THE PROTON COMPTON EFFECT IN THE DIPOLE APPROXIMATION

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The dipole "phase shifts"[1,2] are made more precise. It is shown that all presently available experimental data on the angular and energy dependence of the differential cross sections[3-9] for energies up to 180 MeV as well as the results of theoretical calculations concerning the threshold anomalies[23,24] and the estimates of the electric and magnetic "polarizabilities" of the proton agree well with the results obtained within the present approximation.

THE investigation of the nucleon Compton effect began more or less systematically after publication of the well known paper by Gell-Mann, Goldberger, and Thirring[1] in which the dispersion relations were for the first time introduced into the quantum field theory and applied to the proton Compton effect. Despite the rough approximations used in evaluating the dispersion integrals (the authors themselves emphasized the qualitative character of their conclusions) the paper established the following features, which were confirmed by subsequent experiments:[3-9] (a) the smallness of the differential and total cross sections below the meson photoproduction threshold, owing to the interference between the Thomson and dispersion amplitudes; (b) the fast rise of the cross sections above threshold (at ~ 350 MeV we have $\sigma \sim 10\sigma_T$, $\sigma_T = 8\pi r_0^2/3$ is the classical total scattering cross section); c) the strong asymmetry in the angular distribution at threshold—this fact has as yet not been experimentally investigated.

In the same paper it was proposed that the considered effects can be satisfactorily described keeping just a few basic scattering "phase shifts," namely the dipole "phase shifts." This proposal and the results of the papers by Low, Gell-Mann, and Goldberger[11] were utilized by Capps,[2] whose papers will be discussed more fully below.

Several attempts were made to describe the nucleon Compton effect semiphenomenologically by means of the nucleon isobars,[11,12] the strong $(^3/_2, {}^3/_2)$ resonance,[13] and the fixed source meson theory.[14] Hyman et al[9] have described the Compton amplitude as a combination of the Thompson amplitude, the amplitudes associated with the $(^3/_2, {}^3/_2{}^+)$ and $(^1/_2, {}^1/_2{}^-)$ resonances and the decay of a virtual π^0 meson into two photons (the Low amplitude[15]). This led to a very good agreement with the experimental data on scattering at 90° if one assumed for the lifetime of the π^0 meson a value between 10^{-16} and 10^{-18} sec. However, all these models had the drawback that even though they could be made to agree with some particular kind of experimental data (the angular dependence of the cross sections, the energy dependence at fixed angle etc.) they sometimes disagree violently with another kind. This is obviously a consequence of the present-day inability to describe sufficiently precisely that part of the amplitude which is associated with the meson-nucleon interaction. A more correct description of this part at present can be given only within the framework of the dispersion relations.

Dispersion relations were given by Akiba and Sato[15] in the approximation where only the first power of the photon energy was retained. These were compared with experimental data by Yamagata et al[7] who found that they were not in disagreement. We have obtained[17] exact (in the e_0^2-approximation) dispersion relations which allow the inclusion of nucleon recoil effects (terms quadratic in photon energy in the integrals; we note that these terms can have a strong influence on the form of the magnitudes which are to be compared with experiment).

Jacob and Mathews have derived dispersion relations (unpublished) where they included the Low amplitude. They used these[18] to find limits for the π^0 lifetime. The best agreement with the experimental data of Bernardini et al[8] was obtained with the values $10^{-18} \lesssim \tau(\pi^0) \lesssim 10^{-16}$ sec. However, Lapidus and Chou Kuang-chao[19] have recently shown that the Low amplitude has been introduced by Jacob and Mathews with the wrong sign and that this produced their good agreement with experiment. We note that a correct description of the Low amplitude is possible only within the framework of the double dispersion relations where it appears automatically as a pole in the momentum transfer in one channel. The problem of the double dispersion relations for the Compton effect is rather complicated and has as yet not been solved.

Recently there have been investigated such detailed features of the effect as the "polarizability" [5,20-22,4b] and threshold effects. [23,24] The results of these papers will be briefly discussed below (point 3).

