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MEASUREMENT OF MASER LINE WIDTH

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A technique and apparatus are described for measuring the frequency fluctuations of an ammonia maser ($\lambda = 1.25$ cm). Measurements of the frequency fluctuation spectrum are presented. The experimental values of the absolute natural line width and relative technical line width (per second) are 5.3×10^{-4} cps and 1.7×10^{-11}, respectively.

THE very low power output of masers makes it extremely difficult to measure their line width. The techniques used conventionally to measure the line widths of microwave oscillators[1,2] cannot be employed. Following an analysis of several possible procedures it was decided to measure the line width by investigating the frequency fluctuations of audio-frequency beats between two identical masers.*

The method of Basov and Petrov[3] was used to obtain beat frequencies from the output of a circuit receiving oscillations from both masers simultaneously. For continuous registration of the beat frequency the receiver output was connected to a frequency discriminator. Because of the extremely narrow line it would be technically difficult to employ conventional discriminators (circuits or delay lines), which would require larger initial signal voltages.

The present author's setup for discriminating frequencies is based on the properties of an inductance bridged-T network as shown in Fig. 1. With the bridge balanced, 4Rr = L/C, the network input and output voltages are related by the following differential equation:

$$\ddot{V}(t) + 2\frac{r}{L}\dot{V}(t) + \omega_0^2 V(t) = \ddot{E}(t) + \omega_0^2 E(t), \qquad (1)$$

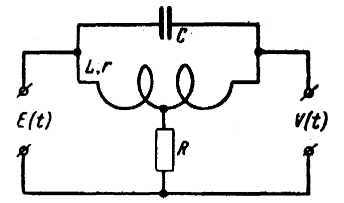

FIG. 1. Bridged-T circuit.

where $\omega_0^2 = 1/LC$. The response of the bridged-T network to weak amplitude and frequency fluctuations

$$E(t) = E_0(1 + \alpha_t)\cos\left(\omega t + 2\pi \int \nu_t dt\right),$$
$$\alpha_t \ll 1, \qquad \nu_t \ll \frac{\omega_0}{2\pi} \qquad (2)$$

can be reduced by the substitution

$$U(t) = \omega_0^{-2}\left[\ddot{E}(t) + \omega_0^2 E(t)\right] \qquad (3)$$

to the already solved problem[4] of the response of the tank circuit represented by the left-hand side of (1) to an equivalent emf U(t) with fluctuating frequency and amplitude.

Writing U(t) and the output voltage V(t) in a form similar to (2):

$$U(t) = U_0(1 + x_t)\cos(\omega t + \varphi_t), \qquad (4)$$
$$V(t) = V_0(1 + y_t)\cos(\omega t + \psi_t), \qquad (5)$$

we substitute (2) and (4) in (3) in order to determine x_t and φ_t and follow the procedure developed by Gorelik and Elkin.[4] We then find that for

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*The investigation of the beat-frequency fluctuations of two masers was suggested to the author by V. S. Troitskiĭ.

slight detuning $D = (\omega - \omega_0)/2\pi \ll \omega_0/2\pi$ the spectral density $\overline{y^2_F}$ of the square of relative amplitude fluctuations in the output voltage is related to the spectra of fluctuating parameters of the initial oscillations (2) by

$$\overline{y^2_F} \approx D^{-2}\overline{\nu^2_F} + \overline{\alpha^2_F} + \overline{(\alpha\nu)}_F / D. \qquad (6)$$

Here $\overline{\nu^2_F}$, $\overline{\alpha^2_F}$ and $\overline{(\alpha\nu)}_F$ are the spectral densities, respectively, of the square of frequency fluctuations, the square of relative amplitude fluctuations, and mixed (amplitude and frequency) fluctuations of the input oscillations E(t).

According to (6) the contribution of the first term (the useful effect) to the spectrum of relative amplitude fluctuations in the output voltage of the bridged T increases as the detuning D decreases. However, a calculation shows that at the same time the absolute value V_0 of the output voltage decreases.

For maximum utilization of the discriminating properties of the bridged-T network, which are manifested when D is small, the author proposed to amplify the discriminated signal V(t) at its carrier frequency ω, after which the envelope of the amplified signal $\sim V_0 yt$ would be discriminated by a detector and its spectrum would be analyzed (using an analyzer similar to that described by Malakhov in[5]). When the initial voltage was $E_0 \approx 1$ V this discriminator measured the spectral densities of the square of frequency fluctuations at frequencies to $10^{-8} - 10^{-10}$ cps, and thereby measured the absolute line width in the same range.

