INVARIANT FORMULATION OF THE THEORY OF THE GRAVITATIONAL WAVE FIELD

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Submitted to JETP editor September 1, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 577-583 (February, 1962)

The Einstein theory of gravitation is formulated in such a way that the fundamental quantity is the fourth-rank gravitational field-strength tensor F_{iklm}, which is linearly connected with the Riemann curvature tensor R_{iklm} and vanishes in a Euclidean space. The equations of gravitation, which enable us to calculate both F_{iklm} and g_{ik} for prescribed sources, are formally the same as the Bianchi identities. The main advantage of the new formulation is the possibility of constructing an invariant theory of weak gravitational waves.

1. INTRODUCTION

It is well known that if there is a gravitational field in the four-dimensional world space the space is noneuclidean and the fourth-rank Riemann tensor R_{iklm} is different from zero. Therefore it is natural to describe the gravitational field by means of the tensor R_{iklm}, and not of the metric tensor g_{ik}, whose components can be different from δ_{ik} even in the absence of a gravitational field.

As the fundamental field quantities of the theory of gravitation we introduce the twenty independent components of the Riemann tensor R_{iklm} and the ten components of the metric tensor g_{ik}, and do not postulate a priori the usual connection between R_{iklm} and g_{ik}.

The thirty quantities R_{iklm} and g_{ik} are connected by ten relations which express the prescribed source tensor T_{ik} in terms of the field quantities:

$$R_{ik} - \tfrac{1}{2} g_{ik} R = T_{ik}, \qquad R_{ik} = T_{ik} - \tfrac{1}{2} g_{ik} T. \tag{1}$$

The field equations for the twenty independent quantities are postulated in a form which is formally the same as the Bianchi identities:

$$R_{iklm;\,n} + R_{ikmn;\,l} + R_{iknl;\,m} = 0. \tag{2}$$

In the system (2) there are twenty-four equations, of which only twenty are independent. Multiplying Eq. (2) by $g^{il}g^{km}$, we get

$$\left(R^l_k - \tfrac{1}{2}\delta^l_k R\right)_{;\,l} = T^l_{k;\,l} = 0, \tag{3}$$

which assures that the covariant divergence of the source tensor T_{ik} is equal to zero.

We can look for the solution of the equations (2) by expressing R_{iklm} in terms of the g_{ik} and their derivatives in the usual way. Then, as is well known, the system (2) is satisfied identically, and the definition (1) becomes the ten Einstein equations for the potentials g_{ik}.

In the scheme described here there is so far nothing new.

An essentially new element is brought in by breaking up the tensor R_{iklm} into a sum of two 10-component tensors M_{iklm} and F_{iklm}, both of which have the symmetry of the Riemann tensor:

$$R_{iklm} = M_{iklm} + F_{iklm}. \tag{4}$$

The fourth-rank matter tensor is expressed in terms of the field-source tensor T_{ik} by the formula

$$M_{iklm} = \tfrac{1}{2}\left[g_{il}T_{km} + g_{km}T_{il} - g_{kl}T_{im} - g_{im}T_{kl}\right] + \tfrac{1}{3}\left(g_{kl}g_{im} - g_{il}g_{km}\right) T \tag{5}$$

and vanishes when there are no sources. According to Eqs. (4) and (5) the fourth-rank gravitational field-strength tensor is given by the formula

$$F_{iklm} = R_{iklm} - \tfrac{1}{2}\left[g_{il}T_{km} + g_{km}T_{il} - g_{kl}T_{im} - g_{im}T_{kl}\right] - \tfrac{1}{3}\left(g_{kl}g_{im} - g_{il}g_{km}\right) T \tag{6}$$

and, as we see, describes a field which can exist in a space without sources.

The equations (2) can be written in an obvious way as inhomogeneous first-order equations for the intensities F_{iklm}:

$$F_{iklm;\,n} + F_{ikmn;\,l} + F_{iknl;\,m} = -\left[M_{iklm;\,n} + M_{ikmn;\,l} + M_{iknl;\,m}\right].$$

Because of the covariant differentiations Eq. (7) involves the gravitational potentials and their derivatives, and therefore these equations are a first-order nonlinear system of equations for the simultaneous determination of the ten intensities F_{iklm} and the ten potentials g_{ik} for given sources T_{ik}.

