A SPACE-TIME APPROACH TO QUANTUM FIELD THEORY

G. V. RYAZANOV

Submitted to JETP editor January 4, 1962; revised July 6, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1281-1287 (October, 1962)

An attempt is made to replace quantum field theory by a statistics of trajectories with changing direction of time. The general expression for quantum-mechanical probabilities [1] is supplemented with arguments about the nature of the random motion of particles on trajectories with changing time direction. The wave properties of microscopic particles follow almost automatically from the idea of particles capable of moving both forward and backward in time (Section 1). In this connection we discuss the possibility of constructing a quantum field theory on the basis of a small number of postulates (Section 2).

1. WAVE PROPERTIES OF MICROSCOPIC PARTICLES

WE shall use the concept of the "trajectory of a microscopic particle" as we did in [1]. It is desirable not to ascribe any properties to the particles beforehand, and in particular not to regard the trajectory $x(t)$ as a single-valued function of the time. This means that at each instant of time there are several particles. The basic idea of the present paper is to assert the possibility that under these conditions there will be periodic oscillations of the particle density. These oscillations are identified with de Broglie waves.

Let us take as an example a case in which the particle number density does not depend on the coordinates. A change of the sign of the time along a path can be regarded as a transition of a particle with positive (negative) direction of time into a particle with the opposite direction of time. From symmetry arguments we introduce two types of transitions: causal transitions, for which the number of conversions is proportional to the number of particles in the initial state, and anticausal transitions, for which the number of conversions is proportional to the number of particles in the final state. In order for the transitions to be half of the causal type and half anticausal, we shall introduce two types of particles with positive time direction (densities n_1 and n_2) and two types of particles with negative time direction (densities m_1 and m_2). Let the transitions $n_1 \rightleftarrows m_1$ and $n_2 \rightleftarrows m_2$, say, be causal, and $n_1 \rightleftarrows m_2$ and $n_2 \rightleftarrows m_1$, anticausal. We denote the mean number of kinks per unit time—the transition probability—by ω.

The change of the number of particles of type n_1 in the time ϵ because of transitions $n_1 \rightleftarrows m_1$ (causal) is given by $-\omega n_1\epsilon + \omega m_1\epsilon$, and the change because of transitions $n_1 \rightleftarrows m_2$ by $-\omega m_2\epsilon + \omega n_1\epsilon$. Thus

$$n_1(t+\varepsilon) = n_1(t) + \omega m_1(t)\,\varepsilon - \omega m_2(t)\,\varepsilon. \tag{1}$$

When we write out the analogous relations for the other types of particles, for $\epsilon \to 0$ we get four equations:

$$\dot{n}_1 = \omega(m_1 - m_2), \quad \dot{m}_1 = \omega(n_2 - n_1),$$
$$\dot{n}_2 = \omega(m_2 - m_1), \quad \dot{m}_2 = \omega(n_1 - n_2). \tag{2}$$

The last two equations are obtained from the first two by replacing t by $-t$ and n_i by m_i. These equations have the solution

$$n_1 = 1 + \cos 2\omega t,\ n_2 = 1 - \cos 2\omega t,$$

$$m_1 = 1 - \sin 2\omega t,\ m_2 = 1 + \sin 2\omega t.$$

If we introduce the function $\psi = n_1 - n_2 + im_1 - im_2$, we have from Eq. (2):

$$\dot{\psi} = i\omega\psi. \tag{3}$$

Thus it is convenient to describe the time-symmetrical system of interconvertible particles by the function ψ, which has no direct physical meaning.

Let us now go on to a system in which the particle number density depends on the coordinates. In order to include spin effects from here on, we shall ascribe to a particle, besides its coordinates, three Euler angles which give the orientation of an orthogonal frame attached to the particle. Let us introduce the notations: $u = n_1 - n_2$, $v = m_1 - m_2$; α is a set of three Euler angles, and $f_\epsilon(\alpha, \alpha')$ is the probability of transition from orientation α to orientation α' in the time ϵ.

