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# *ON THE EXPERIMENTAL VERIFICATION OF UNITARY SYMMETRY OF STRONG INTERACTIONS*

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The possibilities of an experimental verification of unitary symmetry of strong interactions are discussed. For this purpose, the relations between the amplitudes of various processes are established for each of the two unitary symmetry variants.

## 1. INTRODUCTION

EIGHT of the recently observed resonances in the $2\pi$, $3\pi$, and $K + \pi$ systems have strikingly close mass values: $m_\rho = 750$ MeV ($\rho \rightarrow \pi + \pi$), $m_\omega = 785$ MeV ($\omega \rightarrow \pi^+ + \pi^- + \pi^0$), and $m_{K*} = 885$ MeV ($K^* \rightarrow K + \pi$). Regardless of the meaning that one can ascribe to these resonances as new "particles," the agreement between the masses apparently points to a certain approximate symmetry in strong interactions, higher than isotopic invariance. The relatively small splitting of the masses must be ascribed in this case to a less strong interaction, which disturbs the high symmetry, but is isotopically invariant as before. By now a rather large number of possible high symmetry variants have been proposed for strong interactions, but only two of these can be used to explain the experimental situation with the resonances. Both these variants are based on symmetry under transformations of a unitary unimodular group in three dimensions, frequently called simply unitary group or unitary symmetry.

Unitary symmetry was first considered in the Sakata model by Ikeda, Ogawa, and Ohnuki [1], whose paper was followed by many others [2]. In the Sakata model, as is well known, the starting point is a triplet of baryons, possibly p, n, and $\Lambda$. The remaining particles are regarded as compound ones. The unitary symmetry signifies that both in the initial triplet and in the "compound" multiplets the masses and interactions of all particles are the same. In recent papers by Gell-Mann and Ne'eman [3] a second variant of unitary symmetry was proposed, in which one starts out from two triplets of certain "conceptual" particles; in this case all eight variants are compound and in the presence of unitary symmetry they form a degenerate octet. We shall henceforth call these two variants of unitary symmetry the 3-symmetry and 8-symmetry respectively.

Within the framework of the 3- or 8-symmetry one can have, generally speaking, different multiplets of mesons or baryons. In both variants, however, the simplest meson multiplets are singlets and octets. It is natural to think that the observed septet of $\pi$ and K mesons can be classified as an octet of this type. Here, obviously, there should exist an eighth particle, hitherto not observed, the so-called $\chi^0$ meson (pseudo-scalar in ordinary space and scalar in isotopic space). The question of the position that this particle occupies in the unitary symmetry scheme and concerning its decay properties was considered in many pepers [4]. In some papers it was called a $\sigma$ meson.

Resonances can be ascribed to a second octet (it is not very clear whether this octet should contain the $\omega^0$ resonance or the $\eta^0$ resonance, which is similar to the $\omega^0$ resonance in the $\pi^+$, $\pi^-$, $\pi^0$ system with mass 550 MeV). One can note that in transformations of the unitary group the law of transformation of particles within the octet is the same for both the 3- and the 8-symmetry.

The masses of the baryons, and particularly those of mesons, differ quite strongly from one another. This means that the unitary symmetry is approximate, if it exists at all. At low energies such an approximation should certainly be poor. Owing to the differences in the masses it is not even clear at what energies various amplitudes and cross sections can be compared, a factor essential for the verification of unitary symmetry. At high energies the mass difference is not so essential, but the scattering is principally of the diffraction type in this case, and is independent of the detailed structure of the interaction. It is possible just the same that a structure of this type does manifest itself in collisions (both elastic and inelastic) accompanied by large momentum transfer. In such nondiffraction processes small distances are significant, at which a unitarily-symmetrical interaction probably predominates.

If the described situation does indeed obtain, it is of interest to establish the relations between the cross sections or between the probabilities of different processes which follow from the unitary symmetry. A direct experimental verification of these relations will show the extent to which the unitary symmetry is correct and in which of its two variants. We shall write down below the "unitary" relations between the amplitudes of the observed processes of the type meson + nucleon → meson + baryon and baryon + nucleon → baryon + baryon. The resultant equations will be compared with the available experimental data. In the concluding section we shall discuss (in analogy with what was done in [5]) the possible manifestations of unitary symmetry in lepton decays of strange particles.

## 2. AMPLITUDES

In the case of meson + nucleon → meson + baryon processes we can write out 27 amplitudes, which are independent from the point of view of isotopic invariance:

$$\omega_1 = \langle p\pi^+ \mid p\pi^+\rangle = \langle n\pi^- \mid n\pi^-\rangle,$$
$$\omega_2 = \langle p\pi^- \mid p\pi^-\rangle = \langle n\pi^+ \mid n\pi^+\rangle,$$
$$\omega_3 = \langle pK^0 \mid pK^0\rangle = \langle nK^+ \mid nK^+\rangle,$$
$$\omega_4 = \langle pK^+ \mid pK^+\rangle = \langle nK^0 \mid nK^0\rangle,$$
$$\omega_5 = \langle pK^- \mid pK^-\rangle = \langle n\bar{K}^0 \mid n\bar{K}^0\rangle,$$
$$\omega_6 = \langle p\bar{K}^0 \mid p\bar{K}^0\rangle = \langle nK^- \mid nK^-\rangle,$$
$$\omega_7 = \langle n\pi^0 \mid p\pi^-\rangle = -\langle p\pi^0 \mid n\pi^+\rangle,$$
$$\omega_8 = \langle n\chi^0 \mid p\pi^-\rangle = \langle p\chi^0 \mid n\pi^+\rangle,$$
$$\omega_9 = \langle nK^+ \mid pK^0\rangle = \langle pK^0 \mid nK^+\rangle,$$
$$\omega_{10} = \langle n\bar{K}^0 \mid pK^-\rangle = \langle pK^- \mid n\bar{K}^0\rangle,$$
$$\omega_{11} = \langle \Lambda K^0 \mid p\pi^-\rangle = \langle \Lambda K^+ \mid n\pi^+\rangle,$$
$$\omega_{12} = \langle \Lambda\pi^+ \mid p\bar{K}^0\rangle = \langle \Lambda\pi^- \mid nK^-\rangle,$$
$$\omega_{13} = \langle \Lambda\pi^0 \mid pK^-\rangle = -\langle \Lambda\pi^0 \mid n\bar{K}^0\rangle,$$
$$\omega_{14} = \langle \Lambda\chi^0 \mid pK^-\rangle = \langle \Lambda\chi^0 \mid n\bar{K}^0\rangle,$$
$$\omega_{15} = \langle \Sigma^-K^+ \mid p\pi^-\rangle = \langle \Sigma^+K^0 \mid n\pi^+\rangle,$$
$$\omega_{16} = \langle \Sigma^-\pi^+ \mid pK^-\rangle = \langle \Sigma^+\pi^- \mid n\bar{K}^0\rangle,$$
$$\omega_{17} = \langle \Sigma^+K^+ \mid p\pi^+\rangle = \langle \Sigma^-K^0 \mid n\pi^-\rangle,$$
$$\omega_{18} = \langle \Sigma^+\pi^- \mid pK^-\rangle = \langle \Sigma^-\pi^+ \mid n\bar{K}^0\rangle,$$
$$\omega_{19} = \langle \Sigma^+\pi^0 \mid p\bar{K}^0\rangle = -\langle \Sigma^-\pi^0 \mid nK^-\rangle,$$
$$\omega_{20} = \langle \Sigma^+\chi^0 \mid p\bar{K}^0\rangle = \langle \Sigma^-\chi^0 \mid nK^-\rangle,$$
$$\omega_{21} = \langle \Sigma^0K^0 \mid p\pi^-\rangle = -\langle \Sigma^0K^+ \mid n\pi^+\rangle,$$
$$\omega_{22} = \langle \Sigma^0\pi^+ \mid p\bar{K}^0\rangle = -\langle \Sigma^0\pi^- \mid nK^-\rangle,$$
$$\omega_{23} = \langle \Sigma^0\pi^0 \mid pK^-\rangle = \langle \Sigma^0\pi^0 \mid n\bar{K}^0\rangle,$$
$$\omega_{24} = \langle \Sigma^0\chi^0 \mid pK^-\rangle = -\langle \Sigma^0\chi^0 \mid n\bar{K}^0\rangle$$
$$\omega_{25} = \langle \Xi^0K^0 \mid pK^-\rangle = -\langle \Xi^-K^+ \mid n\bar{K}^0\rangle,$$
$$\omega_{26} = \langle \Xi^-K^+ \mid pK^-\rangle = -\langle \Xi^0K^0 \mid n\bar{K}^0\rangle,$$
$$\omega_{27} = \langle \Xi^0K^+ \mid p\bar{K}^0\rangle = -\langle \Xi^-K^0 \mid nK^-\rangle. \quad (1)$$

Equations (1) do not include amplitudes of reactions in which $\pi^0$ or $\chi^0$ participate in the initial state, since such reactions are practically unobservable.

Analogously, we can write for the amplitudes of processes such as baryon + nucleon → baryon + baryon

$$\Omega_1 = \langle pn \mid pn\rangle = \langle np \mid np\rangle,$$
$$\Omega_2 = \langle p\Sigma^- \mid p\Sigma^-\rangle = \langle n\Sigma^+ \mid n\Sigma^+\rangle,$$
$$\Omega_3 = \langle p\Sigma^+ \mid p\Sigma^+\rangle = \langle n\Sigma^- \mid n\Sigma^-\rangle,$$
$$\Omega_4 = \langle p\Xi^- \mid p\Xi^-\rangle = \langle n\Xi^0 \mid n\Xi^0\rangle,$$
$$\Omega_5 = \langle p\Xi^0 \mid p\Xi^0\rangle = \langle n\Xi^- \mid n\Xi^-\rangle,$$
$$\Omega_6 = \langle n\Xi^0 \mid p\Xi^-\rangle = \langle p\Xi^- \mid n\Xi^0\rangle,$$
$$\Omega_7 = \langle \Sigma^+\Sigma^- \mid p\Xi^-\rangle = -\langle \Sigma^-\Sigma^+ \mid n\Xi^0\rangle,$$
$$\Omega_8 = \langle \Lambda\Lambda \mid p\Xi^-\rangle = -\langle \Lambda\Lambda \mid n\Xi^0\rangle,$$
$$\Omega_9 = \langle \Sigma^0\Sigma^0 \mid p\Xi^-\rangle = -\langle \Sigma^0\Sigma^0 \mid n\Xi^0\rangle,$$
$$\Omega_{10} = \langle \Lambda\Sigma^0 \mid p\Xi^-\rangle = \langle \Lambda\Sigma^0 \mid n\Xi^0\rangle,$$
$$\Omega_{11} = \langle \Sigma^0\Lambda \mid p\Xi^-\rangle = \langle \Sigma^0\Lambda \mid n\Xi^0\rangle,$$
$$\Omega_{12} = \langle \Sigma^+\Lambda \mid p\Xi^0\rangle = -\langle \Sigma^-\Lambda \mid n\Xi^-\rangle,$$
$$\Omega_{13} = \langle \Sigma^+\Sigma^0 \mid p\Xi^0\rangle = \langle \Sigma^-\Sigma^0 \mid n\Xi^-\rangle,$$
$$\Omega_{14} = \langle n\Lambda \mid p\Sigma^-\rangle = \langle p\Lambda \mid n\Sigma^+\rangle,$$
$$\Omega_{15} = \langle n\Sigma^0 \mid p\Sigma^-\rangle = -\langle p\Sigma^0 \mid n\Sigma^+\rangle,$$
$$\Omega_{16} = \langle n\Sigma^+ \mid p\Lambda\rangle = \langle p\Sigma^- \mid n\Lambda\rangle,$$
$$\Omega_{17} = \langle n\Sigma^+ \mid p\Sigma^0\rangle = -\langle p\Sigma^- \mid n\Sigma^0\rangle,$$
$$\Omega_{18} = \langle p\Lambda \mid p\Lambda\rangle = \langle n\Lambda \mid n\Lambda\rangle,$$
$$\Omega_{19} = \langle p\Sigma^0 \mid p\Lambda\rangle = -\langle n\Sigma^0 \mid n\Lambda\rangle,$$
$$\Omega_{20} = \langle p\Lambda \mid p\Sigma^0\rangle = -\langle n\Lambda \mid n\Sigma^0\rangle,$$
$$\Omega_{21} = \langle p\Sigma^0 \mid p\Sigma^0\rangle = \langle n\Sigma^0 \mid n\Sigma^0\rangle. \quad (2)$$