The proposed scheme of equations for gravitation theory shows a formal resemblance to the equations of electrodynamics for the electromagnetic field intensities F_{ik}. If in analogy with Eq. (1) we regard the first system of Maxwell equations

$$\partial F_{ik}/\partial x^k = s_i \tag{8}$$

as the definition of the sources of the electromagnetic field, then the second system of Maxwell equations

$$\frac{\partial F_{ik}}{\partial x^l} + \frac{\partial F_{kl}}{\partial x^i} + \frac{\partial F_{li}}{\partial x^k} = 0 \tag{9}$$

is the analog of the system (7).

The equations (9) are identically satisfied by the substitution

$$F_{ik} = \frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}. \tag{9.1}$$

If we substitute Eq. (9.1) in Eq. (8), the definition (8) is converted into a system of second-order equations

$$\frac{\partial}{\partial x^k}\left(\frac{\partial A_k}{\partial x^i} - \frac{\partial A_i}{\partial x^k}\right) = s_i \tag{8.1}$$

for the determination of the four potentials A_i for given sources s_i.

The deep difference between gravitation theory and electrodynamics is that besides the intensities F_{iklm} the field equations (7) also contain the potentials g_{ik}, whereas in electrodynamics the equations for the intensities do not contain the potentials A_i.

Of course we might not solve the equations (7), but first determine the potentials g_{ik} from the usual gravitational equations (1) and then calculate the intensities F_{iklm} of the gravitational field by the formula (6). The "Maxwellized" form of the gravitational equations shows its superiority, however, in the theory of a weak gravitational field, where we can legitimately replace covariant differentiation by ordinary differentiation and take $g_{ik} = \delta_{ik}$.

2. THE FOURTH-RANK DUAL RIEMANN TENSOR

Using the symmetry of the Riemann tensor, let us denote its mixed components by R^{ik}_{lm}. By definition we give the name of its dual to the tensor with the components

$$\overset{*}{R}{}^{ik}_{lm} = \tfrac{1}{4} E^{ik\sigma\tau} E_{lm\lambda\mu} R^{\lambda\mu}_{\sigma\tau}, \tag{10}$$

where (cf., e.g., Sec. 83 of [1])

$$E^{ik\sigma\tau} = \varepsilon^{ik\sigma\tau}/\sqrt{-g}, \qquad E_{lm\lambda\mu} = \sqrt{-g}\,\varepsilon_{lm\lambda\mu}. \tag{11}$$

By the definition of the completely antisymmetric fourth-rank tensor $\epsilon^{ik\sigma\tau}$ we have

$$E^{ik\sigma\tau} E_{lm\lambda\mu} = \begin{vmatrix} \delta^i_l & \delta^i_m & \delta^i_\lambda & \delta^i_\mu \\ \delta^k_l & \delta^k_m & \delta^k_\lambda & \delta^k_\mu \\ \delta^\sigma_l & \delta^\sigma_m & \delta^\sigma_\lambda & \delta^\sigma_\mu \\ \delta^\tau_l & \delta^\tau_m & \delta^\tau_\lambda & \delta^\tau_\mu \end{vmatrix}. \tag{12}$$

Expanding the determinant and substituting Eq. (12) in Eq. (10), we get

$$\overset{*}{R}{}^{ik}_{lm} = R^{ik}_{lm} - [\delta^i_l R^k_m + \delta^k_m R^i_l - \delta^k_l R^i_m - \delta^i_m R^k_l] + \tfrac{1}{2}(\delta^i_l \delta^k_m - \delta^k_l \delta^i_m) R, \tag{13}$$

or, going over to the covariant components

$$\overset{*}{R}_{iklm} = R_{iklm} + [g_{kl} R_{im} + g_{im} R_{kl} - g_{il} R_{km} - g_{km} R_{il}] + \tfrac{1}{2}(g_{il} g_{km} - g_{kl} g_{im}) R. \tag{13.1}$$

Contracting with respect to the indices i and l, we get

$$\overset{*}{R}_{km} = -\left\{R_{km} - \tfrac{1}{2} g_{km} R\right\} = -T_{km}. \tag{14}$$

Equation (13) enables us to draw the important conclusion that for a space in which all of the R_{ik}, and consequently also the T_{ik}, are equal to zero the Riemann tensor is equal to its dual. In such an "empty" space, however, there can still exist a field of gravitational radiation free of sources.

