DIPOLE EXCITATIONS ON THE SUPERFLUID MODEL

V. V. BALASHOV, V. B. BELYAEV, and B. N. ZAKHAR'EV

Nuclear Physics Institute, Moscow State University

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The dipole state of the nucleus is treated as a superposition of a large number of two-quasiparticle excitations. Inclusion of nucleon pairing produces practically no change in the energy of this state (the position of the "giant resonance" for photoabsorption) from its single-particle value. The required increase in energy is achieved by introducing, in addition to an interaction of the pairing type, a dipole-dipole interaction between nucleons. Ways of further developing the shell model of the giant resonance in deformed nuclei are discussed.

THE generating method for describing the main macroscopic characteristics of the collective dipole excitation of a nucleus [1] allows one to include the residual interactions between nucleons, which are so important for the formation of the dipole state, [2] and at the same time avoid the diagonalization difficulties which are inherent in the traditional computations on the shell model. Use of the generator model for describing the giant resonance for photoabsorption in the nuclei C^{12}, O^{16}, Ca^{40}, Pb^{208} [1] and Zr^{90} [3] gave good agreement with experiment. One is faced with the problem of extending the range of application of this model. In going over to nonmagic nuclei and, beyond that, to deformed nuclei, one must first investigate the question of the role of nucleon correlations in the ground state, which are of little importance when one considers magic nuclei.

Great successes along this line have been achieved by applying to the nucleus the methods of the theory of superfluidity. [4] Within the framework of the superfluid model, a whole series of effects—the presence of a gap in the spectra of even-even nuclei, certain features of the spectra of odd nuclei, the influence of pair correlations on the probability of β and γ transitions and of direct nuclear reactions—have been given a clear physical explanation. Especially fruitful is the inclusion, in addition to interactions of the pairing type, of long-range correlations between nucleons which are describable by a quadrupole pair interaction. [5] If one treats only the low-lying levels, one can disregard the dipole-dipole part of the effective interaction between nucleons. The latter interaction, which couples only distant states, separated by an interval of order 10—12 MeV (the distance between neighboring shells), only produces a weak polarization of the nuclear ground state.

In the present paper (Sec. 1) the dipole state of the nucleus is described as a superposition of a large number of two-quasiparticle states. Including nucleon pairing produces practically no change in the energy of this state from its value on the single-particle model (Sec. 2). We know that such estimates (Wilkinson model) give much too low values of the energy of the giant resonance for photoabsorption. The required raising of the energy of the dipole state is accomplished by introducing the dipole-dipole interaction (Sec. 3). Thus it is possible, without spoiling the concept of the superfluid model for describing the lowest excitations, also to describe within a single scheme the high dipole excitations of the nucleus.

1. THE DIPOLE STATE OF THE NUCLEUS

According to [1], the wave function $\Psi_{\rm dip}$ of the collective dipole state of the nucleus is obtained by applying the electric dipole operator $\hat{D}$ to the ground state wave function Ψ_0:

$$\Psi_{dip} = \langle \hat{D}\Psi_0 | \hat{D}\Psi_0 \rangle^{-1/2} \hat{D}\Psi_0. \quad (1)$$

In accordance with the requirements of the superfluid model, [4] the function Ψ_0 represents the quasiparticle vacuum:

$$\alpha_{s\rho}\Psi_0 = 0 \quad (2)$$

($\alpha_{s\rho}$ is the quasiparticle annihilation operator, s numbers the single-particle states and ρ is the magnetic quantum number).

The dipole moment operator

$$\hat{D} = \sum_{s\rho s'\rho'} d\,(s\rho|s'\rho')\, a^+_{s\rho} a_{s'\rho'} \quad (3)$$

can be transformed by the Bogolyubov canonical transformation

$$a_{s\rho} = u_s \alpha_{s-\rho} + \rho v_s \overset{+}{\alpha}_{s\rho}, \qquad \overset{+}{a}_{s\rho} = u_s \overset{+}{\alpha}_{s-\rho} + \rho v_s \alpha_{s\rho} \tag{4}$$

and expressed in terms of quasiparticle operators:

$$\hat{D} = \sum d\,(s\rho|s'\rho')\, u_s \rho' v_{s'} \overset{+}{\alpha}_{s-\rho} \overset{+}{\alpha}_{s'\rho'} \tag{5}$$

