POLARIZATION EFFECTS IN THE RADIATION DECAY OF PIONS

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An investigation is made of the polarization effects in the $\pi \to e(\mu) + \nu + \gamma$ decay for the V, A variants of weak direct interaction. Selection rules are found for the correlations of longitudinal polarizations of spin of the decay products. It is shown that the electrons (positrons) corresponding to the diagrams a and b (Fig. 1) are longitudinally polarized in opposite directions. By measuring the energy spectrum of electrons (positrons) and the angular distribution of photons (electrons) alone it is possible to verify the correctness of the selection rules obtained and to get information about the predominance of V − A and V + A variants of weak interaction. Such information cannot be obtained from the photon energy distribution.

1. The decay of $\pi^{\pm}$ mesons in the $\pi \to e(\nu) + \nu + \gamma$ mode was considered in many papers [1-9]. This decay was investigated in [1-3] with account of longitudinal polarization of the decay-product spin and the anomalous magnetic moment of the electron (muon) for the S and P variants of the interaction in accord with diagram a (Fig. 1). The

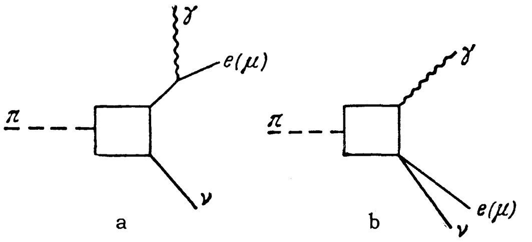

FIG. 1. Decay diagrams

investigations show that the calculation results do not depend on this case on the interaction mechanism [4-10]. The tensor variant for this process, without account of the decay-product polarization, was investigated in [7,11]. Radiative decay of $\pi^{\pm}$ mesons was investigated in [5,7-9] both in accord with diagram a and in accord with diagram b (Fig. 1), without account of polarized states of the particles. It was shown in these papers that the matrix element describing the emission of a photon in accord with diagram b has a vector and axial-vector form, while the vector variant in accord with diagram a vanishes identically.

We investigate in the present paper the $\pi \to e(\mu) + \nu + \gamma$ decay with account of longitudinal polarization of the spin of the produced fermions and gamma quanta for the V ± A variants of weak direct interaction.

The Hamiltonian of the interaction of the pion field with the spinor field and the gamma-quantum field can be written as (see [5,7-9])

$$\hat{H} = \frac{\partial\varphi}{\partial x_{\mu}}(\bar{\psi}_e O_{\mu}\psi_{\nu}) - \frac{ei}{\hbar c}\varphi A_{\mu}(\bar{\psi}_e O'_{\mu}\psi_{\nu}).$$

Here φ, ψ_e, ψ_ν, and A_μ are respectively the field operators of the pion, electron, neutrino, and photon, while O_μ and O'_μ are matrices that determine the type of the interaction; the first and second terms describe pion decay in accord with the first and second diagrams, respectively.

Radiative pion decay in accord with diagram a, with account of longitudinal polarization of the decay product spin, was considered by us earlier [1-3]. Vaks and Ioffe [5] have shown that these diagrams do not interfere when $m_{0e} \to 0$. We therefore confine ourselves here to a detailed examination of diagram b only, and compare the results obtained with the corresponding results for diagram a.

The probability of pion decay in accord with diagram b is determined by the formula

$$dW = \frac{e^2 k_{0\pi}\varkappa}{(2\pi)^3}\frac{(d^3\varkappa)(d^3k)}{\hbar^2 c}|S|^2\delta(k_{0\pi} - K - k_\nu - \varkappa),$$

$$S = \overset{+}{u}_e\{bg_V(\rho_1\sigma\overset{+}{\mathbf{a}}_e) + ag_A\,\boldsymbol{\sigma}[\mathbf{n}\overset{+}{\mathbf{a}}]\}u_\nu, \qquad (1)^*$$

where $k_0 = m_0c/\hbar$ is the rest mass of the particle, $c\hbar K$ and $\hbar\mathbf{k}$ are the energy and momentum of the fermion, $c\hbar\varkappa$ and $\hbar\boldsymbol{\varkappa}$ are the energy and momentum of the photon, $\mathbf{n}$ is a unit vector in the direction of $\boldsymbol{\varkappa}$, $\mathbf{a}_e$ is the photon polarization vector, a and b are structural constants for the pion, u_e and u_ν are the spinor amplitudes of the elec-

*$[\mathbf{na}] = \mathbf{n} \times \mathbf{a}$.

tron (positron) and the neutrino, while ρ_1 and σ are four-row Dirac matrices.