In the general case in which there are sources ($T_{ik} \neq 0$) the Riemann tensor can be represented as the sum of a selfdual tensor D_{iklm} and an antidual tensor A_{iklm}:

$$D_{iklm} = \tfrac{1}{2}(R_{iklm} + \overset{*}{R}_{iklm}), \tag{15}$$

$$A_{iklm} = \tfrac{1}{2}(R_{iklm} - \overset{*}{R}_{iklm}). \tag{16}$$

From Eqs. (13) and (16) we have

$$A_{iklm} = \tfrac{1}{2}[g_{il} R_{km} + g_{km} R_{il} - g_{kl} R_{im} - g_{im} R_{kl}) - \tfrac{1}{4}(g_{il} g_{km} - g_{kl} g_{im}) R, \tag{17}$$

$$A^{ik}_{lm} = \tfrac{1}{2}\left[\delta^i_l R^k_m + \delta^k_m R^i_l - \delta^k_l R^i_m - \delta^i_m R^k_l\right] - \tfrac{1}{4}(\delta^i_l \delta^k_m - \delta^k_l \delta^i_m) R. \tag{17.1}$$

Since the fourth-rank tensor A_{iklm} is constructed from the symmetric tensors R_{ik} and g_{ik}, the pseudoscalar

$$\tfrac{1}{8}\varepsilon^{iklm} A_{iklm} = A_{1234} + A_{1342} + A_{1423} \equiv 0 \tag{18}$$

is identically equal to zero.

Let us count the numbers of independent components of the tensors D^{ik}_{lm} and A^{ik}_{lm}. To do this we note that it follows from Eq. (10) that the three components of the tensor R^{ik}_{lm} that have all four indices different are selfdual:

$$\overset{*}{R}{}^{23}_{14} = R^{23}_{14}, \qquad \overset{*}{R}{}^{31}_{24} = R^{31}_{24}, \qquad \overset{*}{R}{}^{12}_{34} = R^{12}_{34},$$

and the remaining eighteen components can be combined into nine pairs $\{R^{ik}_{lm}, \overset{*}{R}{}^{ik}_{lm}\}$. Consequently the tensor A^{ik}_{lm} has $(21-3)/2 = 9$ components and the tensor D^{ik}_{lm} has 12 components. It then follows from the identity

$$R_{1234} + R_{1342} + R_{1423} = 0 \tag{19}$$

and Eq. (18) that the components of the tensor D_{iklm} satisfy the relation

$$D_{1234} + D_{1342} + D_{1423} = 0, \tag{20}$$

which puts one condition on the 12 components of the tensor D_{iklm} and reduces the number of independent components to 11.

3. THE FOURTH-RANK GRAVITATIONAL FIELD TENSOR F_{iklm} AND THE MATTER TENSOR M_{iklm}

Let us consider the fourth-rank tensor

$$\delta^{ik}_{lm} = \delta^i_l\,\delta^k_m - \delta^k_l\delta^i_m, \tag{21}$$

which has the symmetry of the Riemann tensor. It is easy to see that it is selfdual:

$$\overset{*}{\delta}{}^{ik}_{lm} = \tfrac{1}{4}\,\varepsilon^{ik\sigma\tau}\,\varepsilon_{lm\lambda\mu}\,(\delta^\lambda_\sigma\delta^\mu_\tau - \delta^\mu_\sigma\delta^\lambda_\tau) = \tfrac{1}{2}\,\varepsilon^{ik\lambda\mu}\varepsilon_{lm\lambda\mu}$$
$$= \delta^i_l\delta^k_m - \delta^k_l\delta^i_m = \delta^{ik}_{lm}. \tag{22}$$

Its invariant is

$$\delta^{ik}_{ik} = 12. \tag{23}$$

Let us now introduce two new fourth-rank tensors F^{ik}_{lm} and M^{ik}_{lm} by the formulas

$$R^{ik}_{lm} = F^{ik}_{lm} + M^{ik}_{lm}, \tag{24}$$
$$F^{ik}_{lm} = D^{ik}_{lm} - \tfrac{1}{12}(\delta^i_l\delta^k_m - \delta^k_l\delta^i_m)\,R, \tag{25}$$
$$M^{ik}_{lm} = A^{ik}_{lm} + \tfrac{1}{12}(\delta^i_l\delta^k_m - \delta^k_l\delta^i_m)\,R. \tag{26}$$

Being the sum of two selfdual tensors, F^{ik}_{lm} is selfdual. It is reasonable to call it the gravitational field-strength tensor. The tensor M^{ik}_{lm} is the sum of an antidual tensor and a selfdual tensor. We shall call it the matter tensor.