In the present paper the Compton effect on the proton is treated in the so-called dipole approximation. [1,2]

1. Low and Gell-Mann and Goldberger [10] have shown that the scattering amplitude of a photon on a system with spin $1/2$, charge e, mass M, and anomalous magnetic moment λ is given in the approximation linear in photon energy in the laboratory coordinate system (l.s.) by the expression:

$$R_l = r_0 \{ee' + k_0 \frac{2\lambda+1}{2} i\sigma [ee'] + \frac{(\lambda+1)^2}{2} \frac{i\sigma [[e'k'][ek]]}{k_0} + \frac{\lambda+1}{2} \frac{i\sigma [ke](e'k) - i\sigma [k'e'](ek')}{k_0}\}_l, \qquad (1)^*$$

where $\hbar = c = 1$; $r_0 = e^2/M$ is the classical nucleon radius; the index "l" implies that the quantities are taken in the l.s.; **e** and **e′** are the polarization vectors of the incoming and outgoing photon respectively; **k** and **k′** are their momenta, and σ is the nucleon spin. For the following we will need the scattering amplitude in the center-of-mass system (c.m.s.). This is obtained from (1) by a Lorentz transformation of all the quantities (**e**, **e′**, **k**, **k′**, k_0) and with the additional gauge transformation

$$e'_l \rightarrow e'_l + f(\delta, n, n', e')\, n' \qquad (2)$$

(e'_l, e′, and n′ are 4-vectors). Here $\delta = k_0/M$, $\mathbf{n} = \mathbf{k}/M$; $\mathbf{n}' = \mathbf{k}'/M$; all quantities are taken in the c.m.s. The function f is determined by the condition

$$(e'_0)_l + f\cdot(k'_0)_l = 0, \qquad (3)$$

since (1) was derived with just this condition. [10] This gives

$$f = -(e'_0)_l/(k'_0)_l = -e'n/(\varepsilon + \delta\,(nn')); \qquad (4)$$

further ϵ, δ and ω denote the energy of the nucleon, the photon, and the total energy ($\omega = \epsilon + \delta$) respectively in units M = 938 MeV.

The relations (2) and (4) lead to the necessity of transforming $(\mathbf{e}')_l$ in (1) in the form

$$(e')_l = e' - \delta \frac{e'n}{\varepsilon + \delta\,(nn')} \left(n' + \frac{\delta}{\varepsilon+1} n\right). \qquad (5)$$

This way we obtain with (5) the scattering amplitude in the approximation linear in δ in the c.m.s.

$$R = r_0 \Big\{ee' + \delta \Big[-(e'n)(en') + \frac{2\lambda+1}{2} i\sigma [ee'] + \frac{(\lambda+1)^2}{2} i\sigma [(e'n')[en]] + \frac{\lambda+1}{2} (i\sigma [ne](e'n) - i\sigma [n'e'](en'))\Big]\Big\}. \qquad (6)$$

We note that the immediate evaluation of R in the c.m.s. by the method of Low, Gell-Mann, and Goldberger [10] also leads to expression (6). Capps originally [2a] did not take into account the transformations (2) and (5). His corresponding formula thus lacks the term $r_0\delta(\mathbf{e}'\cdot\mathbf{n})(\mathbf{e}\cdot\mathbf{n}')$. He further did not separate off the quadrupole phases (see p. 2). This leads to wrong values for the dipole phases (the difference reaches 30 to 40%) and the differential cross sections (up to 50%). In [2b] the expressions for the dipole phases were corrected and they are in agreement with our expressions [see (9) below]. However, the contribution of the quadrupole phases was omitted [see (9), (10) below]. This changes the value of S_1^e (15) by 3 to 8%, S_2^e and S_1^m by 25 to 30% (they are, however, small in absolute value: $\sim 0.05\, r_0$), and S_2^m by 1 to 2%. This results in a change of the results of Capps [2b] by 9 to 10% (see also Figs. 9, 10 below).