When we set up kinetic equations analogous to Eq. (1) we get

$$u(t+\varepsilon, \mathbf{x}, \alpha) = \int f_\varepsilon u(t, \mathbf{x} - \mathbf{v}\varepsilon, \alpha')\, d\alpha' + \omega\varepsilon \int f_\varepsilon v(t, \mathbf{x}, \alpha')\, d\alpha',$$

$$v(t-\varepsilon, \mathbf{x}, \alpha) = \int f_\varepsilon v(t, \mathbf{x} + \mathbf{v}\varepsilon, \alpha')\, d\alpha' + \omega\varepsilon \int f_\varepsilon u(t, \mathbf{x}, \alpha')\, d\alpha'. \quad (4)$$

Here the velocity $\mathbf{v}$ depends on α, α': $\mathbf{v} = \mathbf{v}(\alpha, \alpha')$.

We introduce the following notations: $D^j_{m,n}(\varphi, \theta, \psi)$ are generalized spherical functions, and $\Gamma^j_{mm'}$ are the operators (matrices) for an infinitesimal rotation, taken in the representation j (for $j = 1/2$ these are the Pauli matrices). Furthermore, we set

$$u^j_{m,k}(\mathbf{x}, t) = \int u(\mathbf{x}, t, \alpha) D^j_{m,k}(\alpha)\, d\alpha,$$

$$v^j_{m,k}(\mathbf{x}, t) = \int v(\mathbf{x}, t, \alpha) D^j_{m,k}(\alpha)\, d\alpha. \quad (5)$$

We can obtain the equations which describe particles with various spins if we set

$$\lim_{\varepsilon\to 0} f_\varepsilon(\alpha, \alpha') = \delta(\alpha - \alpha'),$$

$$\lim_{\varepsilon\to 0} \mathbf{v}_\varepsilon(\alpha, \alpha') f_\varepsilon(\alpha, \alpha') = \sum_{j,k,m,m'} D^{*j}_{m,k}(\alpha)\, \Gamma^j_{mm'} D^j_{m'}(\alpha'). \quad (6)$$

We note that the sum on the right is different from zero only for $\alpha = \alpha'$.

Substituting Eqs. (5) and (6) in Eq. (4), we get for $\epsilon \to 0$:

$$\dot{u}^j_{m,k} - \Gamma^j_{mm'} \nabla u^j_{m',k} = \omega \mathbf{v}^j_{m,k},$$

$$\dot{v}^j_{m,k} + \Gamma^j_{mm'} \nabla v^j_{m',k} = -\omega u^j_{m,k}. \quad (7)$$

For $j = 1/2$ let us introduce the function $\psi = \begin{pmatrix} u \\ v \end{pmatrix}$. Using the Dirac matrices ρ_1, ρ_2, ρ_3 we get from Eq. (7)

$$\partial\psi/\partial t - \rho_3 \boldsymbol{\sigma} \partial\psi/\partial \mathbf{x} = i\omega\rho_2\psi. \quad (8)$$

If we replace ρ_3 by ρ_1 and ρ_2 by ρ_3 we get the usual form of the Dirac equation.

If we use the four densities (instead of the wave function), then when we go from the kinetic equations (2) or (4) to the corresponding sums over paths the (nonrelativistic) "unified principle" formulated in [1],

$$W(a) = \int_a \exp\{iS/h\}\, \delta x(t) \quad (9)$$

will in the relativistic case take the form (for details see below)

$$W(a) = \int_a (-1)^s\, W\{x(t)\}\, \delta x(t). \quad (10)$$

Here W(a) is the experimentally observed distribution of any quantity a; $\int_a \delta x(t)$ is the integral over paths having the property a; S is the classical action along the path; s is the number of anticausal transitions along the path; and $W\{x(t)\}$ is the probability for the path to be x(t).

The idea of trajectories with changing sign of time enables us to understand certain features of quantum mechanics. We shall elucidate the meaning of quantum-mechanical operators with the operator $\hat{p} = (h/i)\,\partial/\partial x$ as an example. The wave function is a sum over paths, and differentiation of the wave function means multiplication by the corresponding physical quantity under the summation sign. The result gives automatically the required averaging of this quantity over all paths (for details see [1]).

As can easily be shown, the factor i in the expression for the operator $\hat{p}$ means that when we calculate the mean momentum the only contributions to the sum are from paths for which the sign of the time does not change at the instant t (i.e., products of the type $n_i m_i$).

