ENERGY LOSSES OF CHARGED PARTICLES IN A PLASMA

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The differential probabilities for radiation and absorption of longitudinal and transverse quanta by a charged particle in media with spatial dispersion are determined in the presence of radiation. It is shown that in a quantum analysis that allows for the recoil effect[11] the energy losses depend on the radiation density. Energy losses of relativistic particles in a high-temperature media and particularly in an ultrarelativistic plasma are considered.

PARTICLE energy losses in a plasma were first analyzed by Akhiezer and Sitenko,[1] who used the kinetic equation. Their Green's-function calculations were refined by Larkin[2] who determined, in particular, the coefficients under the logarithm signs in the expressions.[1] Larkin's results contain an approximate polarization operator. At the same time Lienhard[3], Silin,[4] and others (see, for example, [5]) obtained the particle energy losses in media with spatial dispersion, involving only the dielectric-constant tensor $\epsilon_{ij}(\omega, k)$. The latter can be readily related with the polarization operator (see [6]). A comparison of Silin's and Larkin's results shows, however, that the expressions obtained by Larkins contain additional temperature-dependent factors.

The appearance of temperature factors is closely connected with the dispersion relations between the real and imaginary parts of the photon Green's functions (see [7-9]). The problem solved by Larkin actually differs from the problem considered in [1] and [4], although this fact is not stipulated. The point is that the temperature Green's function technique used by Larkin presupposes that the system is in complete equilibrium particularly in equilibrium with the radiation, whereas Akhiezer and Sitenko did not consider the equilibrium radiation.

The presence of equilibrium radiation gives rise to induced emission and absorption proportional to the radiation density. The energy lost by a charged particle is the difference between the emitted and absorbed energy. If $\gamma^{+}_{\mathbf{p},\,\mathbf{p}-\mathbf{k}}$ is the probability of spontaneous emission of a quantum with transition from a state with momentum **p** to a state with momentum **p** − **k**,* then the total radiation probability is $\gamma^{+}_{\mathbf{p},\,\mathbf{p}-\mathbf{k}}(1 + N_{\omega,\,\mathbf{k}})$ where $N_{\omega,\,\mathbf{k}}$ is the number of quanta of frequency ω (equal to the transition frequency) and momentum **k**. Let the probability of absorption with transition from **p** − **k** to **p** be $\gamma^{-}_{\mathbf{p}-\mathbf{k},\,\mathbf{p}} N_{\omega,\,\mathbf{k}}$, with $\gamma^{-}_{\mathbf{p}-\mathbf{k},\,\mathbf{p}} = \gamma^{+}_{\mathbf{p},\,\mathbf{p}-\mathbf{k}}$ (see [10]). The absorption from the state with momentum **p** is thus specified by the quantity $\gamma^{+}_{\mathbf{p}+\mathbf{k},\,\mathbf{p}} N_{\omega,\,\mathbf{k}}$. The spectral density of the energy loss

$$W_\omega = \omega\{(N_{\omega,\mathbf{k}} + 1)\gamma^{+}_{\mathbf{p},\mathbf{p}-\mathbf{k}} - N_{\omega,\mathbf{k}}\gamma^{+}_{\mathbf{p}+\mathbf{k},\,\mathbf{p}}\}$$

is independent of the radiation density $N_{\omega,\,\mathbf{k}}$ only if the quantum corrections (recoil) can be neglected, $|\mathbf{k}| \ll |\mathbf{p}|$, and if one can assume approximately $\gamma^{+}_{\mathbf{p},\,\mathbf{p}-\mathbf{k}} \approx \gamma^{+}_{\mathbf{p}+\mathbf{k},\,\mathbf{p}} \approx \gamma^{+}_{\mathbf{p},\,\mathbf{p}}$. Attention should be called to the fact that although the quantum corrections are small,* the fraction of the particle-energy change proportional to the radiation density may in the case of large radiation density turn out to exceed the losses without radiation.

In an account of the spatial dispersion of the dielectric constant there is no need to distinguish between losses connected due to long-range and short-range collisions (see, for example, [4]). In short-range collisions with large momentum transfer **k** is large, indicating merely that spatial dispersion cannot be neglected in this region. The problem thus reduces to a successive account of the quantum effects both in the probability for the emission of the quantum and in the dielectric constant of the medium. For an analysis of the motion of relativistic particles it is necessary, in addition, to carry out the relativistic calculations.

*The spin states, say, of the electron are disregarded for simplicity.

