a finite limit as $\Delta \to 0$. One can establish this fact by carrying out the calculation in the same way as in the computation of $\partial F/\partial\Delta^2$. Similar calculations, with unimportant complications, can also be carried out in the anisotropic case.

Thus we have shown that the derivative $d\gamma_s/dT$ is finite for $T = T_c$. This conclusion is found in qualitative agreement with the results of Bezuglyĭ, Galkin, and Korolyuk.[8]

Omitting the straightforward but rather cumbersome calculations, we write down the results of the calculation of $d\gamma_s/dT$ for $T = T_c$, which have been carried out for three limiting cases (s is the sound velocity):

a) $$\frac{\hbar\omega}{kT_c}\ll\frac{s}{v_0},\qquad \frac{d\gamma_s(T_c)}{dT}=\frac{2}{3}ac^2\left(\frac{s}{v_0}\right)^3\frac{1}{kT_c}; \qquad (17)$$

b) $$s/v_0\ll\hbar\omega/kT_c\ll 1,$$
$$d\gamma_s(T_c)/dT=\frac{1}{24}ac^2(\hbar\omega)^2/(kT_c)^{-3}(\ln 4-1); \qquad (18)$$

c) $$\hbar\omega\gg kT_c,\qquad \frac{d\gamma_s(T_c)}{dT}=-\frac{8ac^2}{\hbar\omega}\ln\frac{\hbar\omega}{kT_c}. \qquad (19)$$

The coefficient of absorption in the normal metal does not depend on the temperature. Thus the derivative $d\gamma/dT$ experiences a jump in the transition from the normal state to the superconducting state. Under experimental conditions[8] ($\omega \sim 10^8\ \text{sec}^{-1}$, which corresponds to the case a) the jump $d\gamma/dT$ is very small:

$$\frac{T_c}{\gamma_n}\frac{d\gamma^s}{dT}\sim 10^{-4}.$$

In the case of high frequencies ($\hbar\omega \gg kT_c$), the quantity $d\gamma_s/dT$ is negative [see (17)]. Consequently, close to T_c, the absorption coefficient increases with decrease in temperature.

3. We proceed to investigate the temperature dependence of the absorption coefficient. We first consider the case of comparatively low ultrasonic frequencies $\hbar\omega \ll kT$ (and not very low temperatures). With an aim toward allowance for the threshold effect, we carry out the computations with accuracy up to terms of second order of $\hbar\omega/kT$. In the general expression (8), we study separately the components corresponding to the absorption of the phonon by the electron and the decay of the phonon into a pair of quasi-particles. We denote these components by γ_1 and γ_2, respectively. In the integrand for γ_1 [see Eq. (8)], the quantity

$$\frac{\varepsilon(\hbar\omega+\varepsilon)-\Delta^2}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega+\varepsilon)^2-\Delta^2}}$$

can be replaced in the main region of integration $\epsilon - \Delta \sim kT$ by unity with accuracy up to terms of order $(\hbar\omega/kT)^2$. Then the expression for γ_1 takes the simple form:

$$\gamma_1 = 4a\int_\Delta^\infty [f(\varepsilon) - f(\hbar\omega+\varepsilon)]\,d\varepsilon. \tag{20}$$

Limiting ourselves to quantities of order $f(\Delta)\hbar\omega/kT$, we have

$$\gamma_1/\gamma_n = 2f(\Delta) + \hbar\omega f'(\Delta). \tag{21}$$

The threshold absorption exists only at temperatures satisfying the condition $\hbar\omega \geq 2\Delta$. From the assumption $\hbar\omega \ll kT$, the value of $1 - f(\epsilon) - f(\hbar\omega - \epsilon)$ is approximately equal to $\hbar\omega/4kT_c$.

The function

$$\frac{\varepsilon(\hbar\omega-\varepsilon)+\Delta^2}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega-\varepsilon)^2-\Delta^2}}$$

in the integrand of the expression for γ_2 cannot be replaced by unity since the interval of integration is small in comparison with kT. Such a substitution was made by Geĭlikman and Kresin.[2] As a result, the jump in the absorption coefficient at $\hbar\omega = 2\Delta$ was eliminated.