Equations (2) do not include the amplitude $\langle pp \mid pp\rangle = \langle nn \mid nn\rangle$, since it obviously coincides with the amplitude of scattering of a proton by a neutron in state with isotopic spin 1, i.e., in an odd spin-orbit state.

Any new symmetry, higher than isotopic invariance, will lead to the appearance of several relations between the written amplitudes. The method used to obtain these relations is described in the Appendix. The results are presented below.

### 3. 3-SYMMETRY

If 3-symmetry holds true, then only three of the first 14 amplitudes are independent. The remainder are expressed in terms of these three in accordance with the equations

$$\omega_4 = \omega_1, \quad \omega_5 = \omega_2, \quad \omega_6 = \omega_3, \quad \omega_7 = (\omega_1 - \omega_2)/\sqrt{2},$$
$$\omega_8 = (\omega_1 + \omega_2 - 2\omega_3)/\sqrt{6},$$
$$\omega_9 = \omega_{12} = \sqrt{2}\,\omega_{13} = \omega_1 - \omega_3, \quad \omega_{10} = \omega_{11} = \omega_2 - \omega_3,$$
$$\omega_{14} = (\omega_1 + \omega_3 - 2\omega_2)/\sqrt{6}. \tag{3}$$

Analogously

$$\Omega_1 = \Omega_{18}. \tag{4}$$

Relations of the type (3) and (4) can be obtained also for amplitudes of other processes. To find the most interesting relations it is sufficient to use only the invariance with respect to (A.2). In particular, the following relations hold true

$$\langle n\pi^+\pi^+ \mid p\pi^+\rangle = \langle \Lambda K^+K^+ \mid pK^+\rangle,$$
$$\langle n\pi^+\pi^- \mid p\pi^-\rangle = \langle \Lambda K^+K^- \mid pK^-\rangle,$$
$$\langle nK^+K^- \mid p\pi^-\rangle = \langle \Lambda\pi^+\pi^- \mid pK^-\rangle,$$
$$\langle pK^-K^0 \mid p\pi^-\rangle = \langle p\pi^-\bar{K}^0 \mid p\bar{K}^-\rangle, \quad \langle pn\pi^+ \mid pp\rangle = \langle p\Lambda K^+ \mid pp\rangle,$$
$$\langle \pi^+\pi^- \mid \bar{p}p\rangle = \langle K^+K^- \mid \bar{p}p\rangle, \quad \langle 2\pi^+2\pi^- \mid \bar{p}p\rangle = \langle 2K^+2K^- \mid \bar{p}p\rangle \tag{5}$$

### 4. 8-SYMMETRY

In a case of 8-symmetry, only five of all the 27 amplitudes $\omega_i$ are independent (this circumstance was already noted in [3]). The relations between the $\omega_i$ have the following form (it is convenient to regard $\omega_1$, $\omega_4$, $\omega_5$, $\omega_7$, and $\omega_8$ as independent):

$$\omega_3 = \omega_1, \quad \omega_2 = \omega_6 = \omega_1 - \sqrt{2}\,\omega_7, \quad \omega_9 = \omega_{17} = \omega_4 - \omega_1,$$
$$\omega_{10} = \omega_{18} = \omega_5 - \omega_1 + \sqrt{2}\,\omega_7, \quad \omega_{11} = (\sqrt{3}\omega_7 - \omega_8)/2,$$
$$\omega_{12} = \sqrt{2}\omega_{13} = \omega_{20} = \sqrt{2}\,\omega_{24} = -(\sqrt{3}\omega_7 + \omega_8)/2,$$
$$\omega_{14} = \omega_5 - \omega_1 + (3\sqrt{3}\,\omega_7 - 5\omega_8)/2\sqrt{6},$$
$$\omega_{15} = \omega_{27} = \omega_4 - \omega_1 - (\omega_7 + \sqrt{3}\,\omega_8)/2,$$
$$\omega_{16} = \omega_{25} = \omega_5 - \omega_1 + (\sqrt{3}\,\omega_7 - \omega_8)\sqrt{3/2},$$
$$\omega_{19} = -\omega_{22} = (\omega_7 - \sqrt{3}\,\omega_8)/2, \quad \omega_{21} = (\omega_7 + \sqrt{3}\,\omega_8)/2,$$
$$\omega_{23} = \omega_5 - \omega_1 + (5\omega_7 - \sqrt{3}\omega_8)/2\sqrt{2},$$
$$\omega_{26} = 2\omega_1 - \omega_4 - \omega_5 - \sqrt{2}\omega_7 + \sqrt{6}\,\omega_8. \tag{6}$$

Out of the 21 amplitudes $\Omega_i$, three are independent. The equations relating the $\Omega_i$ have the form:

$$\Omega_3 = \Omega_1, \quad \Omega_5 = \Omega_2, \quad \Omega_6 = \Omega_9 = \Omega_7, \quad \Omega_{13} = 0,$$
$$\Omega_4 = \Omega_2 - \Omega_7, \quad \Omega_8 = \tfrac{2}{3}(\Omega_1 - \Omega_2) + \Omega_7,$$
$$\Omega_{18} = -(\Omega_1 + 5\Omega_2)/6, \quad \Omega_{21} = -(\Omega_1 + \Omega_2)/2,$$
$$\sqrt{3}\Omega_{10} = \sqrt{3}\Omega_{11} = -\sqrt{3/2}\,\Omega_{12} = \sqrt{6}\Omega_{14} = \sqrt{2}\Omega_{15} = \sqrt{6}\Omega_{16}$$
$$= -\sqrt{2}\Omega_{17} = 2\sqrt{3}\Omega_{19} = 2\sqrt{3}\Omega_{20} = \Omega_1 - \Omega_2. \tag{7}$$