*For Cerenkov radiation (emission of a transverse quantum) the relative correction in the visible region, as shown by Ginzburg[11] is $\approx 10^{-5}$–10^{-6}. For the emission of a longitudinal quantum (plasmon) the quantum corrections may be more appreciable.[12]

A relativistic quantum expression for the dielectric constant in an isotropic medium, with account of the spatial dispersion, was obtained earlier[6] in the e^2 approximation. In the present paper we obtain quantum differential radiation probabilities in the e^2 approximation. Unlike Larkin[2], we do not confine ourselves to the relativistic region. This enables us, in particular, to obtain the energy losses of relativistic particles in a highly heated equilibrium plasma.

1. DIFFERENTIAL PROBABILITY OF EMISSION AND ABSORPTION OF LONGITUDINAL AND TRANSVERSE QUANTA BY A CHARGED PARTICLE IN AN ISOTROPIC MEDIUM

1. To determine the emission probability γ it is sufficient, as shown in [12],* to know the effective energy spectrum $E(\mathbf{p})$ of the particle in the medium. Then

$$\gamma = -2\,\mathrm{Im}\,E(\mathbf{p}). \tag{1}$$

In the e^2 approximation we can disregard in the losses the macroscopic mass-renormalization effects considered in [12]; we then obtain for $\gamma/\epsilon_p \ll 1$, where ϵ_p is the particle energy,

$$\gamma = 2\,(\delta E'' - \mathbf{p}\delta\mathbf{p}''/\varepsilon_p - m\delta m''/\varepsilon_p). \tag{2}$$

Here $\delta E''$, $\delta \mathbf{p}''$, and $\delta m''$ are the components of the anti-Hermitian part of the mass operator:†

$$\delta\hat{M}_R = i\boldsymbol{\gamma}\,(\delta\mathbf{p}' + i\delta\mathbf{p}'') - \gamma_4\,(\delta E' + i\delta E'') + \delta m' + i\delta m''.$$

In the e^2 pproximation we have

$$\delta\hat{M}_c = -\frac{ie_1^2}{(2\pi)^4}\int \gamma_\mu G_c(\varepsilon_p - \omega,\ \mathbf{p}-\mathbf{k})\,\gamma_\nu D^c_{\mu\nu}(\omega, \mathbf{k})\,d\omega\,d\mathbf{k}. \tag{3}$$

The causal photon Green's function $D^c_{\mu\nu}$ can be expressed in terms of the imaginary part of the retarded function $D^{R''}_{\mu\nu}$ with the aid of the relations derived, for example, by Dzyaloshinskiĭ and Pitaevskiĭ[9]:

$$D^c_{\mu\nu}(\omega, \mathbf{k}) = \frac{1}{\pi}\int_0^\infty D^{R''}_{\mu\nu}(\omega', \mathbf{k})\left\{\mathrm{P}\frac{1}{\omega'-\omega} - \mathrm{P}\frac{1}{\omega'+\omega} + i\pi\,\mathrm{cth}\frac{\omega'\beta}{2}\,[\delta(\omega-\omega') - \delta(\omega+\omega')]\right\}d\omega', \tag{4‡}$$

where P is the principal-value symbol and $1/\beta$ is the temperature.

The electron Green's function is best written in the form (see [6])

$$G_c(E, \mathbf{p}) = \Lambda^+_\mathbf{p}\mathrm{P}\frac{1}{\varepsilon_\mathbf{p} - E} + \Lambda^-_\mathbf{p}\mathrm{P}\frac{1}{\varepsilon_\mathbf{p}+E} + \Lambda^+_\mathbf{p}\,i\pi\delta(\varepsilon_\mathbf{p} - E) + \Lambda^-_\mathbf{p}\,i\pi\delta(\varepsilon_\mathbf{p} + E);$$

$$\Lambda^\pm_\mathbf{p} = (m - i\hat{p}^\pm)/2\varepsilon_\mathbf{p},\quad p^\pm_\mu = \{\mathbf{p}^\pm,\ i\varepsilon_\mathbf{p}\},\qquad \hat{p} = p_\mu\gamma_\mu, \tag{5}$$

where $\epsilon_\mathbf{p}$ is the modulus of the energy.

Upon substituting (4) and (5) in (3) we can integrate in the Hermitian part of $\delta M'$ by using δ functions. The anti-Hermitian part of the mass operator is then determined (see [12]) by replacing the resultant energy denominators with δ functions.