2. For the following it will be convenient to change to the following spin "structures": [17]

$$n_1 = (ee'), \quad n_2 = (e'n)(en'), \quad n_3 = (\sigma n')(\sigma n)(ee'),$$
$$n_4 = (\sigma n')(\sigma n)(e'n)(en'), \quad n_5 = (\sigma e')(\sigma e),$$
$$n_6 = (\sigma\, [e'n'])(\sigma\, [en]). \qquad (7)$$

The amplitude in the c.m.s. $\Phi(\omega, \mathbf{n}\cdot\mathbf{n})$ [17] can be written by means of (7) and (6) as follows:

$$\Phi(\omega, x) = -\frac{1}{\omega} R = -\frac{r_0}{\omega} \Big\{\Big[1 + \delta\Big(\frac{\lambda}{2} - x\frac{(\lambda+1)^2}{2}\Big)^2\Big] n_1 + \delta\Big[\Big(\frac{(\lambda+1)^2}{2} - 1\Big) n_2 + \frac{\lambda+1}{2} x n_3 - \frac{\lambda+1}{2} n_4 - \frac{\lambda}{2} n_5 + \frac{\lambda+1}{2} (\lambda + 1 - x)\, n_6\Big]\Big\}, \qquad (8)$$

where $x = \mathbf{n}\cdot\mathbf{n}' = \cos\theta_{c.m.s.}$. In [17] a "phase-shift analysis" was performed and expressions were obtained for the scalar amplitudes $\Phi_i(\omega, x)$ of the process. They were given as functions of the probabilities ("phase shifts") $S(J, \Pi_L, \Pi_{L'},$

*ee′ = e·e′; [ee′] = e×e′.

L, L′) of the transitions of given total angular momentum J, the multipolarities of the incoming and outgoing photons Π_L and $\Pi_{L'}$, with corresponding angular momenta L and L′ in terms of derivatives of Legendre polynomials. Below the photoproduction threshold and at energies where (8) is valid we can compare the corresponding expressions for $\Phi_j(\omega, x)$ with the coefficients of n_j in (8). This yields a compatible system of equations with unique solutions which lead to the following expressions for the phases:

$$\delta_1^e = S\,({}^1/_2E1) = -\frac{2}{3}\frac{r_0}{\omega}(1-\delta\lambda),$$

$$\delta_3^e = S\,({}^3/_2E1) = -\frac{2}{3}\frac{r_0}{\omega}\left(1+\delta\frac{\lambda}{2}\right),$$

$$\delta_1^m = S\,({}^1/_2M1) = -\frac{2}{3}\frac{r_0}{\omega}\delta\left[(\lambda+1)^2+\frac{1}{2}\right],$$

$$\delta_3^m = S\,({}^3/_2M1) = \frac{r_0}{3\omega}\delta\,[(\lambda+1)^2-1],$$

$$\delta_3^{em} = S\,({}^3/_2EM12) = -\frac{r_0}{\sqrt{3}\omega}\delta\,\frac{\lambda+1}{2},$$

$$\delta_3^{e_2} = S\,({}^3/_2E2) = \frac{r_0}{5\omega}\delta,$$

$$\delta_5^{e_2} = S\,({}^5/_2E2) = \frac{r_0}{5\omega}\delta; \qquad (9)$$

all other phases vanish.

