CERENKOV RADIATION AND TRANSITION RADIATION FROM ELECTROMAGNETIC WAVES

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A new radiation effect is considered. This effect is Cerenkov radiation or transition radiation from wave fronts or electromagnetic wave packets (a three-dimensional wave train or a modulated wave train) moving in a medium: at the edge of the packet the wave field gives rise to an average (with respect to the carrier frequency) gradient force that acts on the electrons in the medium, polarizing the medium. This region of polarization of the medium surrounding the wave packet moves together with the wave packet, i.e., the polarization effect moves with the group velocity of the wave. If this velocity is greater than the phase velocity in the frequency region being considered, which is lower than the carrier frequency of the wave packet, then "superluminal" radiation is emitted at the characteristic Cerenkov angle. Similar transition radiation effects should appear when a wave packet moves through a boundary between two media. The magnitudes of these effects are estimated.

The effects considered here represent a new class of radiation phenomena—Cerenkov radiation and transition radiation produced by electromagnetic beams and photons rather than by moving particles.

1. INTRODUCTION

It is usually assumed that Cerenkov radiation or transition radiation can be produced only by passage of fast particles in a medium. It is of interest, however, to investigate the possibility that radiation can also be produced by another rapidly moving effect that acts on electrons in a medium; we refer here to the average polarization of a medium by a wave front or a wave packet, that is to say, a train of electromagnetic oscillations propagating in the form of a modulated beam with finite transverse dimensions. It is this kind of wave packet that is actually produced in most cases of practical interest. The fact that radiation beams are usually collimated and transmitted for finite time intervals and that the radiation is modulated in amplitude, frequency, or aperture justifies an analysis of three-dimensional wave packets; this analysis is even more important when one considers that the majority of the effects considered below do not apply in the hypothetical case of an ideal plane wave in a medium or in an unmodulated beam. These effects imply the existence of gradients and are connected with both transverse and longitudinal variations in the mean square intensity of the field associated with the beam.

2. AVERAGE POLARIZATION OF A MEDIUM AND COMMON DISPLACEMENT OF POLARIZATION AND A WAVE PACKET

Let us assume that a beam with a high-frequency carrier (hf) propagates in a real dispersive medium. We are interested in the variation of the average polarization of the medium or the variation in the amplitude or frequency of the oscillations. (Hereinafter by "average" we mean averages taken over the hf oscillations at the carrier frequency.)

The hf field will exert on the electrons in the medium an average force given by

$$\mathbf{f}_{\mathrm{av}}(t) = \frac{e^2}{2m}\frac{\omega_0^2-\omega^2}{(\omega_0^2-\omega^2)^2+\gamma^2}\nabla(E^2)_{\mathrm{av}},$$

where ω is the carrier frequency, ω_0 is a characteristic frequency (due to atomic coupling, plasma resonance, or Larmor resonance) and γ is the damping factor for the electron oscillations. For example, when $\omega \gg \omega_0$ we can assume that the electrons are free and that the force due to the gradient is [1]

$$\mathbf{f}_{\mathrm{av}}(t) = -(e^2/2m\omega^2)\nabla(E^2)_{\mathrm{av}}.$$

This force acts primarily on the electrons. It can produce a volume polarization of the medium

$$\mathbf{P}(\mathbf{r}, t) = \alpha E_{\mathrm{eq}}(\mathbf{r}, t),$$

where $\mathbf{E}_{eq} = \mathbf{f}/e$ is the field equivalent to the average force and α is the dynamic polarizability of the medium. (In general the polarizability due to electrons alone may be different from the conventional polarizability due to the effect of an electric field on the electrons and nuclei; however, if one considers averages over rapidly varying effects the ion inertia will act to make both polarizabilities approximately the same.)

In a modulated beam the force due to the average effect on the electrons in the medium is displaced along the beam at the group velocity:

$$\mathbf{E}_{eq} \approx \Phi(\rho)\Psi(t - z/V_{gr}),$$

where V_{gr} is the group velocity of the signal, which depends on the carrier frequency ω associated with the wave vector $k(\omega)$: $V_{gr} \approx \Delta\omega/\Delta k \sim \Omega/K$. Here, Ω and K are the frequency and wave vector of the modulated wave (for simplicity we assume that the modulation is not too rapid, that is to say, $\Omega \ll \omega$).