$$\delta\hat{M}'' = \frac{e_1^2\pi}{(2\pi)^4}\int_0^\infty d\omega\int d\mathbf{k}D^{R''}_{\mu\nu}(\omega, \mathbf{k})\Big\{\gamma_\mu\Lambda^+_{\mathbf{p}-\mathbf{k}}\gamma_\nu\Big[\delta(\omega + \varepsilon_{\mathbf{p}-\mathbf{k}} - \varepsilon_\mathbf{p})\times\Big(1 + \mathrm{cth}\frac{\omega\beta}{2}\Big) + \delta(\omega + \varepsilon_\mathbf{p} - \varepsilon_{\mathbf{p}-\mathbf{k}})\Big(1 - \mathrm{cth}\frac{\omega\beta}{2}\Big)\Big] + \gamma_\mu\Lambda^-_{\mathbf{p}-\mathbf{k}}\gamma_\nu\Big[\delta(\omega + \varepsilon_{\mathbf{p}-\mathbf{k}} + \varepsilon_\mathbf{p})\Big(1 + \mathrm{cth}\frac{\omega\beta}{2}\Big) + \delta(\omega - \varepsilon_\mathbf{p} - \varepsilon_{\mathbf{p}-\mathbf{k}})\Big(1 - \mathrm{cth}\frac{\omega\beta}{2}\Big)\Big]\Big\}. \tag{6}$$

The radiation described by the first term of (6), proportional to $\delta(\omega + \epsilon_{\mathbf{p}-\mathbf{k}} - \epsilon_\mathbf{p})$, contains a factor $1 + \coth(\omega\beta/2) = 2(N_\omega + 1)$, where $N_\omega = [e^{\omega\beta} - 1]^{-1}$ is the number of equilibrium quanta of frequency ω. The absorption is proportional to $2N_\omega = \coth(\omega\beta/2) - 1$. In addition, losses are caused by the single-photon annihilation, described by the last term of (6).* Within the framework of the e^2 approximation, the losses contain only the number of quanta averaged over the initial statistical ensemble. Therefore in the case of nonequilibrium non-isotropic radiation it is sufficient to replace N_ω by the quantity $N_{\omega,\mathbf{k}}$ which is determined from the radiation density. We shall assume here that $N_{\omega,\mathbf{k}}$ vanishes at large frequencies $\omega \sim \epsilon_\mathbf{p} + m$, where the conservation laws allow annihilation. Further, the integration with respect to the longitudinal-quantum momenta in (6) can be readily carried out with the aid of δ functions, and iteration with respect to the transverse momenta reduces to integration over the scattering angles.

In the case of isotropic media the Green's function $D^{R''}_{\mu\nu}$ can be expressed in terms of the longitudinal and transverse dielectric constants $\epsilon^l(\omega, \mathbf{k})$ and $\epsilon^t(\omega, \mathbf{k})$.[12] We then obtain from (6) and (2) the following probabilities for the emis-

*We shall retain the notation of our earlier paper.[12]

†We are considering a spin-½ particle.

‡cth = coth.

*It is assumed in (6) that the energy-losing particle in the medium is not one of the original particles of the medium. The annihilation is therefore due to the fact that particle pairs identical with the energy-losing particle are in equilibrium with the radiation. Upon annihilation the particle is replaced by a thermal particle.

sion (γ^+) and absorption (γ^-) of a longitudinal or transverse quantum

$$\gamma^{\pm}_{t,l} = \int_0^\infty d\omega \int d\Omega \gamma^{\pm}_{t,l}(\omega, \theta), \tag{7}$$

where $d\Omega$ is the scattering solid angle, $\theta = (\mathbf{p}\cdot\mathbf{p}')/pp'$; $\mathbf{p}'$ is the momentum after scattering. Here $\gamma^{\pm}_{t,l}(\omega, \theta)$ are the differential probabilities:

$$\gamma^{\pm}_{t}(\omega, \theta) = \frac{e_1^2}{\pi^2}\sqrt{p^2+\omega^2\mp 2\varepsilon_p\omega}$$
$$\times\left[\frac{p^2}{\varepsilon_p} + \omega - \frac{p}{\varepsilon_p}\sqrt{p^2+\omega^2\mp 2\varepsilon_p\omega}\cos\theta\right.$$
$$\left. + \frac{p^2\sin^2\theta}{k_\pm^2\varepsilon_p}(p^2+\omega^2\mp 2\varepsilon_p\omega)\right]$$
$$\times \operatorname{Im}\frac{1}{k_\pm^2 - \omega^2\varepsilon^t(\omega, k_\pm)}\begin{cases} N_{\omega,\mathbf{k}}+1 & \text{for } \gamma^+ \\ N_{\omega,\mathbf{k}} & \text{for } \gamma^- \end{cases}, \tag{8}$$

$$\gamma^{\pm}_{l}(\omega, \theta) = -\frac{e_1^2}{2\pi^2}\sqrt{p^2+\omega^2\mp 2\varepsilon_p\omega}\,[\varepsilon_p \mp \omega + \frac{m^2}{\varepsilon_p}$$
$$+\frac{p}{\varepsilon_p}\sqrt{p^2+\omega^2\mp 2\varepsilon_p\omega}\cos\theta]$$
$$\times \operatorname{Im}\frac{1}{k_\pm^2\,\varepsilon^l(\omega, k_\pm)}\begin{cases} N_{\omega,\mathbf{k}}+1 & \text{for } \gamma^+ \\ N_{\omega,\mathbf{k}} & \text{for } \gamma^- \end{cases}; \tag{9}$$

$$k_\pm^2 = (p^2+\omega^2\mp 2\varepsilon_p\omega) - 2p\sqrt{p^2+\omega^2\mp 2\varepsilon_p\omega}\cos\theta + p^2. \tag{10}$$