We also give the expressions for $\Phi_j(\omega, x)$ in terms of δ_j^i:

$$\Phi_1 = \frac{3}{2}\delta_3^e + \frac{3}{2}x\delta_3^m - \frac{1}{2}x\delta_3^{e_2} + 3x\delta_5^{e_2},$$

$$\Phi_2 = -\frac{3}{2}\delta_3^m + \frac{1}{2}\delta_3^{e_2} + 2\delta_5^{e_2},$$

$$\Phi_3 = \delta_3^{e_2} - \delta_5^{e_2} + \sqrt{3}\,x\delta_3^{em}, \qquad \Phi_4 = -\sqrt{3}\,\delta_3^{em},$$

$$\Phi_5 = \frac{1}{2}(\delta_1^e - \delta_3^e) + x(\delta_3^{e_2} - \delta_5^{e_2}),$$

$$\Phi_6 = \frac{1}{2}(\delta_1^m - \delta_3^m) + \frac{1}{2}(\delta_3^{e_2} - \delta_5^{e_2}) - \sqrt{3}\,x\delta_3^{em}. \qquad (10)$$

We did not use the results (9) in obtaining (10) since the dispersion integrals can change the δ_j^i differently, and also in order to emphasize the further approximations. We have, however, kept in (10) the phases whose real part vanishes in the linear approximation.

Following Capps[2b] we now assume that the photoproduction changes in essence three phases: δ_1^e, δ_3^e, and δ_3^m. The change of δ_3^e and δ_3^m is given by one unknown function $M(\delta)$ while the change of δ_1^e is given by another unknown function $E(\delta)$. This takes place in the form

$$\delta_1^e \to \delta_1^e + \frac{r_0}{\omega}E(\delta), \qquad \delta_3^m \to \delta_3^m + \frac{r_0}{\omega}RM(\delta),$$

$$\delta_3^e \to \delta_3^e + \frac{r_0}{\omega}(1-R)\,M(\delta), \qquad (11)$$

where

$$R \approx \begin{cases} 1.18-3\,(k_0)_l\,/\,M & \text{for } 60 \leqslant (k_0)_l \leqslant 180 \text{ MeV} \\ 1.15-5.3\,(k_0)_l\,/\,M & \text{for } 180 \leqslant (k_0)_l \leqslant 280 \text{ MeV} \end{cases} \qquad (12)$$

(in the following we shall denote the changed quantities δ_j^i by the previous symbol δ_j^i).

With (9)–(12) the forward scattering amplitude for the total angular momentum $^1/_2$ and $^3/_2$ is given by (see [17]):

$$f_1\,({}^1/_2, {}^3/_2, x=1) = (\Phi_1+\Phi_3+\Phi_5+\Phi_6)|_{x=1}$$

$$= \frac{r_0}{\omega}\left[M+\frac{1}{2}E-\frac{3\delta}{10}\right]-\frac{r_0}{\omega},$$

$$f_2\,({}^1/_2, {}^3/_2, x=1) = -(\Phi_5+\Phi_6)|_{x=1}$$

$$= \frac{r_0}{\omega}\left[\frac{1}{2}M-\frac{1}{2}E-\frac{\delta}{10}\right]+\frac{r_0}{\omega}\frac{\delta\lambda^2}{2}. \qquad (13)$$

The dispersion relations for f_1 and f_2 are simple[1] and can be easily established and applied for the evaluation of E and M. Comparing with the results of Capps[2] we find that

$$E = \mathscr{E} + \delta/15, \qquad M = \mathscr{M} + 4\delta/15, \qquad (14)$$

where $\mathscr{E}$ and $\mathscr{M}$ are the functions which were introduced by Capps to describe the meson structure of the nucleon. By itself the difference between our and Capps' results is unimportant. However, we need the interference of E and M with the amplitudes δ_j^i (9) [see (16) below].

In Fig. 1 we have shown the values of δ_j^i given by (11) taking into account (14) and[2a]. One sees

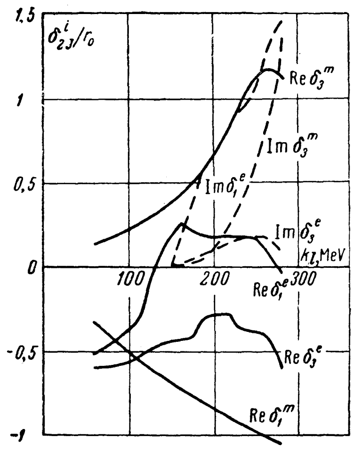

FIG. 1. "Phases" of the Compton effect (i = m, e; J = ½, ³⁄₂).

that while the quantities Re δ_1^m, Re δ_3^m, Im δ_3^m, Im δ_1^e and Im δ_3^e change more or less smoothly the change of Re δ_1^e and Re δ_3^e is nonmonotonic. This will lead to definite nonmonotonic features in the observed quantities (see the discussion in point 3, and also Figs. 2, 3, 6, 7, 11).