We then get for the norm in (1):

$$\langle \hat{D}\Psi_0 | \hat{D}\Psi_0 \rangle = \sum |d\,(s\rho\,|\,s'\rho')|^2\,[u_s^2 v_{s'}^2 + u_s \rho v_s \cdot u_{s'} \rho' v_{s'}]. \tag{6}$$

The second term corresponds to a dipole transition between states within the region of smearing out of the Fermi surface, and should therefore be dropped. We finally get:

$$\langle \hat{D}\Psi_0 \,|\, \hat{D}\Psi_0 \rangle = \sum |d\,(s\rho\,|\,s'\rho')|^2\, u_s^2 v_{s'}^2. \tag{7}$$

Thus, in accordance with (1) and (3), the dipole state Ψ_{dip} is a superposition of a large number of two-quasiparticle excitations; in general it is not an eigenfunction of the nuclear Hamiltonian $\hat{H}$. Thus the dipole state is smeared out in energy. The average of $\hat{H}$ over Ψ_{dip} characterizes the energy of the dipole state, and the average of $(\hat{H} - \overline{H})^2$ (the dispersion) gives its width.

2. ADDITIONAL EXCITATION ENERGY ON THE SUPERFLUID MODEL

Let us write the total Hamiltonian of the nucleus as a sum of single-particle energies (potential of self-consistent field plus kinetic energy) and residual pairing interactions between nucleons:

$$\hat{H} = \hat{E} + \hat{V}; \tag{8}$$

$$\hat{E} = \sum E_s \overset{+}{a}_{s\rho} a_{s\rho},$$

$$\hat{V} = -\frac{1}{4} \sum V\,(s_1\rho_1,\ s_2\rho_2\ |\ s_3\rho_3,\ s_4\rho_4)\, \overset{+}{a}_{s_1\rho_1} \overset{+}{a}_{s_2\rho_2} a_{s_3\rho_3} a_{s_4\rho_4}. \tag{9}$$

For the pairing potential we have

$$V\,(s_1\rho_1,\ s_2\rho_2\,|\,s_3\rho_3,\ s_4\rho_4) = -G\delta_{s_1 s_2} \delta_{\rho_1 - \rho_2} \delta_{s_3 s_4} \delta_{\rho_3 - \rho_4}. \tag{10}$$

Expressing $\hat{E}$ and $\hat{V}$ in terms of quasiparticle operators by using (4), we arrive at an expression for the average energy of dipole excitation:

$$\overline{E} = \langle \Psi_{dip} \,|\, \hat{H} - E_0 \,|\, \Psi_{dip} \rangle$$
$$= \Big[\sum u_{s_1}^2 v_{s_2}^2 |d(s_1\rho_1\,|\,s_2\rho_2)|^2 \{E_{s_1} u_{s_1}^2 - E_{s_2} v_{s_2}^2\}$$
$$+ \{\overline{V}_1 + \overline{V}_2 + \overline{V}_3 + \overline{V}_4 + \overline{V}_5\} \Big]$$
$$\times \Big[\sum |d\,(s_1\rho_1\,|\,s_2\rho_2)|^2\, u_{s_1}^2 v_{s_2}^2 \Big]^{-1}, \tag{11}$$

where

$$\overline{V}_1 = \sum V\,(s_1\rho_1,\ s_2\rho_2\,|\,s_3\rho_3,\ s_4\rho_4)\, u_{s_1}^2 u_{s_2} \rho_2 v_{s_2} u_{s_3}^2 u_{s_4} \rho_4 v_{s_4}$$
$$\times\, d\,(s_1\rho_1\,|\,s_2 - \rho_2)\, d\,(s_3\rho_3\,|\,s_4 - \rho_4),$$

$$\overline{V}_2 = \sum V\,(s_1\rho_1,\ s_2\rho_2\,|\,s_3\rho_3,\ s_4\rho_4)\, u_{s_1} \rho_1 v_{s_1} v_{s_2}^2 u_{s_3} \rho_3 v_{s_3} v_{s_4}^2$$
$$\times\, d\,(s_1\rho_1\,|\,s_2 - \rho_2)\, d\,(s_3\rho_3\,|\,s_4 - \rho_4),$$

$$\overline{V}_3 = -\sum V\,(s_1\rho_1,\ s_2\rho_2\,|\,s_3\rho_3,\ s_4\rho_4)\, u_{s_1}^2 v_{s_2}^2 u_{s_3}^2 v_{s_4}^2$$
$$\times\, d\,(s_1\rho_1\,|\,s_4\rho_4)\, d\,(s_2\rho_2\,|\,s_3\rho_3),$$