ψ is the angle the quantum makes with the initial momentum

$$\cos\psi^{\pm} = \pm[p - \sqrt{p^2+\omega^2\mp 2\varepsilon_p\omega}\cos\theta]/k_\pm. \tag{11}$$

The probabilities given by formulas (8) and (9) differ from zero only if the radicands in (8) and (9) are positive.

2. Let us consider the induced Cerenkov radiation.[13] As is well known, the spatial dispersion in the transverse component ϵ^t plays a small role at ordinary nonrelativistic temperatures. Assuming that $\epsilon^t(\omega, k_\pm) \approx \epsilon(\omega) = n^2(\omega)$ and that the imaginary part of ϵ^t is equal to zero (see [6]) we have

$$\operatorname{Im}[k_\pm^2 - \omega^2 n^2(\omega)]^{-1} = \pi\delta(k_\pm^2 - \omega^2 n^2(\omega)). \tag{12}$$

From this and from (8)

$$\gamma_t^+ = \int_{\cos\psi_+<1} e_1^2 v\left[1 - \frac{1}{n^2v^2} - \frac{\omega}{vp}\left(1-\frac{1}{n^2}\right)\right.$$
$$\left.+\frac{\omega^2n^2}{4p^2}\left(1-\frac{1}{n^4}\right)\right](N_\omega^+ + 1)d\omega, \tag{13}$$

$$\gamma_t^- = \int_{\cos\psi_-<1} e_1^2 v\left[1 - \frac{1}{n^2v^2} + \frac{\omega}{vp}\left(1-\frac{1}{n^2}\right)\right.$$
$$\left.+\frac{\omega^2n^2}{4p^2}\left(1-\frac{1}{n^4}\right)\right]N_\omega^- d\omega;$$

$$\cos\psi_\pm = 1/nv \pm \omega(n^2-1)/2pn, \tag{14}$$

where $v = p/\epsilon_p$ is the particle velocity, $N_\omega^\pm = N(\omega, \psi_\pm)$. Finally,

$$N(\omega, \psi) = \frac{1}{2\pi}\int_0^{2\pi} N(\omega, k, \psi, \varphi)\,d\varphi$$

with $k = \omega n$, where $N(\omega, k, \psi, \varphi)$ — number of quanta $N_{\omega,\mathbf{k}}$ in the spherical system in the space $\mathbf{k}(k, \psi, \varphi)$ with z axis parallel to $\mathbf{p}$.

We consider the simplest example with constant n, isotropic and constant N, and $nv > 1$. We then obtain for the induced part of the "losses"

$$\Delta W_n = \int(\gamma_t^+ - \gamma_t^-)\,\omega d\omega$$
$$= -\frac{8e_1^2 v\varepsilon_p^2 N}{nv(n^2-1)}\left[n^2v^2\left(\frac{5}{3}+\frac{n^2+1}{n^2-1}\right)+\frac{2n^2}{n^2-1}\right]. \tag{15}$$

The condition $nv > 1$ is usually not satisfied for all frequencies. If $N_\omega \neq 0$ in the region where both conditions $\cos\psi_+ < 1$ and $\cos\psi_- < 1$ are satisfied and N_ω is isotropic, then

$$\Delta W = -\frac{2e_1^2}{p}\int N_\omega\,\omega^3\,d\omega\left(1-\frac{1}{n^2}\right). \tag{16}$$

This part of the forces accelerates the particles, rather than slowing them down. In order for this quantity to be comparable with or larger than the ordinary Cerenkov radiation, it is necessary to have $N_\omega \sim \epsilon_p v^2/\omega \sim 10^6$. The order of magnitude of N_ω is $(2\pi)^3\rho(\omega)/n^2\omega^3$, where $\rho(\omega)$ is the radiation density, i.e., $\rho(\omega) \sim n^2\omega^2\epsilon_p/(2\pi)^3$; for $\Delta\omega \sim \omega$ we have

$$\rho \sim \omega^3\varepsilon_p/(2\pi)^3 \sim \varepsilon_p/\lambda^3.$$

For an electron, for example, an energy of the order of 1 MeV should be concentrated in a cube with dimension on the order of the radiation wavelength. We consequently have for $\lambda \sim 5\times 10^3$ Å a value $\rho \sim 10^7$ erg/cm^3, corresponding to a radiation pressure on the order of 10 atm.