Calculating the invariants, we get:

$$F = 0, \qquad M = -R = T. \tag{27}$$

We now calculate the dual components:

$$\overset{*}{F}{}^{ik}_{lm} = F^{ik}_{lm},$$
$$\overset{*}{M}{}^{ik}_{lm} = -A^{ik}_{lm} + \tfrac{1}{12}(\delta^i_l\delta^k_m - \delta^k_l\delta^i_m)\,R$$
$$= -M^{ik}_{lm} + \tfrac{1}{6}(\delta^i_l\delta^k_m - \delta^k_l\delta^i_m)\,R. \tag{28}$$

By Eqs. (26), (28), (17), and (24) we have

$$M^{ik}_{lm} = \tfrac{1}{2}[\delta^i_lR^k_m + \delta^k_mR^i_l - \delta^k_lR^i_m - \delta^i_mR^k_l]$$
$$- \tfrac{1}{6}(\delta^i_l\delta^k_m - \delta^k_l\delta^i_m)\,R,$$
$$F^{ik}_{lm} = R^{ik}_{lm} - \tfrac{1}{2}[\delta^i_lR^k_m + \delta^k_mR^i_l - \delta^k_lR^i_m - \delta^i_mR^k_l]$$
$$+ \tfrac{1}{6}(\delta^i_l\delta^k_m - \delta^k_l\delta^i_m)\,R. \tag{29}$$

By expressing R_{ik} in terms of the field-source tensor by Eq. (1), we can rewrite Eq. (29) in the form

$$M^{ik}_{lm} = \tfrac{1}{2}[\delta^i_lT^k_m + \delta^k_mT^i_l - \delta^k_lT^i_m - \delta^i_mT^k_l]$$
$$+ \tfrac{1}{3}(\delta^k_l\delta^i_m - \delta^i_l\delta^k_m)\,T,$$
$$F^{ik}_{lm} = R^{ik}_{lm} + \tfrac{1}{2}[\delta^k_lT^i_m + \delta^i_mT^k_l - \delta^i_lT^k_m - \delta^k_mT^i_l]$$
$$- \tfrac{1}{3}(\delta^k_l\delta^i_m - \delta^i_l\delta^k_m)\,T. \tag{30}$$

The fact that the invariant F of the tensor F^{ik}_{lm} is zero imposes one condition on the eleven components of the self-dual tensor and reduces the number of its independent components to ten. According to Eq. (26) the matter tensor M^{ik}_{lm} also has ten independent components.

4. THE FIELD EQUATIONS

The field equations in our theory are the Bianchi equations

$$R^{ik}_{lm;n} + R^{ik}_{mn;l} + R^{ik}_{nl;m} = \tfrac{1}{2}\,\varepsilon^{lmk\rho}R^{ik}_{lm;n} = 0. \tag{31}$$

If we multiply Eq. (31) by $\frac{1}{2}\epsilon_{ik\lambda\mu}$ and take into account the definition (10), we get

$$\tfrac{1}{4}\,\varepsilon_{ik\lambda\mu}\,\varepsilon^{lmn\rho}R^{ik}_{lm} = \overset{*}{R}{}^{n\rho}_{\lambda\mu;n} = 0. \tag{31.1}$$

Consequently, the field equations can be written in the two equivalent forms

$$(F^{ik}_{lm;n} + F^{ik}_{mn;l} + F^{ik}_{nl;m}) + (M^{ik}_{lm;n} + M^{ik}_{mn;l} + M^{ik}_{nl;m}) = 0 \tag{32}$$

or

$$\overset{*}{F}{}^{ik}_{lm;k} + \overset{*}{M}{}^{ik}_{lm;k} = 0. \tag{32.1}$$

From these equations one can determine both the ten intensities F^{ik}_{lm} and the ten potentials g_{ik} for given sources T_{ik}. Taking into account Eqs. (28), (30), and (3), we easily get from Eq. (32.1)

$$F^{ik}_{lm;k} = \tfrac{1}{2}(T^i_{m;l} - T^i_{m;l}) - \tfrac{1}{2}(\delta^i_mT_{,l} - \delta^i_lT_{,m}). \tag{33}$$