With the example of the operators $\hat{p}$ and $\hat{x}$ one can show the meaning of the noncommutativity of operators. It means, for example, that in doing the averaging one must complete the definition of the function xp, i.e., must specify the order of action of the operators. The necessity for completing the definition of xp also arises in the path language, since each value at an instant of time t is an average over all particles and antiparticles existing at the instant t, and it is not clear whether we are to take, say, $\overline{xp}$ or $\bar{x}\cdot\bar{p}$.

In this connection we must explain one more interesting fact, which is important for the understanding of the role of trajectories in quantum mechanics. The many attempts to define in quantum mechanics the probability of observing noncommuting quantities, for example a path x(t), have always led to complex or negative probabilities.[2] It is clear that if we have to complete the definition of a quantity, and if the supplementary definitions are different in different problems, then the probability $W\{x(t)\}$ does not exist. If, however, we take some complex expression for the probability, then it turns out that the rules of addition and multiplication of probabilities are satisfied for it, and also the observable probabilities turn out positive. This is due to the property of the sign factor $(-1)^s$ in Eq. (10). The sign factor of a path is the product of the sign factors along the sections of the path. When we go over from Eq. (3) to sums over paths it can be seen that for individual sections of the path these factors can also be complex. For any path the probability multiplied by the sign factor is equal to the product of the same quantities for the sections of the path. Therefore the probability multiplied by the sign factor—the complex "probability" or probability amplitude—has the properties of an ordinary probability.

Thus in quantum mechanics there remain only two nonclassical concepts: paths with changing sign of the time, and the sign factor $(-1)^S$ in Eq. (10).

2. A POSSIBLE PROGRAM FOR CONSTRUCTION OF A THEORY OF ELEMENTARY PARTICLES

We start from the idea of the existence of a universal distribution of paths of primitive particles. To introduce an interaction of the primitive particles we assume that each section where two paths come together introduces an additional factor in the probability.

We introduce these symbols: ν is the additional factor which each point of joining of two paths brings into the probability; q is the number of such points for a given path; r is the number of changes of sign of the time along the path; t_j are the instants of coming together of two paths; and t_i are the instants at which the time direction changes sign.

Let us divide the time axis into intervals of magnitude ϵ. Let $t_k = t_{k-1} + \epsilon$ be the succession of instants of time along a path, x_k be the values of the coordinates at the times t_k, and ΔS_k be the space-time interval for the segment (t_k, t_{k-1}).

The universal distribution can be formulated in the following way:

$$W\{x(t)\}\ \delta x(t) = \nu^q \prod_{-\infty}^{\infty} \delta(\Delta S_k)\, d^3x_k \prod_{i=1}^{r} dt_i \prod_{j=1}^{q} dt_j. \qquad (11)$$

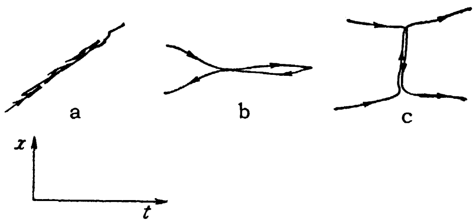
In this expression the factors which assure invariance and normalization are omitted.

If we suppose that the particle can exist only at the lattice points of a space-time lattice, then for transitions with $\Delta S_k = 0$ Eq. (11) takes the form

$$W\{x(t)\} = \nu^q. \qquad (11')$$

We shall now show how a distribution of the type (11) allows us to explain in a qualitative way the basic features of the microscopic world.

a) The sign factor $(-1)^S$ in Eq. (10). This factor connects the quantities in the right member of Eq. (10), which refer to the microscopic particles, with the macroscopic distributions appearing in the left member of Eq. (10). A macroscopic change occurring at some point in a photographic plate can be the result of repeated passage of particles past the plate, and the changes which finally produce the blackening can both accumulate and be dissipated. Causal and anticausal transitions give contributions of opposite signs to the probability of a macroscopic change at the point x, t.



b) Spin. The interaction leads to an increase of the probability of paths of type a (see diagram). For such paths the probability of detecting a particle at the point x, t will depend on the behavior of the thickening in the neighborhood of the point x, t. The kinetic equations involve probabilities which depend on additional variables, and in particular on the orientation of some reference frame. After an averaging by Eq. (3) or Eq. (10) the square of the angular velocity of this frame turns out to be quantized with integral and half-integral eigenvalues (for details see [3]).