It is easy to understand why the particle is accelerated by the induced processes. The smaller the angle the emitted quantum makes with the particle velocity, the lower the intensity of the Cerenkov radiation. Since the emission angle at a given frequency is somewhat smaller than the absorption angle, (by virtue of the conservation laws) induced absorption predominates over induced emission. In the equilibrium state the induced processes cannot predominate for equilibrium radiation, since $N_{\omega,\text{eq}} \gg 1$ when $\omega\beta \ll 1$; in this case the relative magnitude of the induced radiation is $1/\epsilon_p\beta \ll 1$.*

*If $\epsilon_p\beta \ll 1$, then the temperatures are ultrarelativistic, the substance is ionized at equilibrium, $n^2<1$, and Cerenkov radiation is impossible.

For anisotropic highly directional radiation, at frequencies not close to the threshold of Cerenkov absorption and emission, we have

$$N_\omega^\pm \approx N_0 \mp \frac{\partial N_0}{\partial \psi_0} \frac{\omega (n^2-1)}{2pn(1-n^{-2}v^{-2})^{1/2}}\ ; \ \cos\psi_0 = \frac{1}{nv},$$

$$N_0 = N(\omega, \psi_0), \qquad (17)$$

$$\Delta W \approx \frac{2e_1^2}{p}\int \omega^3 d\omega \left(1-\frac{1}{n^2}\right)\left[-N_0 - \frac{1}{2}\sqrt{n^2v^2-1}\,\frac{\partial N_0}{\partial \psi_0}\right]. \qquad (18)$$

If the radiation density changes abruptly with the angle (as is the case when the radiation directivity is high), namely,

$$(\sin\psi_0/N_0)(\partial N_0/\partial\psi_0) \gg 1$$

then the sign of the "loss" is determined by $\partial N_0/\partial\psi_0$. If the radiation density in the region of the Cerenkov angles decreases with increasing angle, then the induced radiation and emission increase the losses.

3. Let us consider the question of induced emission of plasmons. The presence of plasma waves in the system can either be due to nonequilibrium fluctuations or produced artificially, say by a beam in the plasma. We shall therefore assume that $N_{\omega,\,k}$, which is connected with the plasma density, has a nonvanishing value only in the region where the plasmon damping is small. In this region we can approximately neglect the spatial dispersion and put

$$\operatorname{Im}\frac{1}{\varepsilon^l(\omega, k_\pm)} \approx -\pi\delta(\varepsilon(\omega)) = -\pi\sum_s \delta(\omega-\omega_s)\left|\frac{\partial\varepsilon}{\partial\omega}\right|^{-1}_{\omega=\omega_s}, \qquad (19)$$

where ω_s are the zeros of $\epsilon(\omega)$. Let $N_{\omega,\,k}$ be isotropic, independent of k when $0 < k < k_{max}$, and equal to zero when $k > k_{max}$. The induced emission and absorption contributes to the particle energy change per second if the order of ω_s/vp is taken into account. For $\omega_s/vp \ll 1$ we have from (9)

$$\Delta W = -e_1^2\sum_s \frac{2N_{\omega_s}\omega_s^2}{v\varepsilon_p}\left|\frac{\partial\varepsilon(\omega)}{\partial\omega}\right|^{-1}_{\omega=\omega_s}\left[\ln\frac{v^2k_{max}^2}{\omega_s^2} - 2 + \frac{1}{v^2}\right]. \qquad (20)$$

2. CHARGED-PARTICLE ENERGY LOSSES IN AN EQUILIBRIUM PLASMA

1. For an equilibrium isotropic plasma we can obtain for the energy losses a simpler expression which allows for the induced emission and absorption. We introduce $x = \cos\psi = (\mathbf{k}\cdot\mathbf{p})/kp$ and integrate in (6) with respect to the frequency, using δ functions. Radiation will then involve angles x, for which

$$\varepsilon_p > \varepsilon_{p-k} = \sqrt{\varepsilon_p^2 - 2pkx + k^2},$$

i.e., $x > k/2p$, while for absorption $x < k/2p$ $(-1 < x < 1)$.

To find the energy loss we must multiply the differential radiation probability by $\omega = \epsilon_p - \epsilon_{p-k}$, subtract the differential absorption probability multiplied by ω, and integrate the results over all k. Recognizing that $\operatorname{Im}\epsilon(\omega)$ is an odd function of ω, we find that the emission and absorption are described by identical expressions, except that for emission we integrate with respect to x in the region $x > k/2p$, while for absorption we integrate in the region $x < k/2p$. The final expression for the losses, $W = W^t + W^l$, will include integration over all angles $(-1 < x < 1)$. Let us calculate W^l in detail:

$$W^l = \int_0^\infty dk \int_{-1}^{1} dx\, W^l(k, x); \qquad (21)$$

$$W^l(k,x) = -\frac{e_1^2}{2\pi}\left[1 + \operatorname{cth}\frac{\beta(\varepsilon_p - \varepsilon_{p-\kappa})}{2}\right](\varepsilon_p - \varepsilon_{p-k}) \times \left(1 + \frac{\varepsilon_p - kvx}{\varepsilon_{p-k}}\right)\operatorname{Im}\frac{1}{\varepsilon^l(\varepsilon_p - \varepsilon_{p-k}, k)}. \qquad (22)$$