In analogy with (5), the following equations hold true in the case of 8-symmetry

$$\langle n\pi^+\pi^+ \mid p\pi^+\rangle = \langle \Sigma^+K^0K^0 \mid pK^0\rangle,$$
$$\langle n\pi^+\pi^- \mid p\pi^-\rangle = \langle \Sigma^+K^0\bar{K}^0 \mid p\bar{K}^0\rangle,$$
$$\langle nK^+K^- \mid p\pi^-\rangle = \langle \Sigma^+K^+K^- \mid p\bar{K}^0\rangle,$$
$$\langle pK^-K^0 \mid p\pi^-\rangle = \langle pK^-\pi^+ \mid p\bar{K}^0\rangle,$$
$$\langle pn\pi^+ \mid pp\rangle = \langle p\Sigma^+K^0 \mid pp\rangle,$$
$$\langle \pi^+\pi^- \mid \bar{p}p\rangle = \langle K^0\bar{K}^0 \mid \bar{p}p\rangle,$$
$$\langle 2\pi^+2\pi^- \mid \bar{p}p\rangle = \langle 2K^02\bar{K}^0 \mid \bar{p}p\rangle \tag{8}$$

etc.

### 5. COMPARISON WITH EXPERIMENT

As was already noted, it would be desirable to compare relations (3)—(8) with the cross sections of the corresponding processes, accompanied by large momentum transfer. At the present time there are no such data. The equations obtained can therefore be compared for the time being only with experiments in which the integral cross sections of elastic or inelastic processes, occurring upon collisions between mesons or baryons and nucleons, were measured at energies on the order of several BeV, and also with total meson-nucleon and baryon-nucleon cross sections. Of course, we cannot expect good agreement. It is more likely that the character of the "disagreement" is of interest.

At present the most complete data are those pertaining to the integral cross sections of meson-proton interactions. According to [6], in the laboratory-system momentum range from 10 to 20 BeV/c, the total cross section $\sigma_t(\pi^+p)$ ranges between 24.8 and 23.5 mb, while $\sigma_t(\pi^-p)$ ranges from 26.9 to 25.6 mb. In a similar momentum interval, $\sigma_t(K^+p)$ remains approximately constant at about 18 mb, whereas $\sigma_t(K^-p)$ varies from ~24 to ~22 mb.[7] If we compare these figures with the 3-symmetry equations $\omega_4 = \omega_1$ and $\omega_5 = \omega_2$, predicted in (3), whence

$$\sigma_t(pK^+) = \sigma_t(p\pi^+), \quad \sigma_t(pK^-) = \sigma_t(p\pi^-), \tag{9}$$

we see that the second of these equations is accurate to 10—15 per cent, whereas the first is accurate only to 30 per cent. One must note, however, still another circumstance. According to the well known theorem of Pomeranchuk [8], the following asymptotic equations should obtain at very large energies

$$\sigma_t(K^-p) \approx \sigma_t(K^+p), \qquad \sigma_t(\pi^-p) \approx \sigma_t(\pi^+p). \tag{10}$$

(if the cross sections decrease with increase in energy E no faster than $(\ln e)^{-1}$, then (10) can be proved rigorously).

The experimental data presented above for energies ~10—20 BeV show that the corresponding cross sections have a tendency to come close to each other. Inasmuch as the approximate equality $\sigma_t(K^-p) = \sigma_t(\pi^-p)$ already takes place at relatively low energy, it is natural to think that in a region in which relations (10) will be valid we can approximately equate also $\sigma_t(K^+p)$ and $\sigma_t(\pi^+p)$.

The 8-symmetry predicts in (6) the equalities $\omega_3 = \omega_1$ and $\omega_6 = \omega_2$, from which it follows that

$$\sigma_t(nK^+) = \sigma_t(p\pi^+), \qquad \sigma_t(nK^-) = \sigma_t(p\pi^-). \tag{11}$$

The cross sections of Kn interactions at energies ~10—20 BeV have not been measured so far. Data on $K^+n$ scattering are given only for momenta 1—2.8 BeV/c. In this region $\sigma_t(K^+n) = 18$ mb, whereas $\sigma_2(\pi^+p) \approx 29$ mb. For $K^-n$ collisions data are available in the momentum range 2.5—4 BeV/c, where $\sigma_t(K^-n)$ changes from 22.5 to 20.5 mb [10]. In the same region, $\sigma_t(\pi^-p)$ varies slightly about 30 mb. The agreement with 8-symmetry is poor, but the energies are still small. Somewhat better agreement is obtained with the 3-symmetry equation $\omega_6 = \omega_3$, i.e., $\sigma_t(K^+n) = \sigma_t(K^-n)$.

The partial cross sections of the different reactions at large energies are likewise still unknown. A comparison of elastic cross sections of the scattering of $K^-$ and $\pi^-$ by a proton at 1.5 BeV, where $\sigma(K^- + p \to K^+ + p) = 8 \pm 1.5$ mb [10] and $\sigma(\pi^- + p \to \pi^- + p) = 9 \pm 1.5$ mb [11], shows too good an agreement with the 3-symmetry relations $\sigma(K^- + p \to K^- + p) = \sigma(\pi^- + p \to \pi^- + p)$ to be regarded as accidental.