c) Mass. The interaction leads to the singling out of paths of the type b of the diagram, in which a kink in the time is connected with a closed loop. The sum over these loops gives a constant ω which in general depends on states of intrinsic rotation. It can be seen from Eqs. (3) and (8) that this constant plays the role of a mass.

d) Charge. The interaction also singles out paths of type c (see diagram). In the language of Feynman graphs this is an interaction of two particles occurring through the exchange of a third particle.

Let us suppose that a partial inclusion of the interaction as described in a) and b) has been carried out and has given instead of Eq. (11) the rule (notations as explained above)

$$W(a) = \int_a \sum_{s,q} (-1)^s \nu^q \prod_k f_\varepsilon(\alpha_k, \alpha_{k-1})\, d\alpha_k \prod_{i=1}^{r} dt_i \prod_{j=1}^{q} dt_j. \qquad (12)$$

This rule can be made the basis of a theory of elementary particles. If we go over from Eq. (12) to an equation for ψ, the equation is of the type of Eq. (8), but with a nonlinear term. Therefore we can say that the content of Eq. (12) is close to that of Heisenberg's nonlinear spinor equation.[4]

Let us introduce ω, the sum over the loops (see diagram, b). After the summation over these loops a factor ω^r appears in Eq. (12).

We shall now show the equivalence of some of the consequences of Eq. (12) to the usual apparatus of quantum field theory.

We make the following assertion without proof: The probability calculated from Eq. (12) for the transition of a system of primitive particles from a state in which certain quantities L_i (characteristics of the system) are given at time t_0 to the state in which other characteristic quantities M_i are given at time t is equal to the square of the absolute value of the quantity $(M_i t \leftarrow L_i t_0)$ —the sum over the paths which begin at the time t_0 and have at that time the property L_i (i.e., are compatible with the prescription of L_i) and break off at the time t, having at this time the property M_i.

This relation is proved in [1,3] for the nonrelativistic case. In the relativistic case nothing is changed, and the part of cos (S/h) is played by the factor $(-1)^S$ in Eq. (12).

We shall now show that the rules for calculating the quantities $(M_i t \leftarrow L_i t_0)$ which follow from Eq. (12) are identical with the Feynman rules for calculating the S matrix of the second-quantized theory.

From Eq. (12) we get as the expression for the quantities $(M_i t \leftarrow L_i t_0)$:

$$(M_i t \leftarrow L_i t_0) = \int\limits_{M_i t \leftarrow L_i t_0} \sum_{s,q,r} (-1)^s \omega^r \nu^q \times \prod_k f_\varepsilon(\alpha_k, \alpha_{k-1})\, d\alpha_k \prod_{i=1}^{r} dt_i \prod_{j=1}^{q} dt_j. \tag{13}$$

The first term of the power series in ν (q = 0) describes the free motion of the particles. The second term of the series (q = 1) describes the various processes in which the paths come together only once. For example, for a case in which at time t_0 there were two particles at the points $x_1\alpha_1$ and $x_2\alpha_2$, and at the time t one double particle at the point $x_3\alpha_3$, we get from Eq. (13)

$$(x_3\alpha_3 \leftarrow x_1\alpha_1;\ x_2\alpha_2) = \nu \int (x_3\alpha_3 \leftarrow y\beta)\,(y\beta \leftarrow x_1\alpha_1)\,(y\beta \leftarrow x_2\alpha_2)\, dy d\beta,$$

where $(x\alpha \leftarrow x'\alpha')$ is the propagation function of a free particle.

In the j, m, k representation we get (the indices m, k are omitted)

$$(j_3 x_3 \leftarrow j_1 x_1;\ j_2 x_2) = \nu \int \sum_{m,k} (x_3 j_3 \leftarrow y j_3) \times (j_3 \mid j_1 j_2)\,(y j_1 \leftarrow x_1 j_1)\,(y j_2 \leftarrow x_2 j_2)\, dy,$$

where $(j_3 | j_1 j_2)$ are Clebsch-Gordan coefficients.