It is easy to verify that the term with $\coth(\omega\beta/2)$ is insignificant if quantum effects (radiation recoil) are neglected. Indeed, we then have $\omega \approx kvx$ and

$$W^l(kx) = -\frac{e^2}{2\pi}\left(1 + \operatorname{cth}\frac{kvx\beta}{2}\right) kvx \operatorname{Im}\frac{1}{\varepsilon^l(kvx, k)}.$$

Since $\operatorname{Im}\epsilon$ is odd, the term with coth $kvx\beta$ being odd in x, drops out from (21).

To find the energy losses of a relativistic particle in a plasma we must know the relativistic quantum expression for $\epsilon^l(\omega, \mathbf{k})$. It was derived previously [6] and has the following form for a Boltzmann electron gas

$$\left(1 + \operatorname{cth}\frac{\omega\beta}{2}\right)\operatorname{Im}\varepsilon_e^l(\omega, k) = \frac{4e^2}{k^3\beta}\left[\frac{m_e^2}{1-\omega^2/k^2} + \frac{2}{\beta}\varkappa_e + \frac{2}{\beta^2}\right]\exp\{(\mu + \omega/2 - \varkappa_e)\beta\};$$

$$\varkappa_e = \sqrt{m_e^2(1-\omega^2/k^2)^{-1} + k^2/4}, \qquad (23)$$

where μ — chemical potential and m_e — electron mass.

2. We consider the passage of a fast particle, particularly a relativistic one, through a plasma with nonrelativistic temperature, $(\epsilon_p - m)\beta \gg 1$. We assume that the Debye radius is large compared with the deBroglie electron wavelength corresponding to thermal velocities

$$d^2 = 1/\beta m_e \omega_{0e}^2 \gg L^2 = \beta/m.$$

As in [2], we break up the integral with respect to k into two, one from zero to k_1 and one from k_1 to ∞, where $1/d^2 \ll k_1^2 \ll 1/L^2$. In the first interval we can neglect the quantum corrections and assume $\omega = kvx$. Using the analyticity of $1/\epsilon^l$ in the upper half-plane of complex ω (see [2]), we can neglect the spatial dispersion in ϵ^l. Then

$$\operatorname{Im}(1/\varepsilon^l) = -\pi\delta(1 - \omega_e^2/\omega^2),$$

and we obtain

$$W_1^l = (e^2\omega_{0e}^2/v)\ln(k_1 v/\omega_{0e}), \quad \omega_e^2 = 4\pi N_{0e}^2 e^2/m_e. \tag{24}$$

In the integral in the second region, ϵ^l is close to unity. In addition, in the case of nonrelativistic temperatures, we need retain in the square brackets of (23) only the first term. Then

$$W_2^l(k, x) \approx \frac{2e^2 e_1^2}{\pi k^3 \beta} \frac{m_e^2}{1 - \omega^2/k^2}\left(1 + \frac{\varepsilon_p - kvx}{\varepsilon_{p-k}}\right)\omega$$

$$\times \exp\{\beta[\mu - m_e - f(k, x)]\};$$

$$\omega = \varepsilon_p - \varepsilon_{p-k}, \quad f(k, x) = \varkappa_e - \omega/2 - m_e. \tag{25}$$

Since β is large, the main contribution is made by the stationary point f. We have f = 0 and $\partial f/\partial k = 0$ when

$$k = k_0 = \frac{2vxm_e\,\varepsilon_p(m_e + \varepsilon_p)}{(\varepsilon_p + m_e)^2 - v^2x^2\varepsilon_p^2}, \quad \omega = \frac{2\varepsilon_p^2 v^2x^2 m_e}{(\varepsilon_p + m_e)^2 - v^2x^2\varepsilon_p^2}, \tag{26}$$

$$\left[m_e \frac{\partial^2 f}{\partial k^2}(k_0, x)\right]^{1/2} = \frac{[(\varepsilon_p + m_e)^2 - \varepsilon_p^2 v^2 x^2]^2}{2\varepsilon_p(\varepsilon_p + m_e)[(\varepsilon_p + m_e)^2(1 - v^2x^2) + m_e^2 v^2 x^2]}. \tag{27}$$