The expression for the wave field associated with a beam diffracted by a circular aperture of radius a [assuming slow modulation ($f'a/c' \ll f$)] is

$$\mathbf{E} = \frac{\mathbf{A}}{R_0}\frac{J_1(ka\rho/R_0)}{(ka\rho/R_0)} f\left(t - \frac{R_0}{V}\right) e^{i(\omega t - kR_0)},$$

where f(t) is the modulation function of the beam. For example, if a beam contains two carriers of approximately the same frequency:

$$f(t) = \sin(\Delta\omega t/2 - \Delta k R_0/2).$$

It is evident from the expression given for the beam field that the scale size for the transverse inhomogeneity of the field is $\rho_{eff} \sim R_0/ka \sim \lambda' R_0/a$. The longitudinal dimension of the field inhomogeneity is given by $z_{eff} \sim V_{gr}T \sim V_{gr}/\Omega \sim 1/K$ where $T \sim f/f'$ is the characteristic modulation time. The ratio of the inhomogeneity scale sizes is then

$$\rho_{eff} \; z_{eff} \sim K\rho_{eff} \sim \Delta k R_0/ka.$$

The wave packet spreads slowly as it moves. The transverse spreading of the wave packet can be neglected for small relative changes in the distance R_0. The effect of longitudinal spreading will be considered below.

3. RADIATION FROM AN HF FIELD PACKET. EQUIVALENT MOVING CHARGE

Uniform motion of an hf field packet, which gives rise to a moving average polarization of the medium, can produce radiation in two cases:
1) when the displacement velocity $V_{gr}(\omega)$ is greater than $V_{ph}(\Omega')$, the phase velocity of plasma waves, then superluminal radiation should be emitted at the Cerenkov angle

$$\cos\theta = V_{ph}/V_{gr};$$

2) when the medium is inhomogeneous, for example when the wave packet is transmitted through the boundary between two media, then transition radiation should be produced.

The average polarization of the medium produced by the equivalent field will be displaced together with the packet. This polarization is such that an electron deficiency is produced inside the wave packet while a layer of compensating negative charge is produced at the boundary. The density of the space charge is $q = -\operatorname{div}\mathbf{P} = -\alpha \operatorname{div}\mathbf{E}_{eq}$. It follows from the nature of the field distribution that a positive charge is concentrated in that region of the wave packet in which the field is inhomogeneous. Assuming that the inhomogeneous region of the field is comparable in size with the dimensions of the packet we can use the Gauss theorem to estimate the surface charge in the central zone:

$$Q \approx \alpha(\Omega')\,\varepsilon' E_{eq}\rho_{eff} z_{eff} \quad \text{for} \quad z_{eff} > \rho_{eff}$$

(prolate packet) and

$$Q \approx \alpha(\Omega')\,\varepsilon' E_{eq}\rho_{eff}^2 \quad \text{for} \quad z_{eff} < \rho_{eff}$$

(oblate packet). In the simplest case $\rho_{eff} \sim z_{eff} \sim \Lambda'/4$ (Λ' is the wavelength of the detected radiation) all the uncompensated charge radiates coherently. Those waves are radiated effectively which are comparable with the dimensions of the localization of the central cloud. The radiation falls off sharply at long wavelengths since the effect of the layer of compensating charge is felt. To estimate the reduction due to interference we make use of a simplified model of the charge distribution. We calculate the interference factors for the two simplest cases:

1) for a charged disc of radius ρ_1 surrounded by an oppositely charged ring with outer radius ρ_2

$$I(\theta) \approx 2\left\{\frac{J_1(K\rho_1\sin\theta)}{(K\rho_1\sin\theta)} - \frac{J_1(K\rho_2\sin\theta)}{(K\rho_2\sin\theta)}\right\}\frac{\rho_2^2}{\rho_2^2 - \rho_1^2}$$

$$\approx J_2(K\rho\sin\theta) \quad \text{for} \quad \rho_2 - \rho_1 \ll \rho_2;$$

when $K\rho \ll 1$ we obtain the factor

$$I(\theta) = \tfrac{1}{16}(K\rho_2\sin\theta)^2 \ll 1;$$

2) for a thin charged cylinder of length $2l_1$ with an oppositely charged outer layer (total length $2l_2$)

$$I(\theta) = \left\{\frac{\sin(Kl_1\cos\theta)}{(Kl_1\cos\theta)} - \frac{\sin(Kl_2\cos\theta)}{(Kl_2\cos\theta)}\right\}\frac{l_2}{l_2 - l_1}$$

$$\approx l\frac{\partial}{\partial l}\frac{\sin(Kl\cos\theta)}{(Kl\cos\theta)} \quad \text{for} \quad l_2 - l_1 \ll l:$$

when $Kl \ll 1$ we have

$$I(\theta) \approx \frac{1}{3} K^2 l_2 (l_1 + l_2) \cos^2 \theta \approx (Kl \cos \theta)^2.$$

The most effective radiation is emitted at wavelengths comparable with the dimensions of the wave packet. If the usual expressions for Cerenkov radiation and transition radiation are to be used in the frequency range of greatest interest it must be shown that the smearing of the electromagnetic packet is small during the time in which the radiation is produced.

We estimate the longitudinal spreading of the wave packet in a dispersive medium. The rate of spreading is associated with the spread in the group velocities for the frequency range characteristic of the packet:

$$V_{\mathrm{spr}} \approx \Delta V_{\mathrm{gr}} \approx \Delta\omega dV_{\mathrm{gr}} / d\omega \approx V_{\mathrm{gr}}^2 \Delta\omega d^2k / d\omega^2 ,$$

since $V_{gr} = 1/(dR/d\omega)$ where $k = n\omega/c$. If the carrier frequency lies in the "plasma" region of $\epsilon(\omega)$, that is to say, if $n^2 = 1 - (\omega_p/\omega)^2$, then $d^2k/d\omega^2 = (n^2 - 1)/cn^3\omega$ and

$$V_{\mathrm{spr}} = V_{\mathrm{gr}}^2 \frac{n^2 - 1}{cn^3} \frac{\Delta\omega}{\omega} .$$

Let us assume that $V_{gr} \sim c$; then, if $\{(n^2 - 1)/n^3\} \times \{\Delta\omega/\omega\} \ll 1$ we have $V_{spr} \ll V_{gr}$, i.e., the variation in the dimensions of the wave packet is small as the packet traverses a path of several wavelengths Λ' (a path of this length is adequate for production of radiation as far as intensity and characteristic asymmetry of the distribution are concerned; an increase of path length has an effect on the sharpness of the directivity but has little effect on the radiation loss per unit length of path).

For the case which is simplest and of greatest interest $\rho_{eff} \sim z_{eff} \sim \Lambda'/4$ we can obtain the superluminal radiation by using the usual formulas for radiation from a charge:

$$\Delta W \approx \frac{Q_{\mathrm{eff}}^2}{c} \left\{ 1 - \left(\frac{V_{\mathrm{ph}}}{V_{\mathrm{gr}}} \right)^2 \right\} \Omega' \Delta\Omega' L ,$$

where L is the path traversed by the wave packet. Since

$$Q \approx \alpha\varepsilon' E_{\mathrm{eq}} \rho_{\mathrm{eff}}^2 \approx \frac{e^2\alpha}{2m\omega^2} \rho_{\mathrm{eff}} (E^2)_{\mathrm{av}},$$

then

$$\Delta W \approx \alpha^2 \left(\frac{e}{2mc} \right)^2 \left(\frac{E_0}{\omega} \right)^4 \rho_{\mathrm{eff}}^2 \left(1 - \frac{V_{\mathrm{ph}}^2}{V_{\mathrm{gr}}^2} \right) \Omega' \Delta\Omega' L$$