The expression for γ_2/γ_n has the form

$$\frac{\gamma_2}{\gamma_n} = \frac{1}{2kT_c}\int_\Delta^{\hbar\omega/2} \frac{\varepsilon(\hbar\omega-\varepsilon)+\Delta^2}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega-\varepsilon)^2-\Delta^2}}\,d\varepsilon. \tag{22}$$

It is easy to compute this value for $T = T_c$:

$$\gamma_2(T_c)/\gamma_n = \hbar\omega/4kT_c. \tag{23}$$

With allowance for (21), it can be seen that for $T = T_c$ we obtained the exact value of the absorption coefficient $\gamma_s = \gamma_n$. We note that in the paper of Tsuneto[5] the negative term $\hbar\omega f'(\Delta)$ was omitted in the expression for γ_1. Therefore, an incorrect conclusion was drawn by the author, namely, that there is a maximum in the absorption coefficient immediately below T_c, which in fact does not exist.

By computing the limit (22) as $\Delta \to \hbar\omega/2$, we find the value for the jump in the absorption coefficient, $\delta\gamma$,

$$\delta\gamma/\gamma_s = \pi\hbar\omega/8kT_c. \tag{24}$$

For an arbitrary temperature, one can express the right hand side of (22) in terms of the complete integrals $E(z)$ and $K(z)$, and the expression for $\gamma_s = \gamma_1 + \gamma_2$ takes the form

$$\frac{\gamma_s}{\gamma_n} = 2f(\Delta) + \hbar\omega f'(\Delta) + \frac{\hbar\omega}{2kT_c}\left[\frac{E(Z)}{1+Z} - \frac{1-Z}{2}K(Z)\right];$$
$$Z = (\hbar\omega - 2\Delta)/(\hbar\omega + 2\Delta). \tag{25}$$

It can be shown that the $\gamma_s(T)$ curve is convex for $\hbar\omega > 2\Delta$ and concave in the opposite case. The inequality $\hbar\omega > 2\Delta$ determines the region of temperatures near T_c in which the dependence $\gamma_s(T)$ differs significantly from that obtained by Bardeen, Cooper, and Schrieffer. This difference was discovered by Bezuglyĭ, Galkin, and Korolyuk,[8] which led them to an incorrect conclusion as to the existence of anisotropy in the critical temperature. However, the region of temperatures in which deviations were observed from the formula of Bardeen, Cooper, and Schrieffer, is, according to the data of the experiment,[8] significantly greater than theory predicts. Obviously, this is explained by the presence of collisions of electrons with impurities, which we have not taken into account. For $\omega < 1/\tau$, where τ is the time between collisions, our considerations are inapplicable.

The dependence $\gamma_s(T)$ changes upon reduction of the temperature. Under the condition $kT \ll \hbar\omega$, one can set $b = \infty$ in Eq. (8), while the quantity $f(\hbar\omega + \epsilon)$ can be neglected in the integrand in comparison with $f(\epsilon)$. In the absence of threshold absorption ($\hbar\omega < 2\Delta(T)$), the absorption coefficient is equal to

$$\gamma_s = 4a\int_\Delta^\infty \frac{\varepsilon(\hbar\omega+\varepsilon)-\Delta^2}{\sqrt{\varepsilon^2-\Delta^2}\sqrt{(\hbar\omega+\varepsilon)^2-\Delta^2}}e^{-\varepsilon/kT}\,d\varepsilon. \tag{26}$$

Asymptotically, we have as $T \to 0$:

$$\gamma_s/\gamma_n = e^{-\Delta/kT}[2\pi\Delta kT/\hbar\omega(\hbar\omega+2\Delta)]^{1/2} \quad (\hbar\omega \gg kT). \tag{27}$$

We note that in the derivation of this formula, we have not assumed the smallness of $\hbar\omega$ in comparison with kT_c. Thus, close to $T = 0$, the absorption coefficient falls off with decrease in temperature more rapidly than according to the Bardeen-Cooper-Schrieffer formula,[1] which was derived under the condition $\hbar\omega \ll kT$.

Let us consider the case $\hbar\omega \gg kT_c$, when the ultrasonic absorption takes place primarily via the process $\alpha^+\alpha^+ b$. In the components corresponding to this process, one can replace the integrand function by unity. As a result, it turns out that $\gamma_s = \gamma_n$ for all temperatures.[1)] This agrees with Eq. (19) for the derivative $d\gamma_s(T_c)/dT$, since this derivative decreases in absolute value to zero with increase in frequency.

For arbitrary relations between $\hbar\omega$ and kT, one can obtain exactly only the value of the absorption coefficient jump at $\hbar\omega = 2\Delta$:

[1)]Upon increase in ω, the difference $\gamma_s - \gamma_n$ decreases, although each of the quantities γ_s and γ_n increases.

$$\delta\gamma / \gamma_n = {}^1/_2\pi \, [1-2f\,(\hbar\omega/2)]. \tag{28}$$

The Fermi function $f(\hbar\omega/2)$ in this formula must be computed at the threshold temperature T_{th}.