$$\overline{V}_4 = \sum V\,(s_1\rho_1,\ s_2\rho_2\,|\,s_4\rho_4,\ s_2\rho_2)\, u_{s_1}^2 v_{s_2}^2 u_{s_4}^2 v_s^2$$
$$\times\, d\,(s\rho\,|\,s_1\rho_1)\, d\,(s\rho\,|\,s_4\rho_4),$$

$$\overline{V}_5 = -\sum V\,(s_1\rho_1,\ s_2\rho_2\,|\,s_4\rho_4,\ s_2\rho_2)\, v_{s_1}^2 v_{s_2}^2 v_{s_4}^2 u_s^2$$
$$\times\, d\,(s\rho\,|\,s_1\rho_1)\, d\,(s\rho\,|\,s_4\rho_4). \tag{12}$$

u_s is equal to zero for states below the Fermi surface E_{s_0} and equal to unity above it; the converse is true for v_s. The changes in the quantities u_s^2, v_s^2 and $u_s v_s$ from their one-particle spectra are shown schematically in Fig. 1.

The matrix element $d\,(s_1\rho_1/s_2\rho_2)$ couples states having an average separation $\hbar\omega$ (the distance between neighboring shells). This quantity far exceeds the value of the "gap" ϵ. We shall assume that

$$\hbar\omega \gg \epsilon. \tag{13}$$

Then, to within terms of order ϵ, the first term in (11) is equal to the dipole excitation energy $\hbar\omega$ on the single-particle model. This value is only slightly changed by correlations of the pairing type. As we see from (12), the pairing potential (10) gives a contribution only to V_5. As a result we find

$$\overline{E} \approx \hbar\omega + G. \tag{14}$$

Thus, in the traditional superfluid model the dipole excitation energy exceeds the single-particle energy only by the amount of energy required to break up a pair.

3. INTRODUCTION OF DIPOLE-DIPOLE INTERACTION. DIAGRAMS OF EXCITATIONS

The result found above corresponds to the "diagonal approximation" in the interaction of particle

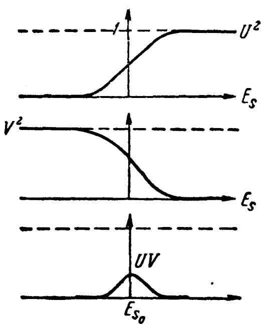

FIG. 1

and "hole," which was described in [6]. To obtain the correct position of the dipole state on the shell model required the inclusion of the nondiagonal part of this interaction. As shown in [1], an effect of coherent addition of amplitudes of dipole transitions, which results in a collective enhancement of the dipole photoabsorption and a shift of the energy of the dipole state, occurs only when certain definite relations are satisfied for the matrix elements of the residual interaction:

$$V_{\lambda\lambda'} \sim \alpha_\lambda^+ \alpha_{\lambda'}, \tag{15}$$

where α_λ are the coefficients in the expansion of Ψ_{dip} in single-particle states Φ_λ of the "hole"-particle configuration. It is easily seen that (15) is satisfied only for the dipole-dipole part of the residual interaction.

In addition to the pairing interaction, let us add to the superfluid model a dipole-dipole interaction

$$V_{DD}(1, 2) = V_0 D^+(1) D(2). \tag{16}$$

Its matrix elements are:

$$V_{D-D}(s_1\rho_1, s_2\rho_2 | s_3\rho_3, s_4\rho_4) = V_0 [d(s_1\rho_1 | s_3\rho_3) d(s_2\rho_2 | s_4\rho_4) - d(s_1\rho_1 | s_4\rho_4) d(s_2\rho_2 | s_3\rho_3)]. \tag{17}$$

As stated above, the potential V_{DD} has practically no effect on the ground state and low-lying states of the nucleus.

Let us investigate the effect of the dipole-dipole interaction on the position of the dipole state. Let us substitute (17) together with (10) into (11).