The differential cross section of an unpolarized

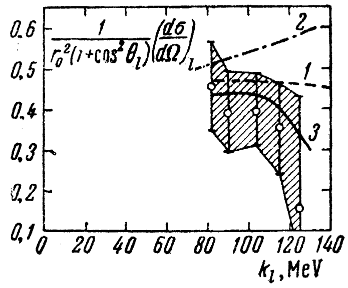

FIG. 2. Differential cross section at 50° (l.s.) as a function of k_l. Experimental points from[9].

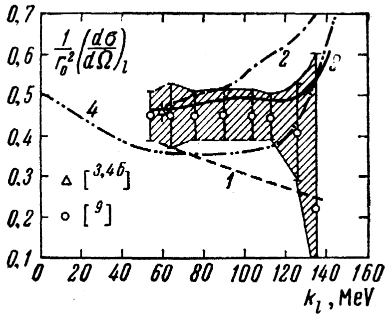

FIG. 3. Differential cross section at 90° (l.s.) as a function of k_l.

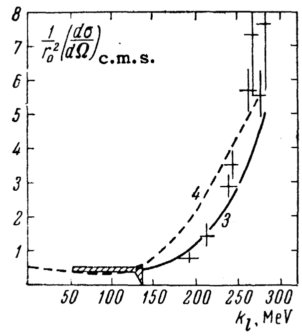

FIG. 4. Differential cross section at 90° (c.m.s.) as a function of k_l. Experimental points from[7, 8], crosshatched region essentially after [9] (cf. Fig. 3).

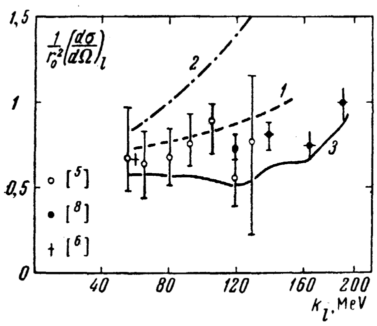

FIG. 5. Differential cross section at 135° (l.s.) as function of k_l.

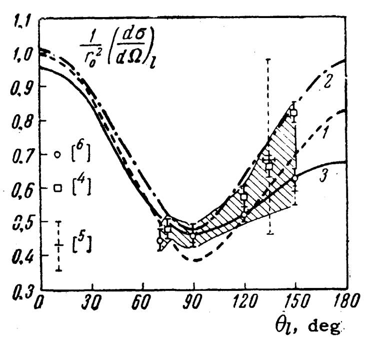

FIG. 6. Angular distributions in the l.s. at k_l = 60 MeV.

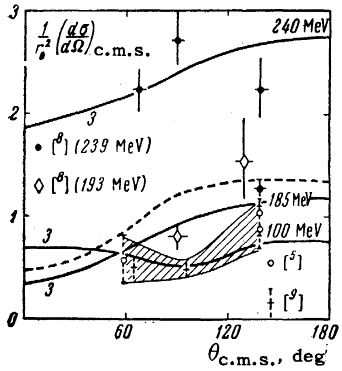

FIG. 7. Angular distributions in the c.m.s. at k_l = 100, 185, and 240 MeV.