We shall show that from the 24 Bianchi equations (33) there follow four identities, so that there are only 20 independent equations. To do this we multiply Eq. (33) by δ^i_m and use the fact that $T^l_{i;l} = 0$. We get

$$F^{ik}_{li;k} = \tfrac{1}{2}(T_{,l} - \tfrac{4}{3}\,T_{,l}) + \tfrac{1}{6}\,T_{,l} \equiv 0 \tag{34}$$

and consequently only 20 equations are independent.

5. THE WEAK GRAVITATIONAL FIELD

In the limiting case of a weak gravitational field it is legitimate to replace covariant differentiation by ordinary differentiation and set $g_{ik} = \delta_{ik}$. The field equations (33) take the form

$$\frac{\partial F_{lmik}}{\partial x^k} = \frac{1}{2}\left(\frac{\partial T_{im}}{\partial x^l} - \frac{\partial T_{il}}{\partial x^m}\right) - \frac{1}{6}\left(\delta_{im}\frac{\partial T}{\partial x^l} - \delta_{il}\frac{\partial T}{\partial x^m}\right). \tag{35}$$

If we introduce three-dimensional notation for the components of the tensor F_{iklm},

$$H_{\alpha\beta} = \begin{pmatrix} F_{2323} & F_{2331} & F_{2312} \\ F_{3123} & F_{3131} & F_{3112} \\ F_{1223} & F_{1231} & F_{1212} \end{pmatrix} = \begin{pmatrix} F_{1414} & F_{1424} & F_{1434} \\ F_{2414} & F_{2424} & F_{2434} \\ F_{3414} & F_{3424} & F_{3434} \end{pmatrix}, \quad (36)$$

$$-iE_{\alpha\beta} = \begin{pmatrix} F_{2314} & F_{2324} & F_{2334} \\ F_{3114} & F_{3124} & F_{3134} \\ F_{1214} & F_{1224} & F_{1234} \end{pmatrix} = \begin{pmatrix} F_{1423} & F_{1431} & F_{1412} \\ F_{2423} & F_{2431} & F_{2412} \\ F_{3423} & F_{3431} & F_{3412} \end{pmatrix}; \quad (37.1)$$

$$\mathrm{Sp} H_{\alpha\beta} = \mathrm{Sp}\, E_{\alpha\beta} = 0, \quad (37.2)$$

the system of Bianchi equations can then be written in the form

$$\varepsilon_{\beta kl} \frac{\partial H_{\alpha l}}{\partial x^k} - \frac{\partial E_{\alpha\beta}}{\partial t} = \frac{1}{2} \varepsilon_{\alpha kl} \frac{\partial T_{\beta l}}{\partial x^k} + \frac{1}{6} \varepsilon_{\alpha\beta k} \frac{\partial T}{\partial x^k},$$

$$\varepsilon_{\beta kl} \frac{\partial E_{\alpha l}}{\partial x^k} + \frac{\partial H_{\alpha\beta}}{\partial t} = \frac{i}{2} \left(\frac{\partial T_{\beta 4}}{\partial x^\alpha} - \frac{\partial T_{\alpha\beta}}{\partial x^4} \right) + \frac{i}{6} \delta_{\alpha\beta} \frac{\partial T}{\partial x^4}; \quad (38)$$

$$\frac{\partial E_{\alpha\beta}}{\partial x^\beta} = \frac{i}{2} \varepsilon_{\alpha\beta\gamma} \frac{\partial T_{4\beta}}{\partial x^\gamma},$$

$$\frac{\partial H_{\alpha\beta}}{\partial x^\beta} = \frac{1}{2} \left(\frac{\partial T_{4\alpha}}{\partial x^4} - \frac{\partial T_{44}}{\partial x^\alpha} \right) + \frac{1}{6} \frac{\partial T}{\partial x^\alpha}$$

$$= -\frac{1}{2} \left(\frac{\partial T_{\alpha\beta}}{\partial x^\beta} + \frac{\partial T_{44}}{\partial x^\alpha} \right) + \frac{1}{6} \frac{\partial T}{\partial x^\alpha}, \quad (38.1)$$

where the indices in these last equations take values from one to three.