The longitudinal polarization of the fermion spin is characterized by an eigenvalue $s = \pm 1$ of the projection operator $\sigma\mathbf{p}/|\mathbf{p}|$, while the circular polarization of the photon is described by the vector [12-15]

$$\mathbf{a}_e = (\mathbf{q} + il\,[\mathbf{nq}]) / \sqrt{2}, \tag{2}$$

where $\mathbf{q}$ is a unit vector perpendicular to the direction of photon motion.

2. The calculations yield for the decay probability

$$dW = \frac{e^2a^2g_A^2\varkappa k_{0\pi}}{(2\pi)^3\hbar^2 c}(d^3\varkappa)(d^3k)(s_\nu - l\lambda)^2\{1 - \beta\cos\theta\cos\theta_1 - ss_\nu(\beta - \cos\theta\cos\theta_1) + ls_\nu(\cos\theta_1 - \cos\theta) + ls(\cos\theta - \beta\cos\theta_1)\}\,\delta(k_{0\pi} - K - k_\nu - \varkappa), \tag{3}$$

where

$$\beta = k/K = v/c,\quad \lambda = bg_V/ag_A,\quad \cos\theta = \boldsymbol{\varkappa}\mathbf{k}/\varkappa k,\quad \cos\theta_1 = \boldsymbol{\varkappa}\mathbf{k}_\nu/\varkappa k_\nu;$$

s and s_ν characterize the total longitudinal polarization of the electron and neutrino spin, $l = 1$ corresponds to right-hand circular polarization of the photon, and $l = -1$ to left-hand polarization.

In summing over all the polarization particle states, formula (3) goes over into the corresponding expression given in [5,9]. (We note, however, that the electron energy spectrum obtained thereby [see formula (6) of the present paper] does not coincide with the results of Vaks and Ioffe [5], but agrees with the result of Bludman and Young [9].)

Calculations show (see, for example, [16]) that in radiative $\pi \to \mu$ decay the internal bremsstrahlung dominates over the structural radiation, while the converse is true for the $\pi \to e$ decay. This circumstance was demonstrated experimentally for the $\pi \to \mu$ decay in [17], where it is emphasized that the existence of a radiative $\pi \to \mu$ decay can be regarded as finally proved. In addition, the electrons (positrons) produced in radiative decay will have limiting relativistic velocities, for which states with total longitudinal polarization of the electron (positron) spin should actually be realized (σp-correlation). We therefore confine ourselves here to a discussion of radiative $\pi \to e$ decay only.

Integrating (3) with respect to the photon and electron energies we obtain for the angular distribution of the photons (electrons) [1)]

$$dW(\alpha, l, s) = \frac{Ak_{0\pi}^3 d\Omega}{2^{10}\alpha^6}(1 - ss_\nu)(s_\nu - l\lambda)^2 \times \Big\{\alpha\Big(45 - \frac{181}{2}\alpha + 48\alpha^2 - \frac{87}{12}\alpha^3\Big) + (1-\alpha)(45 - 63\alpha + 24\alpha^2 - 2\alpha^3)\ln(1-\alpha) + ls\Big[\alpha\Big(25 - \frac{69}{2}\alpha + \frac{46}{3}\alpha^2 - \frac{7}{12}\alpha^3\Big) + (1-\alpha)(25 - 27\alpha + 6\alpha^2)\ln(1-\alpha)\Big]\Big\}, \tag{4}$$

where

$$d\Omega = \sin\theta\, d\theta\, d\varphi,\quad \alpha = \sin^2(\theta/2),\quad A = (eag_A k_{0\pi}/\pi\hbar c)^2.$$

Summing over the spin states of the electrons (positrons) and photons, we obtain from (4) for arbitrary λ

$$\overline{dW}(\alpha) = \frac{Ak_{0\pi}^3 d\Omega}{2^8\alpha^6} \times \Big\{(1+\lambda^2)\Big[\alpha\Big(45 - \frac{181}{2}\alpha + 48\alpha^2 - \frac{87}{12}\alpha^3\Big) + (1-\alpha)(45 - 63\alpha + 24\alpha^2 - 2\alpha^3)\ln(1-\alpha)\Big] + 2\lambda\Big[\alpha\Big(25 - \frac{69}{2}\alpha + \frac{46}{3}\alpha^2 - \frac{7}{12}\alpha^3\Big) + (1-\alpha)(25 - 27\alpha + 6\alpha^2)\ln(1-\alpha)\Big]\Big\}. \tag{4'}$$

If the decay is in accord with diagram a, we obtain for the angular distribution of the photons (electrons) the formula

$$dW_I(\alpha, l, s) = \frac{A_I k_{0\pi} d\Omega}{2^5\alpha^3}(1 + ss_\nu) \times \Big\{\alpha + (1-\alpha)\ln(1-\alpha) + 2\alpha^2(1-\alpha)\Big(\ln\frac{1}{1-y_{max}} - 1\Big) + ls\,[\alpha(1-2\alpha) + (1-\alpha)\ln(1-\alpha)]\Big\}. \tag{5}$$

It follows from (4) and (5) that for diagram b the longitudinal polarization of the electrons (positrons) has a trend that is common to all weak interaction processes: the positrons are polarized in the direction of the motion ($s_+ = 1$, $s_\nu = -1$), while the electrons are polarized oppositely ($s_- = -1$, $s_\nu = 1$), whereas for diagram a we encounter a case which is unusual for weak V, A interaction, namely that the electron spin is oriented along its momentum, while the positron-spin orientation is opposite.