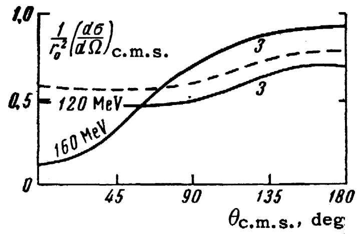

FIG. 8. Angular distributions in the c.m.s. at k_l = 120 and 160 MeV.

photon beam on an unpolarized target can be written in terms of

$$S_1^{e,m} = \delta_3^{e,m} + \frac{1}{2}\delta_1^{e,m}, \qquad S_2^{e,m} = \frac{1}{2}\delta_3^{e,m} - \frac{1}{2}\delta_1^{e,m} \quad (15)$$

as

$$d\sigma/d\Omega = \Big\{\frac{1}{2}\,[|S_1^e|^2 + |S_1^m|^2 + 3|S_2^e|^2 + 3|S_2^m|^2] + 2x\,\mathrm{Re}\,(S_1^{e*}S_1^m + S_2^{e*}S_2^m) + \frac{1}{2}x^2[|S_1^e|^2 + |S_1^m|^2 - |S_2^e|^2 - |S_2^m|^2]\Big\}. \quad (16)$$

By means of (16) and with the values of the phases given in Fig. 1 we have evaluated the cross section $d\sigma/d\Omega$ and plotted it together with all known experimental data[3–8] in Figs. 2—7. Where possible we have indicated the range of values consistent with the errors. The overall agreement with the experimental data is satisfactory (particularly in Figs. 2—6).

For purposes of comparison we have also shown in the figures the curves for the differential cross sections given by the Klein-Nishina-Tamm formula (curve 1), the Powell formula (curve 2) and the fixed source meson theory (curve 4). The results of the dipole approximation are given in all drawings as curve 3. Further, we have given the angular distributions (in the c.m.s. system) for k_l = 120 MeV (the most isotropic case) and for k_l = 160 MeV (the most anisotropic case) (see Fig. 8). The dotted curve in Figs. 7 and 8 represents the results of Capps[2b] and it illustrates the above discussed differences.

It follows from the general principles of quantum mechanics that at the threshold of the reaction a + b → b′ + c there have to appear nonmonotonic features in the reaction a + b → b′ + a′. [23] Lapidus and Chou Kuang-chao have shown that the forward scattering cross section there should show up an anomaly at the threshold (k_l ≈ 150 MeV) of π-mesons which are produced there mainly in an S-state (curve 6 in Fig. 9; curve 5 is taken from [25]).

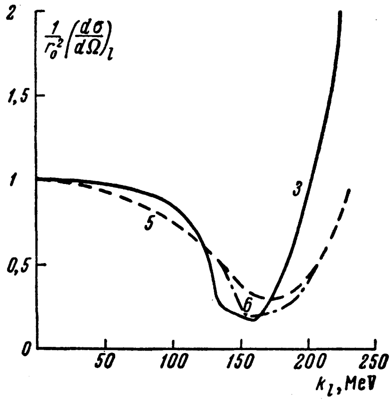

FIG. 9. Differential forward scattering cross section in the l.s. as a function of k_l.

The threshold anomalies as given in the dipole approximation are illustrated in Fig. 1 (curves for Re $\delta^e_{1,3}$). We note that an experimental indication of the existence of threshold anomalies may be contained in the data of [5] at 45°; however, the errors are rather large there. The data of Hyman et al [9] are more precise (see Fig. 2 where the errors are also indicated). They also indicate a decrease of $d\sigma/d\Omega_l$ at approach to the threshold. One also can use the data for 135° [5,8] (Fig. 5) as an indication of the threshold anomaly. Nonmonotonic behavior shows up also in the backward scattering cross sections (Fig. 10). The threshold effect is in itself not large. This is illustrated for the total cross section in Fig. 11 (the cross section is given in units $\sigma_T = {}^8/_3 \pi r_0^2$). A certain deviation from a monotonic variation of the cross section shows up, e.g., in the region 100—140 MeV;

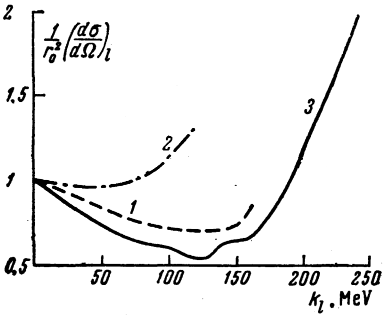

FIG. 10. Differential backward scattering cross section in the l.s. as a function of k_l.