We have obtained eighteen field equations (38) and six supplementary conditions (38.1) which do not involve differentiation with respect to the time. In the derivation we have used the fact that [see Eq. (3)]

$$\frac{\partial T_{\alpha\beta}}{\partial x^\beta} + \frac{\partial T_{\alpha 4}}{\partial x^4} = 0, \qquad \frac{\partial T_{4\beta}}{\partial x^\beta} + \frac{\partial T_{44}}{\partial x^4} = 0. \quad (39)$$

We shall show that only 10 of the 18 equations of the system (38) are independent. To do this we multiply each of the equations (38) sucessively by $\delta_{\alpha\beta}$ and $\epsilon_{\alpha\beta\gamma}$. Taking into account the symmetry of $H_{\alpha\beta}$ and $E_{\alpha\beta}$, and also Eqs. (37.2) and (39), we get as the result of these operations eight identities.

The equations (38) and (38.1) describe the propagation of invariant gravitational waves in space filled with matter.

6. THE PLANE GRAVITATIONAL WAVE IN VACUUM

Let us consider a wave propagated in vacuum in the x^1 direction. We have from Eqs. (38) and (38.1)

$$\frac{\partial H_{\alpha 1}}{\partial x^1} = 0, \quad \frac{\partial H_{\alpha 1}}{\partial t} = 0, \quad \frac{\partial E_{\alpha 1}}{\partial x^1} = 0, \quad \frac{\partial E_{\alpha 1}}{\partial t} = 0,$$

$$\frac{\partial H_{\alpha 3}}{\partial x^1} + \frac{\partial E_{\alpha 2}}{\partial t} = 0, \quad \frac{\partial H_{\alpha 2}}{\partial x^1} - \frac{\partial E_{\alpha 3}}{\partial t} = 0,$$

$$\frac{\partial E_{\alpha 3}}{\partial x^1} - \frac{\partial H_{\alpha 2}}{\partial t} = 0, \quad \frac{\partial E_{\alpha 2}}{\partial x^1} + \frac{\partial H_{\alpha 3}}{\partial t} = 0, \quad \alpha = 1,2,3. \quad (40)$$

It follows from these equations that the tensors $H_{\alpha\beta}$ and $E_{\alpha\beta}$ satisfy the wave equation.

We see that a gravitational wave propagated in the x^1 direction is characterized by the tensors

$$H_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & H_{22} & H_{23} \\ 0 & H_{32} & H_{33} \end{pmatrix}, \qquad E_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & E_{22} & E_{23} \\ 0 & E_{32} & E_{33} \end{pmatrix} \quad (41)$$

and in this sense is transverse, as in the usual theory of gravitational waves.

For a harmonic wave $\sim \exp\{i(kx - \omega t)\}$ the equations (40) give

$$H_{22} = -E_{23} = \alpha, \qquad H_{23} = E_{22} = \beta,$$

and consequently the tensors $H_{\alpha\beta}$ and $E_{\alpha\beta}$ are of the forms

$$H_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & -\alpha \end{pmatrix}, \qquad E_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \beta & -\alpha \\ 0 & -\alpha & -\beta \end{pmatrix}. \quad (42)$$

Setting $\{\alpha \neq 0,\ \beta = 0\}$ and $\{\alpha = 0,\ \beta \neq 0\}$, we get two possible states of polarization of gravitational waves. We recall that in electrodynamics we have for the vectors **H**, **E** the forms $\mathbf{H} = (0, \alpha, \beta)$, $\mathbf{E} = (0, \beta, -\alpha)$.

We see that the proposed theory of weak gravitational waves is very similar to the electromagnetic theory of Maxwell. Just as an electromagnetic wave is determined by two three-component vectors **H** and **E**, a gravitational wave is determined by two five-component tensors $H_{\alpha\beta}$ and $E_{\alpha\beta}$. This is obviously connected with the fact that the photon has spin 1 and the graviton has spin 2.

[1] L. Landau and E. Lifshitz, Teoriya polya (Field Theory), Fizmatgiz, 1960.

Translated by W. H. Furry
87