It must also be noted that when $\lambda = 1$ the photons of diagram b can have only one polarization direction: in π^+ decay the photons should be polarized only along the motion ($l = 1$, $ls = +1$),

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1)The results are presented here and henceforth in the approximation $k_{0e}/K \to 0$ (or $m_{0e} \to 0$).

while in π^- decay they should be polarized against the motion ($l = -1$, $ls = +1$). When $\lambda = -1$ we have the opposite situation. On the other hand, if $\lambda \neq 1$ [18-22], then the produced photons, for example in π^- decays, can have both right-hand and left-hand circular polarization.

An experimental verification of these selection rules could be made by measuring the angular distribution and the energy spectrum of the electrons (photons) or positrons (Figs. 2 and 3), since the

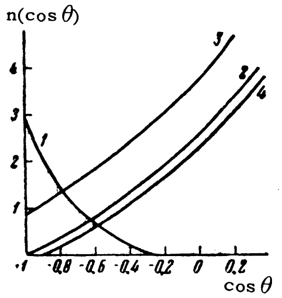

FIG. 2. Angular distribution of electrons (photons) from the $\pi^\pm \to e^\pm + \nu + \gamma$ decay. The curves correspond: 1) to the V + A variant (diagram b), 2) to the case $ls = 1$, 3) to summation over the spin states, 4) to the case $ls = -1$ (diagram a).

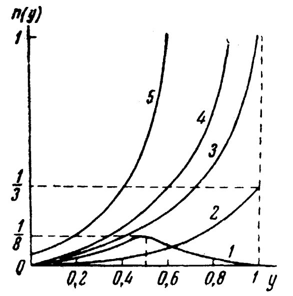

FIG. 3. Energy distribution of electrons (positrons) of the $\pi^\pm \to e^\pm + \nu + \gamma$ decay. The curves correspond: 1) to the V + A variant, 2) to the V – A variant (diagram b), 3) to the case $ls = -1$, 4) to the case $ls = 1$, 5) to summation over the spin states (diagram a).

form of the distribution curves is uniquely determined by the direction of the polarization of the emerging particles. In fact, the electrons (positrons) should in this process, on the one hand, be polarized completely along their momentum, and on the other, if the assumption $\lambda = +1(-1)$ is correct, their spin direction should be fixed. For example, curve 1 of Fig. 3 corresponds to cases in which the electrons (positrons) and photons are polarized circularly in a left-hand (right-hand) direction ($ls = +1$), and curve 2 corresponds to cases in which the electrons (positrons) have left-hand (right-hand) polarization while the photons have right-hand (left-hand) circular polarization ($ls = -1$). As regards the energy distribution of the photons, it is independent of the particle spin states [see formula (8)].

It follows from (4) and (4′) that when $\lambda = 1$ we get $d\overline{W}(\alpha) = \{dW(\alpha)\}_{ls=1}$ for the V + A variant, which corresponds to the corresponding expression of Bludman and Young [9], and when $\lambda = -1$ we get $d\overline{W}(\alpha) = \{dW(\alpha)\}_{ls=-1}$ for the V – A variant. The dependence of the quantity $n_{\lambda=1}(\cos\theta) = d\overline{W}_{\lambda=1}/N d\Omega$ on $\cos\theta$ is shown in Fig. 2 (curve 1).[2)] The same figure shows the dependence $n_I(\cos\theta) = dW_I/N_I d\Omega$ calculated from (5) for different l (curves 2, 3, 4). Here $N = Ak_{0\pi}^3/2^6$ and $N_I = A_I k_{0\pi}/2^5$.

It follows from the curves corresponding to the V + A and V – A variants that if the $\pi^\pm$ meson decay is produced via the V + A interaction, then the decay photons should be emitted predominantly at angles close to $\theta = \pi$; on the other hand, if the decay is via the V – A interaction the angles should be close to $\theta = 0$. Curves 2—4 show that according to diagram a the photons are also emitted at small angles. Consequently, in the region $\pi/2 \leq \theta \leq \pi$ we can distinguish the vector variant from the axial-vector one, whereas in the region $0 \leq \theta \leq \pi/2$ both diagrams give results of similar character. It is possible that a determination of the value of the π-decay constants will permit the character of the decay to be made more exact.