Among the inelastic processes, the most information can be extracted at the present time apparently from experiments on proton-antiproton annihilation. If we select only those cases in which each meson emitted upon annihilation has an energy which is large compared with its rest mass, then the difference in the masses becomes insignificant, and we can expect the equalities of unitary symmetry to be satisfied, particularly the last relation in (5) or (8). Experimental data on the 2-meson annihilation, which is the most favored from this point of view, are available for a momentum 1.61 BeV/c [12]. According to these data $\sigma(\bar{p} + p \to \pi^+ + \pi^-) = 0.1 \pm 0.025$ mb, $\sigma(\bar{p} + p \to K^+ + K^-) = 0.055 \pm 0.018$ mb, and with 90 per cent probability we have $\sigma(\bar{p} + p \to K^0 + \bar{K}^0) < 0.05$ mb. At the same time, according to (5) and (6) we should have

$$\sigma(\bar{p} + p \to \pi^+ + \pi^-) = \sigma(\bar{p} + p \to K^+ + K^-) \quad \text{(3-symmetry)}$$
$$\sigma(\bar{p} + p \to \pi^+ + \pi^-) = \sigma(\bar{p} + p \to K^0 + \bar{K^0}) \quad \text{(8-symmetry)} \tag{12}$$

From this, however, we can still not conclude that the 3-symmetry or the 8-symmetry is incorrect, since the experimental errors are large and the total energy is too small (at a momentum of 1.61 BeV/c, the c. m. s. energy per meson is 1.14 BeV, which is merely 2.3 times the rest energy of the K meson).

From the foregoing comparison with the experimental data we see, first, that the data are still insufficient and, second, that they agree somewhat better with 3-symmetry than with 8-symmetry. Some verification of relations (3)—(8) or their analogs will become possible only after the cross sections of different processes are measured at large energies and large momentum transfers.

## 6. UNITARY SYMMETRY AND LEPTON DECAYS OF STRANGE PARTICLES.

It was shown in several papers [5,13,14] that in the presence of unitary symmetry simple numerical relations exist between the nonrenormalized constants of the weak interaction responsible for the lepton decays of strange particles. Since unitary symmetry is violated in strong interactions, the decay constant will change as a result of the renormalization. In this case no trace can remain of the indicated relations. Nonetheless, there is the known case of the axial constant in ordinary beta decay, when the influence of renormalization is small, although it should appear there because of the unitary symmetrical interaction. We can therefore attempt to compare the ''unitary'' relations between the constants and the experimental data. This was precisely the procedure used by Kobzarev and Okun' [6]. We show below that the relations between the decay constants is conveniently formulated in terms of conserving currents, and derive several new relations.

Unitary 8-symmetry or 3-symmetry corresponds to the vanishing of the divergences of the current (A.1) or (A.3). It can be stated that the first (charged) components of this current coincide with the current contained in the vector part of weak interactions and leading to baryon or meson decay with change of strangeness. In the Sakata model (3-symmetry) this is unavoidable, since the theory contains only one iso-spinor current $\overline{N}\gamma_\lambda\Lambda$ [the current (A.1) is its Yukawa version, which realizes the same isotopic transformation]. In this case the non-renormalized vector constants $C_V^0$ in the lepton decays of strange particles will be related to one another simply as the coefficients of the different terms in the current (A.3) or (A.1).

In the case of 3-symmetry, however, the operators of the $\Sigma$ and $\Xi$ hyperons are not contained explicitly in the current. It is therefore necessary to employ additional considerations here. It is natural to think that in the unitary symmetry scheme the $\Sigma$ and $\Xi$ consist of a minimum possible number of bare particles, i.e., of triplets. Then the $\Xi$ are defined uniquely:

$$\Xi^0 = (\bar{n}\Lambda\Lambda), \qquad \Xi^- = -(\bar{p}\Lambda\Lambda), \tag{13}$$

whereas for $\Sigma$ we have two possibilities: Either

$$\Sigma^+ = (\bar{n}p\Lambda + \bar{n}\Lambda p)/\sqrt{2}, \qquad \Sigma^- = (\bar{p}n\Lambda + \bar{p}\Lambda n)/\sqrt{2},$$
$$\Sigma^0 = (\bar{p}p\Lambda + \bar{p}\Lambda p - \bar{n}n\Lambda - \bar{n}\Lambda n)/2, \tag{14}$$

or

$$\Sigma^+ = (\bar{n}p\Lambda - \bar{n}\Lambda p)/\sqrt{2}, \qquad \Sigma^- = (\bar{p}n\Lambda - \bar{p}\Lambda n)/\sqrt{2},$$
$$\Sigma^0 = (\bar{p}p\Lambda - \bar{p}\Lambda p - \bar{n}n\Lambda + \bar{n}\Lambda n)/2. \tag{15}$$

Relations (13) and (14) pertain to one and the same presentation of the unitary group (the so-called 15-representation), whereas (15) pertains to another representation (the 6-representation). Therefore in a transition generated by the current $\bar{p}\gamma_\lambda\Lambda$ or its analog (A.1), the operators (13) are transformed into (14), but not into (15). If $\Sigma$ were to realize the representation (14), then by considering $\Sigma$ and $\Xi$ as "elementary," one could ascribe to (A.1) a term

$$-\sqrt{2}\,[-(\bar{\Sigma}^0\gamma_\lambda\Xi^-) + \sqrt{2}\,(\bar{\Sigma}^+\gamma_\lambda\Xi^0)]. \tag{16}$$

The current $\bar{p}\gamma_\lambda\Lambda$ leads not only to the transition $\Lambda \rightarrow p$, the consequence of which is in particular (16), but also to the annihilation of $\bar{p}$ and $\Lambda$; in the latter case transitions of $\Xi^-$ into $\Lambda$ will take place, and also transitions of $\Sigma^0$ and $\Sigma^-$ into nucleons. If we denote the vector constant in the amplitude of the transition $\Xi^- \rightarrow \Lambda - \sqrt{x}$, then in the case of (14) the constants of the transition of $\Sigma^-$ into n and of $\Sigma^0$ into p are respectively equal to $\sqrt{x/2}$ and $\sqrt{x}/2$. In the case of (15) the transitions of the latter type occur, generally speaking, with a different constant.