From this we get

$$W_2^l = \frac{e_1^2\omega_{0e}^2}{v}\int_{x_1}^{1}\frac{dx}{x}\frac{(\varepsilon_p + m_e)^2(1 - v^2x^2)}{(\varepsilon_p + m_e)^2 - v^2x^2\varepsilon_p^2},$$

$$x_1 = \frac{\varepsilon_p + m_e}{\varepsilon_p}\frac{\sqrt{m_e^2 + k_1^2} - m_e}{vk_1}. \tag{28}$$

Calculating this integral under the condition $x_1 \ll 1$, which follows from $k_1^2 \ll m/\beta$, and adding to (24), we obtain the final formula for the longitudinal part of the energy loss of a charged particle in a plasma

$$W^l = \frac{e_1^2\omega_{0e}^2}{v}\left\{\ln\frac{2m_e\,\varepsilon_p v^2}{(\varepsilon_p + m_e)\,\omega_{0e}\hbar} - \frac{1}{2}\left[1 - \left(1 + \frac{m_e}{\varepsilon_p}\right)^2\right]\ln\left[1 - \frac{\varepsilon_p^2 v^2}{(\varepsilon_p + m_e)^2}\right]\right\}; \tag{29}$$

$\epsilon_p = (p^2 + m_1^2)^{1/2}$, m_1 is the mass and e_1 the charge of the particle under consideration.

For nonrelativistic velocities $v \ll 1$ we can neglect the last term in (29) and set ϵ_p equal to m_1. Then (29) coincides with Larkin's result[2].

In the classical region $0 < k < k_1$ there are no transverse losses W^t, and in the quantum region we obtain by similar calculations

$$W^t = \frac{1}{2}e_1^2\omega_{0e}^2 v\left\{\frac{(\varepsilon_p + m_e)^2}{v^2\varepsilon_p^2}\ln\frac{(\varepsilon_p + m_e)^2}{(\varepsilon_p + m_e)^2 - v^2\varepsilon_p^2} - 1 + \frac{m_e^2\varepsilon_p^2 v^2}{[(\varepsilon_p + m_e)^2 - \varepsilon_p^2 v^2]^2}\right\}.$$

For ultrarelativistic velocities we get

$$W = W^t + W^l = e_1^2\omega_{0e}^2\begin{cases}\ln\dfrac{2m_e\varepsilon_p}{m_1\omega_{0e}} - \dfrac{1}{2} & \text{for } \dfrac{\varepsilon_p}{m_1} \ll \dfrac{m_1}{m_e}\\[2mm] \dfrac{1}{2}\ln\dfrac{2m_e\varepsilon_p}{\omega_{0e}^2} - \dfrac{3}{8} & \text{for } \dfrac{\varepsilon_p}{m_1} \gg \dfrac{m_1}{m_e}\end{cases}$$

3. Let us consider the passage of particles through an ultrarelativistic plasma, $\beta m_e \ll 1$, subject to the condition that the particle energy is much greater than the mean energy of the plasma electrons, $m_1/m_e \gg \beta\epsilon_p \gg 1$. The condition $d \gg L$ is retained. If the particle velocity is small compared with the mean electron velocity, $v \ll 1$, then we can neglect W^t with accuracy to v^2, and assume the real part of ϵ^l equal to $(1 + k^2d^2)^{-1}$; then

$$\operatorname{Im}\frac{1}{\varepsilon^l} \approx -\frac{\operatorname{Im}\varepsilon^l k^4}{(k^2 + d^{-2})^2}.$$

Since $\beta m_e \ll 1$, the quantity in the exponent of (23) will not be small compared with unity, provided $k \gg m_e$, and the most significant values of k do not exceed $1/\beta$. Therefore, expanding ω in powers of k ($\omega \approx kvx + k^2/2\epsilon_p$) and using the condition $\beta\epsilon_p \gg 1$, we can write with sufficient accuracy $\omega = kvx$, and by virtue of $v \ll 1$ this means that $\omega/k \ll 1$ in the entire region. We can therefore replace the exponential function in (23) by

$$(1 + \beta kvx/2)\,e^{-\beta/2k},$$

and retain in the square bracket of (23) only the last term. We substitute in the result of the integration the value of the chemical potential of the ultrarelativistic gas.

The final formula for the energy lost in the collision of a charged particle with the plasma electrons has the form

$$W^l = e_1^2\omega_{0e}^2 v^2[\ln(2d_e/\beta) - C - 1/2], \tag{32}$$

where C = 0.58 — Euler's number, $\omega_{0e}^2 = 4\pi N_e e^2\beta/3$ — natural frequency, and $d_e = (4\pi N_e e^2\beta)^{-1/2}$ — Debye radius of the ultrarelativistic plasma.