3. Integrating (3) over the photon angles and energies, we obtain the electron energy spectrum

$$dW(y, l, s) = \frac{\pi A k_{0\pi}^3}{2^9} y^2 dy\,(1 - ss_\nu)(s_\nu - l\lambda)^2 \times \left\{\left[(1-y)^2 + \frac{y^2}{6}\right] + ls\left(1 - 2y + \frac{5}{6}y^2\right)\right\}, \tag{6}$$

where $y = 2k/k_{0\pi}$.

On the other hand, in the case of diagram a, the energy spectrum of the electrons has the form

$$dW(y, l, s) = \frac{\pi A_I k_{0\pi}}{2^5}(1 + ss_\nu)\,dy\left\{\frac{2y}{1-y}\left(\ln\frac{4y^2}{d^2} - 2\right) + (1-y)\ln\frac{4y^2}{d^2(1-y)} + ls\left[2\left(\ln\frac{4y^2}{d^2} - 1\right) - (1-y)\ln\frac{4y^2}{d^2(1-y)}\right]\right\}, \tag{7}$$

where

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2)By rotating curve 1 through 180° about the point $\cos\theta = 0$ we readily obtain the curve corresponding to $dW_{\lambda=-1}/Nd\Omega$. The curves obtained (like the curves 1 and 2 on Fig. 3) describe the angular (energy) distributions of both electrons and positrons, inasmuch as $ls = 1$ for both $\pi^\pm$ decays (see above).

$$y = 2k/k_{0\pi},\ d = 2k_0/k_{0\pi}.$$

From (6) we get

$$d\overline{W}(y) = \{dW(y)\}_{ls=1}$$

when $\lambda = 1$ and

$$d\overline{W}(y) = \{dW(y)\}_{ls=-1}$$

when $\lambda = -1$. The dependence of the quantities

$$n_{\lambda=1}(y) = d\overline{W}_{\lambda=1}/(N/2\pi)dy,$$

$$n_{\lambda=-1}(y) = dW_{\lambda=-1}/(N/3\pi)\,dy$$

on y is shown in Fig. 3 (curves 1 and 2). The same figure shows curves for

$$n_l(y) = dW_l(y)/(N_l/\pi)\,dy$$

at different values of l (curves 3, 4, 5). The energy distribution of the electrons (positrons) in the case of the V + A and V − A interaction variants are essentially different. In the V + A case the distribution curve (curve 1) is symmetrical about the point y = 0.5; the decay electrons are emitted predominantly with energy in the interval $0.3 \le y \le 0.7$. On the other hand, the distribution curve obtained for the V − A interaction (curve 2) is similar in character to the curves of diagram a. In this case the electrons emitted should have an energy close to maximal ($y \sim 1$), while the probability of electron emission with energy less than y = 0.5 is insignificant.

4. As a result of integrating (3) over the emission angles and energies of the electrons we obtain the energy spectrum of the pion decay photons

$$dW(x, l, s) = \frac{\pi A k_{0\pi}^3}{3\cdot 2^9}(1 - ss_\nu)(s_\nu - l\lambda)^2 x^3 (1 - x)\,dx, \quad (8)$$

where $x = 2\kappa/k_{0\pi}$. The photons are radiated for the most part with energies in the region $0.7 \le x \le 0.8$. The emission of such hard gamma rays indicates that the decay occurs more readily via V + A rather than V − A interaction, inasmuch as in the latter cases the photons emitted should be softer, as is the case for diagram a.[5]

Consequently, an experimental measurement of the energy spectrum of the photons, of the sign of polarization of the electrons and positrons cannot yield any information whatever on whether the V + A or the V − A variant of weak interaction predominates in the radiative decay of pions. Such information can be obtained, as in the verification of the selection rules, by measuring the energy spectrum of the electrons (positrons) and angular distribution of the photons (electrons, positrons) of the decay. Although radiative $\pi \rightarrow e$ decay is a rare process, the formulas (4)—(8) derived in this paper for the angular and energy distributions, with account of longitudinal polarization of the fermion and gamma-quantum spins, can be used in addition in future experiments on the study of polarization correlations. Longitudinal polarization of electrons can be measured by investigating the circular polarization of bremsstrahlung produced by a longitudinally polarized electron in the Coulomb field of the nucleus, while circular polarization of gamma quanta can be measured by the Compton effect on polarized electrons (see [18]).

Finally, we note that if we take in (5), (7), and (8) the sum over the spin states of the electron and photon, we obtain the results of Vaks and Ioffe[5].

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