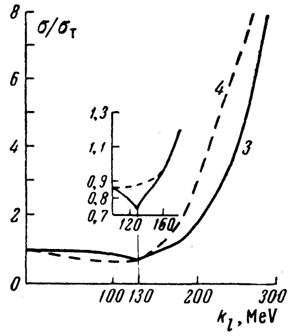

FIG. 11. Total cross section [in units $\sigma_T = ({}^8/_3)\,\pi r_0^2$] as a function of k_l.

there lies a minimum of the cross section at k_l ~ 130 MeV ($\sigma \sim 0.73\,\sigma_T$) and the dip has a half-width ~ 14 MeV. In Fig. 11 the threshold region has been drawn also in more detail. Curve 4 of Fig. 11 shows the results of Karzas et al. [14] We note that in order to give a quantitative description of the threshold anomalies one has to use a more precise evaluation of the dispersion integrals and a more careful consideration of the contribution of the photoproduction to the phases.

In conclusion we remark that it is possible to estimate within the framework of the dipole approximation the electric, and, more essentially, the magnetic polarizability of the proton. Indeed, denoting by $\Delta\Phi^j$ the change of Φ_j from (10) which is due to the structure of the nucleon and assuming that at small energies this will lead to the appearance of two new constants [20] we easily find

$$\frac{k_l^2}{\omega}\alpha = \Delta(\Phi_1 + x\Phi_2 + x\Phi_3 + x^2\Phi_4 + \Phi_5),$$
$$\frac{k_l^2}{\omega}\beta = \Delta(-\Phi_2 - x\Phi_4 + \Phi_6). \qquad (17)$$

In our approximation there exist just three phases (11). Inserting (10) and (11) into (17) and using (15) we obtain

$$\frac{k_l^2}{\omega}\alpha = \Delta S_1^e = \Delta(\delta_3^e + \tfrac{1}{2}\delta_1^e),$$
$$\frac{k_l^2}{\omega}\beta = \Delta S_1^m = \Delta\left(\delta_3^m + \tfrac{1}{2}\delta_1^m\right). \qquad (18)$$

Equation (18) is already completely clear: it shows that the polarizability is given by the change of the electric (S_1^e) and the magnetic (S_1^m) parts of the amplitudes which are independent of the spin dependent interactions. Expressing α and β in terms of $M(\delta)$ and $E(\delta)$ we have

$$\alpha = r_0\frac{R(\delta)M(\delta) + {}^1/_2E(\delta)}{k_l^2}, \quad \beta = r_0\frac{(1 - R(\delta))M(\delta)}{k_l^2}. \qquad (19)$$

An estimate of α and β at k_l^2 = 60 MeV ($1/k^2 = 107 \times 10^{-27}$ cm^2) leads to the following result:

$$\alpha \approx 126\cdot10^{-44}\ \text{cm}^3, \quad \beta \approx 1.1\cdot10^{-44}\ \text{cm}^3; \quad \alpha \gg \beta. \qquad (20)$$

If one assumes that α is the sum of a "true" polarizability α_0 and $(r_0/3)\langle r_e^2 \rangle = 32.6 \times 10^{-44}$ cm^3 ($\sqrt{\langle r_e^2 \rangle} \approx 0.8 \times 10^{-13}$ cm)[22] then one finds for α_0 the value

$$\alpha_0 = \alpha - \frac{r_0}{3} \langle r_e^2 \rangle = 93.4 \cdot 10^{-44} \text{cm}^3. \qquad (21)$$