For $j_1 = j_2 = 1/2$, $j_3 = 1$ the quantities $(x\tfrac{1}{2} \leftarrow x'\tfrac{1}{2})$ obey the Dirac equation (see Section 1), and in the sense of sums over paths they describe the propagation of a Dirac particle; the quantities $(x1 \leftarrow x'1)$ describe the propagation of a particle with spin 1; it is easily shown (one must take into account transitions with change of sign of the time) that the coefficients $(1 | \tfrac{1}{2} - \tfrac{1}{2})$ become the matrices γ_ν —and the result is that we get the usual expression for the matrix element describing annihilation of the particles with spin 1/2 and creation of the particle with spin 1.

Similarly it can be shown that for q = 2 the sum over paths of type c (see diagram) will give the matrix element for the scattering of a particle with spin 1/2 by a particle of spin 1/2 owing to the exchange of a particle with spin 1 or 0. One can also trace in this example (see [1], Section 6) the appearance of antisymmetry under interchanges of particles described by an odd number of merged paths. If the path of a particle is formed from an even number of paths of primitive particles, this antisymmetry is replaced by symmetry.

The merging and separation of particles can also be taken into account in kinetic equations of the type of Eq. (4). Instead of Eq. (8) we get the Dirac equation in an external field.

3. SUMMARY

The main idea of this paper is to assert the existence of a universal distribution—the unified principle (11). This principle was proposed in [1] and [3] for nonrelativistic quantum mechanics. In the present work the concept of a unified principle is extended and includes, in addition to the laws of quantum mechanics, the law of interaction of elementary particles. When we go over to the primitive particles the laws of motion and laws of interaction become universal and therefore indistinguishable.

Unlike [1] and [3], this paper makes an attempt at an intuitive interpretation of the unified principle based on the idea of the self-oscillations of the system of particles and antiparticles. A consistent inclusion of paths reversed in time allows us to trace what paths make a negative contribution to the macroscopic distribution; to explain in principle the possibility of a negative contribution; and also to explain why it is convenient to describe the microscopic world by complex quantities (simultaneous inclusion of the motions of both particles and antiparticles).

The relations (2) and (11) are presented as examples. A wide variety of concrete models fit into the framework of the scheme described here. In particular, one can use the idea of branching or discontinuous paths or can completely replace the concept of a path by the idea of a manifold of space-time events.

The relation (11′) contains only one constant—the probability for the coming together of paths. The choice of any ideas about the motion of the primitive particles means the implicit prescription of a number of additional constants of the type of probabilities for changes of sign of the time and of spatial reversals or turnings. A rigorous mathematical formulation of Eqs. (11)—(13) will evidently require the introduction of an elementary length. Other constants, however—Planck's constant (h), the mass of the primitive particles (m), their charge (e) —will not be involved. The theory contains only ratios of the type of h/mc, e^2/hc, e^2/mc^2. There is no contradiction with the usual formulation of quantum mechanics, since in the last analysis only these ratios, and not h, m, e themselves, are observed experimentally.

In this work we have used the ideas of many authors. The Feynman language of paths [5] and of diagrams [6] is the basis of all the constructions. Paths reversed in time were first considered in [6] and [7]. The idea of motion as a succession of creations and annihilations is due to Frenkel. [8]

In conclusion I thank V. Ya. Faĭnberg and V. L. Berezinskiĭ for criticism and advice.

1 G. V. Ryazanov, JETP **35**, 121 (1958), Soviet Phys. JETP **8**, 85 (1959).

2 P. A. M. Dirac, Revs. Modern Phys. **17**, 195 (1945).

3 G. V. Ryazanov, Dissertation, Moscow State University, 1958.

4 W. Heisenberg, Revs. Modern Phys. **29**, 269 (1957).

5 R. P. Feynman, Revs. Modern Phys. **20**, 367 (1948).

6 R. P. Feynman, Phys. Rev. **76**, 749 (1949).

7 G. A. Zisman, JETP **10**, 1163 (1940); JETP **11**, 631 (1941).

8 Ya. I. Frenkel', UFN **42**, 69 (1950); JETP **21**, 466 (1951); DAN SSSR **64**, 507 (1949).

Translated by W. H. Furry
224