The various matrix elements $\overline{V}_1, \ldots \overline{V}_5$ are conveniently represented by special excitation diagrams. Let the interval $\hbar\omega$ contain N single-particle states, and suppose that the Fermi surface is smeared over an interval n. From (13) it follows that $n/N \ll 1$. For simplicity let us assume that the intensities of all single-particle transitions are equal:

$$d(s_1\rho_1 | s_2\rho_2) \sim \delta_{s_1,\, s_2 \pm N}. \tag{18}$$

Then

$$\langle \hat{D}\Psi_0 | \hat{D}\Psi_0 \rangle \sim N. \tag{19}$$

We represent each matrix element $\overline{V}_1, \ldots \overline{V}_5$ in expression (12) as the sum of three diagrams (a, b, c) corresponding to the two terms in the dipole-dipole matrix element (17) and to the pairing potential (10). The diagrams are drawn on the background of the schematically smeared out Fermi surface.

Taking the distributions of u^2, v^2, and uv (Fig. 1) we get the excitation diagrams shown in Fig. 2. The table gives the quantities characterizing the order of magnitude of the various matrix elements (diagrams). From the table one sees that the dipole-dipole interaction gives a coherent contribution, proportional to N^2, in diagram $\overline{V}_3$, case b.

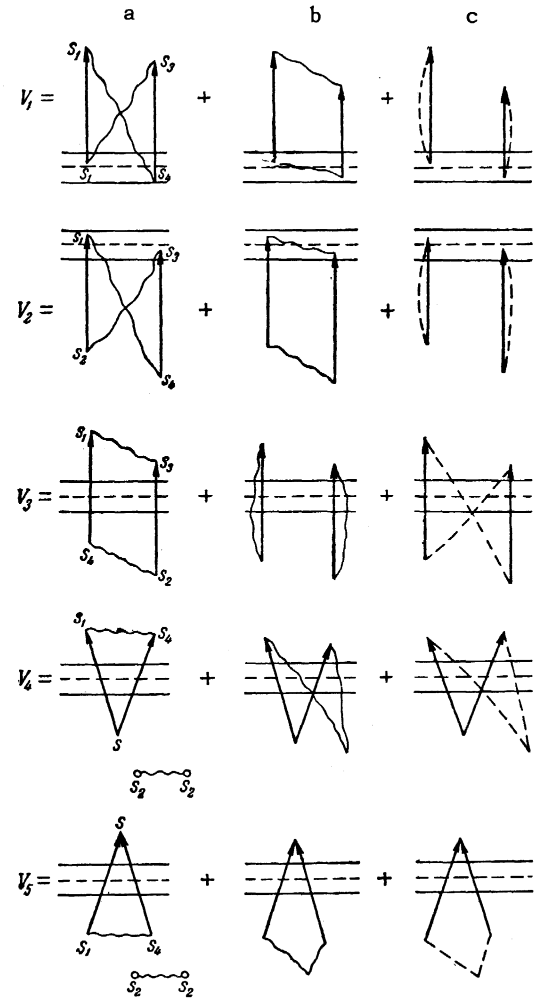

FIG. 2

	a	b	c
$\overline{V}_1$	$V_0 n$	—	—
$\overline{V}_2$	$-V_0 n$	—	—
$\overline{V}_3$	—	$V_0 N^2$	—
$\overline{V}_4$	—	$V_0 N$	—
$\overline{V}_5$	—	$-V_0 N$	GN

Using (19) and dropping small terms of order n/N in (11), we get the final expression for the average energy of dipole excitation:

$$\overline{E} \approx \hbar\omega + NV_0 + G. \tag{20}$$

Here V_0 is the average value of the matrix element of the dipole-dipole interaction and G is the pairing amplitude.

4. CONCLUSIONS

1. Effects of collective enhancement of dipole transitions and sizable shifts in energy of the dipole state from its single-particle value are caused by nucleon correlations of the dipole-dipole type. Long-range correlations of other multipolarities mainly give a contribution to the dispersion of the dipole excitation.

2. The increase in energy of the dipole excitation is proportional to the number of states in the last filled shell. Thus, in accordance with the results found earlier on the shell model, this effect is important for heavy nuclei and unimportant for light nuclei.

3. Interactions of the pairing type have little effect on the location of the dipole excitation. They increase the energy of the dipole state only by the amount of energy required for breaking up a pair. The pairing gives a contribution of order c^2 to the dispersion of the dipole excitation.

From these remarks one sees the way for developing the shell model of the giant resonance in deformed nuclei, which was formulated by Mottelson and Nilsson.[7] Inclusion of pairing does not eliminate the main defect of this model: too low values for the energy of the giant resonance. To rectify this one must include the nondiagonal dipole-dipole interaction between single-particle states of the Nilsson scheme. Results of such computations will be presented in later papers.

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