These results agree well with the evaluation of the experiments on the Compton effect at 60 MeV[4b] which yields

$$\alpha^e \approx (90 \pm 20) \cdot 10^{-44} \text{ cm}^3, \qquad \beta^e \lessapprox (20 \mp 20) \cdot 10^{-44} \text{ cm}^3, \qquad (22)$$

and also with different theoretical estimates.[20,21]

We note that a further improvement in the precision of the account of the influence of the photoproduction on the δ_j^i will barely have a noticeable influence on the magnitudes α: the next phases which contribute to it are $\Delta\delta_{5,3}^{e2}$. However, the numerical value of the quantity β may change since it contains first order contributions from $\Delta\delta_1^m$ and also $\Delta\delta_3^{m2}$ and $\Delta\delta_5^{e2}$.

[1] Gell-Mann, Goldberger, and Thirring, Phys. Rev. **95**, 1612 (1954).

[2] a) R. H. Capps, Phys. Rev. **106**, 1031 (1951);
b) R. H. Capps, Phys. Rev. **108**, 1032 (1957).

[3] C. L. Oxley and V. L. Telegdi, Phys. Rev. **100**, 435 (1955).

[4] a) Goldanskiĭ, Govorkov, Karpukhin, Kutsenko, and Pavlovskaya, DAN SSSR **111**, 988 (1956), Soviet Phys. Doklady **1**, 735 (1957);
b) Goldanskiĭ, Karpukhin, Kutsenko, and Pavlovskaya, JETP **38**, 1695 (1960), Soviet Phys. JETP **11**, 1223 (1960).

[5] Pugh, Gomez, Frisch, and Janes, Phys. Rev. **105**, 982 (1957).

[6] C. L. Oxley, Phys. Rev. **110**, 733 (1958).

[7] Yamagata, Auerbach, Bernardini, Filosofo, Hanson, and Odian, Bull. Am. Phys. Soc. II **1**, 350 (1956), CERN Symposium **2**, 291 (1956).

[8] Bernardini, Hanson, Odian, Yamagota, Auerbach, and Filosofo, Nuovo cimento **18**, 1203 (1960).

[9] Hyman, Ely, Frisch, and Wahlig, Phys. Rev. Lett. **3**, 93 (1959)

[10] F. E. Low, Phys. Rev. **96**, 1428 (1954); M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

[11] V. I. Ritus, JETP **30**, 1070 (1956), Soviet Phys. JETP **3**, 926 (1957).

[12] N. F. Nelipa, JETP **35**, 662 (1958), Soviet Phys. JETP **8**, 460 (1959).

[13] N. Austern, Phys. Rev. **100**, 1522 (1955); B. T. Feld, Ann. Phys. **4**, 189 (1958).

[14] Karzas, Watson, and Zachariasen, Phys. Rev. **110**, 253 (1958).

[15] F. Low, Preprint.

[16] T. Akiba and J. Sato, Progr. Theor. Phys. **19**, 93 (1958).

[17] V. K. Fedyanin, DAN SSSR **140**, 347 (1961), Soviet Phys. Doklady **6**, 791 (1962).

[18] M. Jacob and J. Mathews, Phys. Rev. **117**, 854 (1960).

[19] L. I. Lapidus and Chou Kuang-chao, JETP **41**, 294 (1961), Soviet Phys. JETP **11**, 147 (1960).

[20] A. M. Baldin, Nucl. Phys. **18**, 310 (1960).

[21] V. S. Barashenkov and G. Yu. Kaiser, Preprint J. Inst. Nucl. Res. P-771 (1961).

[22] V. A. Petrun'kin, JETP **40**, 1148 (1961), Soviet Phys. JETP **13**, 808 (1961).

[23] A. I. Baz', JETP **33**, 923 (1957), Soviet Phys. **6**, 709 (1958).

[24] L. I. Lapidus and Chou Kuang-chao, Preprint Joint Inst. Nucl. Res. R-372 (1959); JETP **38**, 201 (1960), Soviet Phys. JETP **11**, 147 (1960).

[25] M. Cini and R. Stroffolini, Nucl. Phys. **5**, 684 (1958).

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