The principal limitations on the sensitivity of the entire setup were receiver noise ahead of the discriminator, thermal noise in the bridged T, and the shot effect in the amplifier of V(t). Noise in the detector following the amplifier and noise in the envelope analyzer in our case imposed practically no limitation on the sensitivity, unlike previous techniques.[1,2]

In the experiments with masers it was noted that when the bridged T was bypassed the separately measured amplitude fluctuations and circuit noise induced in the analyzer output a combined effect $10^{-3} - 10^{-2}$ times smaller than the discriminated signal. This shows the high reliability of the data obtained on the spectrum of maser frequency fluctuations.

The line width was measured using two ammonia masers that generated a 3-kcs beat frequency. As in[3], the masers were decoupled by a hybrid ring. The E_{010}-wave resonators made of invar were stabilized at about 40° C to within ± 0.02° C.

The spectral densities of the square of beat frequency fluctuations $\overline{\nu^2_F}$ are shown in Fig. 2.

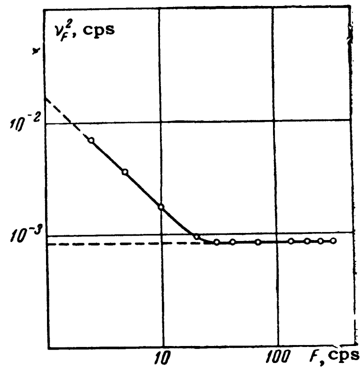

FIG. 2. Spectrum of beat-frequency fluctuations of two masers.

From 2.5 to 30 — 35 cps the spectrum follows a $1.6 \times 10^{-2}/F$ law, which then becomes a horizontal line up to at least 280 cps. It is reasonable to take $\overline{\nu^2_F}/2$ as the measure of the frequency fluctuation of each maser.

The natural maser line width[6] is determined by extrapolating the horizontal portion of the spectrum to F = 0, thus obtaining

$$\Delta f_{\text{nat}} = 5.3 \cdot 10^{-4} \text{cps}.$$

The initial descending portion gives the so-called technical line width.[6] For each maser this line width, in our notation, is

$$\Delta f_{\text{tech}} = (\pi\overline{\nu^2})^{1/2},$$

where $\overline{\nu^2}$ is obtained by integrating $\overline{\nu^2_F}$ over the descending portion of the spectrum. Extrapolating this portion to F = 1 cps and integrating, we obtain the technical line width per second

$$\Delta f_{\text{tech}} = 0{,}4 \text{ cps}.$$

The relative technical line width per second is therefore

$$\Delta f_{\text{tech}}/f_{\text{maser}} = 1{,}7 \cdot 10^{-11}.$$

The natural line width obtained experimentally is very close to the theoretical value 6.6×10^{-4} cps calculated from Troitskiĭ's formulas.[7] The rms phase drift of a maser in time $\tau = 1$ sec, calculated* from Troitskiĭ's formula[8]

$$(\overline{\Delta^2_\tau})^{1/2} = 2\left[\int_0^\infty w_\nu(F)\Omega^{-2}\sin^2\frac{\Omega}{2}\tau dE\right]^{1/2} \quad \text{for } \Omega = 2\pi F$$

(where in our notation $w_\nu(F) = 4\pi^2\overline{\nu^2_F}$) gives ~120° C. By observing the behavior of Lissajous figures when the maser beat frequency was compared with the frequency of a stable audio oscillator this result was confirmed approximately.

In conclusion I wish to thank V. S. Troitskiĭ for a thorough discussion of this work.

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*In our calculation we assumed that in the spectrum $\overline{\nu^2_F} = 1.6 \times 10^{-2}/F$, $\alpha = 0.9$ for $F < 0.5$ cps and $\alpha = 1$ for $F > 0.5$ cps.

[1] V. S. Troitskiĭ and V. V. Khrulev, Radiotekhnika i élektronika (Radio Engineering and Electronics) 1, 832 (1956).

[2] W. M. Gottschalk, Trans. IRE, ED-1, No. 4, 91 (1954).

[3] N. G. Basov and A. P. Petrov, op. cit. [1], 1, 298 (1958).

[4] G. S. Gorelik and G. A. Elkin, op. cit. [1], 1, 28 (1957).

[5] A. N. Malakhov, PTÉ (Instr. and Exptl. Techniques) No. 1, 79 (1958).

[6] A. N. Malakhov, op. cit. [1], 2, 1295 (1957).

[7] V. S. Troitskiĭ, op. cit. [1], 3, 1298 (1958).

[8] V. S. Troitskiĭ, op. cit. [1], 1, 818 (1956).

Translated by I. Emin
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