The lower the temperature at which the jump takes place, the larger is its value. For sufficiently high frequencies, $\delta\gamma$ exceeds γ_n. This means that up to T_{th} the absorption coefficient increases upon decrease in temperature. When $\hbar\omega \gg kT_{th}$, the absorption coefficient is exponentially small at $T < T_{th}$ [see (27)].

The increase in γ_s upon decrease in temperature close to T_c, and the subsequent rapid fall off of γ_s, were discovered experimentally by Bömmel,[11] who used hypersound of frequency $\omega/2\pi = 3 \times 10^{10}\ \text{sec}^{-1}$. Unfortunately, a detailed summary of the results of Bömmel has not yet been published. A brief exposition of these results is contained in the reviews[12,13].

We shall write down the dependence of $\gamma_s = \gamma_2$ on ω for $T = 0$, which generalizes the results of the research of Pokrovskiĭ.[3] Just as in (25), γ_s is computed in terms of the complete elliptic integrals:

$$\gamma_s = \gamma_n \{2E\,(Z)/(1+Z) - (1-Z)\,K\,(Z)\}. \tag{29}$$

4. In conclusion, we shall discuss the problem of the effect of anisotropy on the properties of the threshold ultrasonic absorption. For $T = 0$, this problem was considered by Pokrovskiĭ and Ryvkin.[4] The conclusions of this research are easily generalized to the case of finite temperatures. In an anisotropic superconductor, the general formula for the absorption coefficient has the same *form* as in the isotropic case [see (3)], with only this difference that the interaction constant g depends on the angles and appears as a factor in the expression under the integral. The threshold frequency and temperature are determined from the condition $\hbar\omega = z_{min}$, where $z = \epsilon + \epsilon'$, and z_{min} is the minimum of this quantity. Repeating the discussions given in [4], it is easy to see that $z_{min} \approx 2\Delta^0_{min}$, where Δ^0_{min} is the minimum value of the energy gap along the line $\mathbf{q}\cdot\mathbf{v} = 0$ on the Fermi surface, $\mathbf{v}$ equals the velocity of the "normal" electron. The threshold values of the frequency and temperature thus depend on the direction of propagation of the sound.

We now determine the character of the temperature and frequency dependence of γ_2 close to threshold. For this purpose, we transform in the second component in (3) to integration with respect to z and over the level surfaces of the function z(p):

$$d\mathbf{p} = |\partial z/\partial\mathbf{p}|^{-1}\, dz\, dS_z. \tag{30}$$

After integration with respect to z we get

$$\gamma_2 = \frac{\pi\omega}{2\,(2\pi\hbar)^3}\int \frac{dS}{|\partial z/\partial\mathbf{p}|}\, g^2\,(1-f-f')\left(1-\frac{\xi\xi'-\Delta\Delta'}{\varepsilon\varepsilon'}\right). \tag{31}$$

Integration in (31) is carried out over the surface $z = \hbar\omega$.

In the approach to the threshold, the surface $z = \hbar\omega$ contracts to a point, and near threshold it is represented by the ellipsoid

$$\hbar\omega = z_{min} + \beta_{ik}\,(p_i - p_{0i})\,(p_k - p_{0k});$$

p_{0i} is the coordinate of the point $z = z_{min}$ and β_{ik} is a function of $\mathbf{q}$. The value of $|\partial z/\partial\mathbf{p}|$ vanishes for $z = z_{min}$ linearly with respect to $p_i - p_{0i}$, while the area of the surface $z = \hbar\omega$ is proportional to the square of the dimensions of the ellipsoid. The length of the semi-axis of the ellipsoid is proportional to $\sqrt{\hbar\omega - z_{min}}$. Therefore, close to threshold,

$$\gamma_2 = \text{const}\cdot\sqrt{\hbar\omega - z_{min}}. \tag{32}$$

This result for $T = 0$ was obtained by Pokrovskiĭ and Ryvkin.[4]

When $\hbar\omega = z_{min}$, the curves $\gamma_s(T)$ and $\gamma_s(\omega)$ have vertical tangents to the right ($\hbar\omega > z_{min}$) and oblique lines to the left. The curve $\gamma_s(T)$ is convex for $T > T_{th}$ and concave for $T < T_{th}$. This in principle permits us to determine the values of ω_{th} and T_{th} by experiment. Knowing these quantities, we can find the dependence of Δ^0_{min} on the direction of the sonic wave vector, and from it determine the anisotropy of the energy gap everywhere with the exception of special "blank spots."[14] Consequently, detailed experimental investigation of ultrasonic absorption in superconductors can serve as a method of establishing the angular and temperature dependence of the energy gap.

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231