| Decay | 3-symmetry $[C_V^0/(C_V^0)_K]^2$ | 8-symmetry $[C_V^0/(C_V^0)_K]^2$ | Experiment $(\sqrt{2}\,C_V/G)^2$ |
|---|---|---|---|
| $K^{\mp} \rightarrow \pi^0 + l^{\pm} + \nu$ | 1 | 1 | 1/30 |
| $K^0_{1,2} \rightarrow \pi^{\mp} + l^{\pm} + \nu$ | 1 | 1 | 1/30 |
| $\chi^0 \rightarrow K^{\mp} + l^{\pm} + \nu$ | 3 | 3 | |
| $\Lambda \rightarrow p + l^- + \bar{\nu}$ | 2 | 3 | 1/20—1/10 |
| $\Sigma^- \rightarrow n + l^- + \bar{\nu}$ | $x/2$ *, $y$ ** | 2 | 1/40—1/20 |
| $\Sigma^0 \rightarrow p + l^- + \bar{\nu}$ | $x/4$ *, $y/2$** | 1 | |
| $\Xi^- \rightarrow \Lambda + l^- + \bar{\nu}$ | $x$ | 3 | |
| $\Xi^- \rightarrow \Sigma^0 + l^- + \bar{\nu}$ | 2 *, 0 ** | 1 | |
| $\Xi^0 \rightarrow \Sigma^+ + l^- + \bar{\nu}$ | 4 *, 0 ** | 2 | |

*Corresponds to relation (14).
**Corresponds to relation (15).

The relative values of $(C_V^0)^2$ in the case of both 3- and 8-symmetry have been written out in the table. The 3-symmetry relations for the decays of K, $\chi$, and $\Lambda$ were obtained earlier in [5,13], and the 8-symmetry relations in the case of hyperon decay have been obtained in [14]. In the table $l$ denotes an electron or a muon, while x and y are unknown constants.

The experimental values of the squares of the constants in $G^2/2$ units, where $G = 1.41 \times 10^{-49}$ erg-cm$^3$, are listed in the last column of the table. In the case of the $K^+$ decay, the corresponding number was obtained in several investigations, particularly in [15]. The equality of the constants in lepton decays $K^+$ and $K^0$, which already follows from the hypothesis of the correctness of the $\Delta T = 1/2$ rules in decays with change of strangeness, is confirmed by experiment [16]. The values of the vector constants in the decays of the hyperons have been derived from the data of Humphrey et al [17], and furthermore we put $C_A = -1.25\ C_V$, inasmuch as within the framework of unitary symmetry the relation $C_A/C_A^0 \approx -C_A/C_V^0$ should be the same as in ordinary beta decay. It is seen from the table that the experimental data exhibit a barely noticeable better agreement with 3-symmetry than with 8-symmetry.

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## APPENDIX

## RELATIONS BETWEEN AMPLITUDES

In establishing relations between amplitudes of different processes it is necessary, obviously, to start out from transformations with respect to which the theory is invariant. In the case of a continuous group, such transformations are known if the generators of infinitesimally small transformations are specified, or else if one specifies the conserving currents from which the generators are constructed. The question of finding various symmetry properties in the theory of strong interactions in the Yukawa form and the corresponding conserving currents was solved in the papers by Behrends and Sirlin [18] and by the author [19], in the latter case account being taken of the possible existence of the $\chi^0$ meson (there denoted as the $\rho^0$ meson). The Yukawa form turned out to be convenient, because all the particles are considered elementary there and enter directly into the expressions for the generators or the currents.

Corresponding to the 3-symmetry is a current which can be obtained as a solution of the algebraic system (28) written out in [19] in the absence of Σ and Ξ interactions, when only $d_1$, $d_5$, $d_6$, $g_1$, $\equiv g_{N\pi}$, $g_5 \equiv g_{\Lambda K}$, $g_9 \equiv g_{N\chi}$; and $g_{10} \equiv g_{\Lambda\chi}$ are different from zero. Such a solution turns out to be unique: When

$$g_{N\pi} = (1/\sqrt{2})\, g_{\Lambda K} = \sqrt{3} g_{N\chi} = -(\sqrt{3}/2)\, g_{\Lambda\chi}$$

the current conserved has components

$$(j_\lambda^3)_{1/2} = -\sqrt{2}(\bar{p}\gamma_\lambda\Lambda) - (\overline{K^+}\partial_\lambda\pi^0) - \sqrt{2}(\overline{K^0}\partial_\lambda\pi^-) - \sqrt{3}(\overline{K^+}\partial_\lambda\chi^0),$$
$$(j_\lambda^3)_{-1/2} = -\sqrt{2}(\bar{n}\gamma_\lambda\Lambda) + (\overline{K^0}\partial_\lambda\pi^0) - \sqrt{2}(\overline{K^+}\partial_\lambda\pi^+) - \sqrt{3}(\overline{K^0}\partial_\lambda\chi^0). \qquad (A.1)$$

The letters denote here the annihilation operators for the corresponding quantities; the symbol $\partial_\lambda$ denotes the operation

$$\Phi_2\partial_\lambda\Phi_1 \equiv \Phi_2\frac{\partial\Phi_1}{\partial x_\lambda} - \frac{\partial\Phi_2}{\partial x_\lambda}\Phi_1.$$

One can also note that in case of 3-symmetry the theory is invariant with respect to the transformations

$$p \to p, \quad n \to \Lambda, \quad \Lambda \to -n,$$
$$\pi^+ \to K^+, \quad K^+ \to -\pi^+, \quad \pi^- \to K^-, \quad K^- \to -\pi^-,$$
$$K^0 \to -\overline{K^0}, \quad \overline{K^0} \to -K^0, \quad \pi^0 \to (\pi^0 + \sqrt{3}\chi^0)/2,$$
$$\chi^0 = (\sqrt{3}\pi^0 - \chi^0)/2. \qquad (A.2)$$

The meaning of (A.2) is as follows. If unitary symmetry holds true, then arbitrary unitary and unimodular transformations of the triplet p, n, Λ are possible, particularly the spinor transformations of the pair n, Λ. Equation (A.2) is equivalent to the transformation of this pair ($n \to \Lambda$, $\Lambda \to -n$), corresponding to rotation of 180° about the second axis in the corresponding isotopic space. The "compound" mesons are likewise transformed.