If the particle velocity is close to the velocity of light in the classical region $0 < k < k_1$, we can use the values of $\epsilon^{l,t}$ for the ultrarelativistic plasma; for example, for ϵ^l we have

$$\varepsilon^l(kx, k) = 1 + \frac{1}{k^2d^2}\left[1 + \frac{x}{2}\left(\ln\frac{1-x}{1+x} + i\pi\right)\right]; \tag{33}$$

The imaginary part of ϵ^l differs from zero for all k and x, by virtue of the time-like character of the vector k_μ ($\omega < k$). Substituting (33) in (22) and (21) and neglecting terms of order $k/\epsilon_p \leq 1/\beta\epsilon_p \ll 1$, we obtain

$$W_1^l = \frac{1}{2} e_1^2 \omega_{0e}^2 (\ln k_1^2 d_e^2 - A); \quad (34)$$

$$W_1^t = \frac{e_1^2\omega_{0e}^2}{4}\left(\ln 2k_1^2 d_e^2 + \frac{5}{3} + \frac{3}{2}\ln(1 - v^2) - 2.64\right);$$

$$A = 3\int_0^1 x^2 dx \left\{\frac{1}{2}\ln\left(f^2 + \frac{\pi^2 x^2}{4}\right) + \frac{2f}{\pi x}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{2f}{\pi x}\right)\right\} = 0.38,$$

$$f = 1 + \frac{x}{2}\ln\frac{1-x}{1+x}. \quad (35)^*$$

In the quantum region $k_1 < k < \infty$ and we can assume that $\mathrm{Re}\,\epsilon^{t,l} \approx 1$. Using (23), we obtain

$$2W_2^t = W_2^l = \omega_{0e}^2 e_1^2 (\ln(2/\beta k_1) - C). \quad (36)$$

Finally,

$$W = W^t + W^l = \frac{3}{4} e_1^2 \omega_{0e}^2 (\ln(2d_e^2 m_1/\beta^2 \varepsilon_p) - 0.82). \quad (37)$$

The fraction of the loss due to collisions with ions is calculated in similar fashion. If the thermal ion velocities are nonrelativistic, $\beta m_i \gg 1$, we can write $\mathrm{Im}\,(1/\epsilon^l)$ for the relativistic particles in the form

$$-|\varepsilon^l|^{-2}(\mathrm{Im}\,\varepsilon_e^l + \mathrm{Im}\,\varepsilon_i^l),$$

and use Eq. (33) for $|\epsilon^l|$; the relative error incurred is of order $v_i^2 \sim 1/\beta m_i \ll 1$. We obtain

$$W_i^l(kx) = \frac{e_1^2 e^2 m_i^2 N_i}{\pi\beta}\left(\frac{2\pi\beta}{m_i}\right)^{3/2}\frac{\omega}{1 - \omega^2/k^2}\left(1 + \frac{\varepsilon_p - kvx}{\varepsilon_{p-k}}\right) \times \frac{k}{(k^2 + f/d^2)^2 + \pi^2 x^2/4d^4} e^{\omega/2 + m_i - x_i}. \quad (38)$$

The main contribution to the integral with respect to k is made by the stationary point of the exponential, so that

$$W_i^l = \frac{e_i^2 \omega_{0i}^2}{v}\int_0^1 \frac{dx}{x}\frac{k_0^4}{(k_0^2 + 1/d^2)^2}\frac{(\varepsilon_p + m_i)^2(1 - v^2x^2)}{[(\varepsilon_p + m_i)^2 - v^2x^2\varepsilon_p^2]};$$

$$\omega_{0i}^2 = \frac{4\pi N_i e^2}{m_i}, \quad k_0 = \frac{2vxm_i\varepsilon_p(m_i + \varepsilon_p)}{(\varepsilon_p + m_i)^2 - v^2x^2\varepsilon_p^2}. \quad (39)$$

In addition, we put f = 1 in (39) and neglected the term $\pi^2 x^2/4d^2$ in the denominator of (38), thereby discarding terms of relative order $1/m_i^2 d^2 \ll 1$, i.e., of the order of the ratio of the ion Compton length to the electron Debye radius.

*arctg = $\tan^{-1}$.

The calculation of the integral (39) leads to the following result:

$$W_i^l = e_1^2 \omega_{0i}^2 \left\{\ln 2m_i d_e - \frac{1}{2} - \frac{1}{2}\left(1 + \frac{(\varepsilon_p + m_i)^2}{\varepsilon_p^2}\right) \times \ln\frac{\sqrt{m_i^2 + \varepsilon_p^2} - m_i}{\varepsilon_p} + \frac{1}{2}\left(1 - \frac{(\varepsilon_p + m_i)^2}{\varepsilon_p^2}\right)\ln\frac{\varepsilon_p}{2m_i}\right\}. \quad (40)$$

Finally, W_i^t coincides with (30), in which m_e is replaced by m_i.

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