$$\approx \alpha^2 \pi^2 \varepsilon'^2 \left(\frac{e}{m} \right)^2 \left(\frac{V_{\mathrm{ph}}}{c} \right)^2 \left(\frac{E_0}{\omega} \right)^4 \left(1 - \frac{V_{\mathrm{ph}}^2}{V_{\mathrm{gr}}^2} \right) L$$

for $\rho_{eff} \sim \Lambda'/2\pi \sim V_{ph}/\Omega$ and $\Delta\Omega \approx \Omega$. These formulas show the sharp dependence of radiation intensity on the amplitude of the carrier field E_0 and the carrier frequency ω.

For example, if $(1 - V_{ph}^2/V_{gr}^2) \sim 1$ and $\epsilon' \sim 1$, using a radar carrier with $E_0 \sim 30$ kV/cm, $\omega \sim 10^{10}$ sec^{-1} ($E_0/\omega \sim 10^{-8}$ cgs esu) we find that the superluminal losses of the wave packet are $\Delta W/\Delta L \sim 0.3$ J/m for an energy density $W \sim 100$ J/m^3 in the wave packet (the Cerenkov condition can be satisfied, for instance, in a plasma in a magnetic field if we choose the frequencies so that $\omega > \omega_H$, $\Omega < \omega_H$ and so on). For a laser beam with $E_0 \sim 3 \times 10^7$ V/cm and $\omega \sim 10^{15}$ sec^{-1} ($E_0/\omega \sim 10^{-10}$ cgs esu) we find $\Delta W/\Delta L \sim 10$ erg/cm. These energies correspond to power levels that can be detected experimentally.

For purposes of illustration we estimate the transition radiation produced by a radiation packet incident on the surface of a medium. Upon entering the medium the packet produces a polarization and the radiation in the range $\Lambda' \approx \rho_{eff}$ will correspond to radiation produced by instantaneous acceleration of a charge Q_{eq} to a velocity V_{gr} in moving from the boundary into the depth of the medium. In the simplest case, where $V_{gr}(\omega) \to V_{ph}(\Omega)$, this radiation is highly directional and the intensity per unit solid angle is given by

$$W_\omega(\theta)\, d\Omega' \approx \{\sqrt{\varepsilon}\, Q_{\mathrm{eff}}^2 V_{\mathrm{gr}}^2 \sin^2\theta / 4\pi^2 c^3 \times [1 - (V_{\mathrm{gr}}/V_{\mathrm{ph}}) \cos\theta]^2\}\, d\Omega'.$$

In addition to the examples given above of radiation from a wave packet, there is the possibility of producing radiation in a medium by means of a periodically modulated beam (the radiation spectrum is a line spectrum), by using the leading or trailing edges of beam intensity, by the motion of a beam across an axis and so on—in all cases of motion or changes in the gradient of an hf field in a medium. The effects we have described can be observed over a wide range of wavelengths—from the radio range to the x-ray range and the gamma-ray range. The nonlinearity we have discussed, i.e., the gradient of a force, is not the only possible one. Other nonlinear effects (for example, the scattering reaction force of a packet on electrons and so on) can give rise to a similar displacement of an average polarization of a medium by a wave field and thus to coherent radiation by virtue of the motion of a polarization center. It is possible to have strong gradients and high field amplitudes near nuclei or atoms that radiate light, x rays, or gamma rays. The change in the gradient effect of the field on neighboring electron shells due to changes in a photon field can produce bursts of long-wave radiation.

An enhancement of the polarizability of the medium by the average hf field would be expected when the modulation frequency of the hf beam coincides with the natural frequency of the electrons. This mode ($\omega \gg \omega_p$, $\Omega \rightarrow \omega_p$) can be used, in particular, for volume heating of electrons in a plasma by the penetration of an hf beam.

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[1] A. V. Gaponov and M. A. Miller, JETP **34**, 242 and 751 (1958), Soviet Phys. JETP **7**, 168 and 515 (1958).

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