As regards the 8-symmetry, it corresponds to conservation of the current (31) from [19] with components

$$(j_\lambda^8)_{1/2} = \sqrt{3}(\bar{p}\gamma_\lambda\Lambda) + (\bar{p}\gamma_\lambda\Sigma^0) + \sqrt{2}(\bar{n}\gamma_\lambda\Sigma^-) + \sqrt{3}(\bar{\Lambda}\gamma_\lambda\Xi^-) + (\Sigma^0\gamma_\lambda\Xi^-) - \sqrt{2}(\overline{\Sigma^+}\gamma_\lambda\Xi^0) - (\overline{K^+}\partial_\lambda\pi^0) - \sqrt{2}(\overline{K^0}\partial_\lambda\pi^-) - \sqrt{3}(\overline{K^+}\partial_\lambda\chi^0),$$
$$(j_\lambda^8)_{-1/2} = \sqrt{3}(\bar{n}\gamma_\lambda\Lambda) - (\bar{n}\gamma_\lambda\Sigma^0) + \sqrt{2}(\bar{p}\gamma_\lambda\Sigma^+) - \sqrt{3}(\bar{\Lambda}\gamma_\lambda\Xi^0) + (\overline{\Sigma^0}\gamma_\lambda\Xi^0) + \sqrt{2}(\overline{\Sigma^-}\gamma_\lambda\Xi^-) + (\overline{K^0}\partial_\lambda\pi^0) - \sqrt{2}(\overline{K^+}\partial_\lambda\pi^+) - \sqrt{3}(\overline{K^0}\partial_\lambda\chi^0). \qquad (A.3)$$

The relations between the Yukawa constants, corresponding to the conservation of the current (A.3), have been written out in [19] [the inessential sign multipliers ε, ε′, − ε″, and ε‴ are set equal to + 1 in (A.3)]. Gell-Mann [3] also uses the additional equality $g_{N\pi} = -g_{\Xi\pi}$, by virtue of which there is additional invariance under the discrete transformation

$$R \equiv \begin{cases} p \leftrightarrow -\Xi^-, & n \leftrightarrow \Xi^0, \quad \Sigma^+ \leftrightarrow \Sigma^-, \quad \Sigma^0 \to \Sigma^0, \quad \Lambda \to \Lambda, \\ K^+ \leftrightarrow K^-, & K^0 \leftrightarrow \overline{K^0}, \quad \pi^+ \leftrightarrow \pi^-, \quad \pi^0 \to \pi^0, \quad \chi^0 \to \chi^0. \end{cases} \qquad (A.4)$$

In the case of 8-symmetry, the analog of (A.2) is the transformation

$$S_2 \equiv \begin{cases} p \to -\Sigma^+, \quad \Sigma^+ \to p, \quad \Sigma^- \to -\Xi^-, \quad \Xi^- \to \Sigma^-, \\ n \to -\Xi^0, \quad \Xi^0 \to -n, \quad \Sigma^0 \to (\Sigma^0 + \sqrt{3}\Lambda)/2, \\ \Lambda \to (\sqrt{3}\Sigma^0 - \Lambda)/2, \\ K^+ \to -\pi^+, \quad \pi^+ \to K^+, \quad K^- \to -\pi^-, \quad \pi^- \to K^-, \\ K^0 \to -\overline{K^0}, \quad \overline{K^0} \to -K^0, \quad \pi^0 \to (\pi^0 + \sqrt{3}\chi^0)/2, \\ \chi^0 \to (\sqrt{3}\pi^0 - \chi^0)/2. \end{cases} \qquad (A.5)$$

The analogy with (A.2) lies in the fact that in the Gell-Mann scheme [3] all the particles are constructed (in the sense of isotopic structure) from three "conceptual" fermions, denoted ν, $e^-$, and $\mu^-$, and also from an analogous triplet of bosons $D^0$, $D^-$, and $S^-$. Equation (A.5) precisely corresponds to the spinor transformation of the pairs $e^-\mu^-$ and $D^-S^-$ ($e^- \to \mu^-$, $\mu^- \to -e^-$, $D^- \to S^-$, $S^- \to -D^-$), analogous to the transformation of n and Λ in (A.2). In exactly the same way, the discrete transformation (A.4) corresponds to invariance under discrete transformation of the charge-conjugation type: $\nu \longleftrightarrow \overline{D^0}$, $e \longleftrightarrow \overline{D^-}$, $\mu^- \longleftrightarrow \overline{S^-}$.

In place of (A.4) and (A.5) it is more convenient to use the transformation $T_2S_2RT_2$ ($T_2$ denotes rotation through 180° in ordinary isospace: $p \to n$, $n \to -p$, $\Lambda \to \Lambda$, $\Sigma^\pm \to -\Sigma^\mp$, $\Sigma^0 \to -\Sigma^0$, $\Xi^0 \to \Xi^-$, $\Xi^- \to -\Xi^0$, etc.), i.e.,

$$p \to p, \qquad \Sigma^0 \to (\Sigma^0 - \sqrt{3}\Lambda)/2,$$
$$K^+ \to K^+,$$
$$n \leftrightarrow -\Sigma^+, \qquad \Lambda \to -(\sqrt{3}\Sigma^0 + \Lambda)/2, \qquad K^0 \leftrightarrow -\pi^+,$$

$$\Xi^0 \leftrightarrow -\Sigma^-, \quad \pi^0 \to (\pi^0 - \sqrt{3}\chi^0)/2, \quad \overline{K}^0 \leftrightarrow -\pi^-,$$
$$\Xi^- \to \Xi^-, \quad \chi^0 \to -(\sqrt{3}\pi^0 + \chi^0)/2, \quad K^- \to K^-. \tag{A.6}$$

For example, to obtain relations between the amplitudes (1) in the case of 8-symmetry we can use the invariance of the theory with respect to (A.6). Application of this transformation yields immediately

$$\omega_1 = \omega_3, \quad \omega_2 = \omega_6, \quad \omega_9 = \omega_{17},$$
$$\omega_{10} = \omega_{18}, \quad \omega_{15} = \omega_{27}, \quad \omega_{16} = \omega_{25},$$
$$\omega_{12} = -(\omega_{11} + \sqrt{3}\omega_{21})/2, \quad \omega_{22} = (-\sqrt{3}\omega_{11} + \omega_{21})/2,$$
$$\omega_{19} = (\omega_7 - \sqrt{3}\omega_8)/2, \quad \omega_{20} = -(\omega_7\sqrt{3} + \omega_8)/2,$$
$$\omega_{13} = \omega_{24} = \sqrt{3}(\omega_{14} - \omega_{23})/2. \tag{A.7}$$

Analogously, in the case of 3-symmetry it follows from the invariance with respect to (A.2) that

$$\omega_1 = \omega_4, \quad \omega_2 = \omega_5, \quad \omega_3 = \omega_6, \quad \omega_9 = \omega_{12}, \quad \omega_{10} = \omega_{11},$$
$$\omega_7 = (\omega_{13} + \sqrt{3}\omega_{14})/2, \quad \omega_8 = (\sqrt{3}\omega_{13} - \omega_{14})/2. \tag{A.8}$$

In order to find the remaining relations between the amplitudes, it is necessary to consider the arbitrary transformations $\nu$, $e^-$, $\mu^-$, $D^0$, $D^-$, $S^-$, or p, n, $\Lambda$. We can use instead a different method. It is easy to see that the vanishing of the divergences of the second components of the currents (A.3) and (A.1) is equivalent to the conservation of the operators

$$-2\hat{N}_{S_1} + 2\hat{N}_{S_2} + \hat{N}_{A_1} - \hat{N}_{A_2} + \hat{N}_{B_1} - \hat{N}_{B_2} + \hat{N}_{L_1} - \hat{N}_{L_2} - 2\hat{N}_M \tag{A.9}$$

(8-symmetry) or

$$-\hat{N}_{R_1} + \hat{N}_{R_2} + \hat{N}_{L_1} - \hat{N}_{L_2} - 2\hat{N}_M \tag{A.10}$$

(3-symmetry), where

$$R_{1,2} = \frac{n \pm \Lambda}{\sqrt{2}}, \quad S_{1,2} = \frac{Q_2 \pm N_2}{\sqrt{2}}, \quad Q_{1,2} = \frac{n \pm \Xi^0}{\sqrt{2}},$$
$$N_1 = \frac{\Lambda + \sqrt{3}\Sigma^0}{2}, \quad N_2 = \frac{\Sigma^0 - \sqrt{3}\Lambda}{2}, \quad A_{1,2} = \frac{p \pm \Sigma^+}{\sqrt{2}},$$
$$B_{1,2} = \frac{\Sigma^- \pm \Sigma^-}{\sqrt{2}}, \quad L_{1,2} = \frac{K^+ \pm \pi^+}{\sqrt{2}}, \quad M = \frac{K_2^0 + \Gamma_2^0}{\sqrt{2}},$$
$$\Gamma_1^0 = \frac{\chi^0 + \sqrt{3}\pi^0}{2}, \quad \Gamma_2^0 = \frac{\pi^0 - \sqrt{3}\chi^0}{2}, \quad K_{1,2}^0 = \frac{K^0 \pm \overline{K}^0}{\sqrt{2}}, \tag{A.11}$$

and the particle number operators are defined for baryons $\psi$, and bosons $\varphi$ as

$$\hat{N}_\psi = \int \psi^+(x)\psi(x)\,d^3x, \quad \hat{N}_\varphi = -\int \varphi^+(x)\,\partial_4\varphi(x)\,d^3x. \tag{A.12}$$

Using (A.9)—(A.11) we can obtain the remaining relations between the $\omega_i$. Considering, for example, in the case of 8-symmetry the interaction of $K^+$ and $\pi^+$ with protons, we have to write out the eigenvalues of the operator (A.9) in the $A_iL_j$ system. These quantum numbers are respectively equal to 2, 0, 0, and − 2 for $A_1L_1$, $A_2L_2$, $A_2L_1$, and $A_2L_2$. Hence

$$\langle A_1L_1 | A_1L_2\rangle = \langle A_1L_1 | A_2L_1\rangle = \langle A_1L_1 | A_2L_2\rangle$$
$$= \langle A_1L_2 | A_2L_2\rangle = \langle A_2L_1 | A_2L_2\rangle = 0. \tag{A.13}$$

Denoting in the amplitudes of the reactions that are allowed with respect to strangeness by

$$a_1 = \langle p\pi^+ | p\pi^+\rangle, \quad a_2 = \langle pK^+ | pK^+\rangle, \quad a_3 = \langle \Sigma^+K^+ | p\pi^+\rangle,$$
$$a_4 = \langle p\pi^+ | \Sigma^+K^+\rangle, \quad a_5 = \langle \Sigma^+\pi^+ | \Sigma^+\pi^+\rangle, \quad a_6 = \langle \Sigma^+K^+ | \Sigma^+K^+\rangle$$
$$(a_1 = \omega_1, \ a_2 = \omega_4, \ a_3 = \omega_{17}), \tag{A.14}$$

we can reduce (A.13) to the system

$$-a_1 + a_2 - a_3 + a_4 - a_5 + a_6 = 0,$$
$$a_1 + a_2 + a_3 - a_4 - a_5 - a_6 = 0,$$
$$-a_1 + a_2 - a_3 - a_4 + a_5 - a_6 = 0,$$
$$a_1 + a_2 - a_3 + a_4 - a_5 - a_6 = 0,$$
$$-a_1 + a_2 + a_3 - a_4 - a_5 + a_6 = 0. \tag{A.15}$$

(A.15) is solved in elementary fashion

$$a_1 = a_6, \quad a_2 = a_5, \quad a_3 = a_4, \quad -a_1 + a_2 - a_3 = 0, \tag{A.16}$$

i.e.,

$$-\omega_1 + \omega_4 - \omega_{17} = 0.$$

The last equation is given in (6). We can obtain in a similar manner the remaining relations in (3)—(8).

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