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EFFECT OF RADIATION ON ELECTRON RELAXATION AND PLASMA ELECTRICAL CONDUCTIVITY IN A STRONG MAGNETIC FIELD

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The effect of radiation and absorption of electromagnetic waves by electrons in a plasma in a strong magnetic field on electron relaxation and plasma electrical conductivity is investigated. In the nonrelativistic case these radiative processes provide a relaxation mechanism for the transverse electron momentum (with respect to the magnetic field). The radiation relaxation time is determined by the ratio of mean electron energy to mean radiation intensity in the magnetic field. If it is smaller than the mean time between Coulomb collisions, the radiation relaxation time determines the time required to establish the equilibrium electron distribution with respect to the absolute value of the transverse momentum. Radiation effects can also be important in plasma transport processes. In particular, the transverse plasma conductivity depends on radiative effects and Coulomb collisions whereas the longitudinal conductivity is determined exclusively by Coulomb collisions.

1. INTRODUCTION

IN analyzing transport phenomena and the mechanisms by which thermal equilibrium is established in a plasma one usually starts with a kinetic equation in which only Coulomb collisions are considered. The Coulomb relaxation time increases as the cube of the electron thermal velocity and can be very large, especially at high temperatures. For this reason it is of interest to study other possible mechanisms by which thermal equilibrium can be established in a plasma.

In the present work we show that the radiation and absorption of electromagnetic waves by electrons in a plasma in a strong magnetic field can have a profound effect on the establishment of thermal equilibrium of the electrons.* Specifically, this mechanism means that an equilibrium state can be reached in the absolute value of the transverse (relative to the direction of the magnetic field **H**) electron momentum at nonrelativistic temperatures ($T \ll m_0c^2$, m_0 is the electron mass) and in the transverse and longitudinal electron momenta at relativistic temperatures ($T \gtrsim m_0c^2$).

Radiation effects do not, however, affect relaxation in the angular variable in the plane defined by the transverse electron momentum. Also, they have no effect on relaxation of the longitudinal momentum in the nonrelativistic case and on the angle between the momentum and the magnetic field in the extreme nonrelativistic case. Relaxation in this variable is achieved via Coulomb collisions.

The radiation relaxation time is given roughly by the ratio of the mean electron energy to the mean intensity of the electron radiation in the magnetic field. If it is smaller than the mean time between Coulomb collisions the radiation relaxation time determines the relaxation in the appropriate variable. In this case the electron gas relaxes by a two-step process: the first step is a rapid relaxation in the variable that is affected by radiation; this is followed by the slow approach of the distribution to a Maxwellian, in a time determined by the Coulomb collisions.

The ratio of the radiation relaxation time to the Coulomb relaxation time is approximately unity when $H = 2 \times 10^5$ G, $T = 10^{-2}m_0c^2$ and $\mathfrak{N} = 10^{13}\,\text{cm}^{-3}$ where $\mathfrak{N}$ is the electron density. This ratio diminishes as H and T increase and as $\mathfrak{N}$ decreases.

Radiation relaxation can affect the electrical conductivity of the plasma; the transverse conductivity is determined by Coulomb collisions and radiation effects but the longitudinal component is determined by the Coulomb collisions exclusively. For this reason the electrical conductivity of a plasma in a strong field can be highly anisotropic.

*The effect of photon radiation on the electron distribution function has been considered by Trubnikov and Bazhanova[1] and by Kudryavtsev[2]. These authors, however, have not examined relaxation of the electron and photon gases.

In addition to the electron relaxation process there is a photon relaxation process. The distinguishing feature of the latter is the rapid establishment of a quasi-equilibrium photon distribution that depends on the instantaneous nonequilibrium electron distribution. This quasi-equilibrium distribution, whose establishment is determined by radiation effects and, possibly, by resonance Thomson scattering in the magnetic field, slowly approaches an equilibrium distribution (in the time required for total electron relaxation) near the emission lines; this is a Rayleigh-Jeans distribution.

2. RADIATION COLLISION INTEGRALS

1. We assume that the electron energy is appreciably greater than $\hbar\omega_H$, where $\omega_H = eH/m_0c$. Although we do not quantize the electron motion it is convenient to use a quantum mechanical description, characterizing the electron state in the magnetic field **H** by a quantum number n, that determines the electron motion in the plane perpendicular to **H**, and by p_z, the projection of the electron momentum along the magnetic field (we shall be dealing only with a spatially uniform plasma so that no quantum numbers are needed to describe the position of the center of the electron orbit).

In radiation (absorption) of a photon in the magnetic field the electron goes from a state $\kappa \equiv n, p_z$ to a state $\kappa'' \equiv n-s$, $p_z - \hbar k_z$ ($\kappa' \equiv n+s$, $p_z + \hbar k_z$, where $\hbar k_z$ is the projection of the photon momentum $\hbar\mathbf{k}$ in the direction of **H**. Denoting the probability for photon absorption in the wave vector interval **k**, **k**+d**k** with an electron transition from the state κ to the state κ' by $W_s(n, p_z; \mathbf{k}) \times \delta(\epsilon' - \epsilon + \hbar\omega) V (2\pi)^{-3} d\mathbf{k}$ we can write the change in the electron distribution function f_κ per unit time brought about by the radiation and absorption of photons:

$$\dot{f}_\kappa^{(r)} = \sum_{s=1}^{\infty} \int d\mathbf{k} \{W_s(n, p_z; \mathbf{k}) \, \delta(\varepsilon' - \varepsilon - \hbar\omega) \, [f_{\kappa'} (1 + N_\mathbf{k}) - f_\kappa N_\mathbf{k}] + W_s(n - s, \; p_z - \hbar k_z; \mathbf{k}) \, \delta(\varepsilon'' - \varepsilon + \hbar\omega) \times [f_{\kappa''} N_\mathbf{k} - f_\kappa (1 + N_\mathbf{k})]\} V (2\pi)^{-3}, \quad (1)$$

where $N_\mathbf{k}$ is the number of photons characterized by wave vector **k** and frequency ω (V is the normalization volume).

Since it is primarily soft photons that are radiated, and the energy of these photons is small compared with the electron energy, the quantity $\dot{f}_\kappa^{(r)}$ can be transformed by the Fokker-Planck technique. We write $\dot{f}_\kappa^{(r)}$ in the form

$$\dot{f}_\kappa^{(r)} = \sum_{s=1}^{\infty} \int d\mathbf{k} \{\Phi_s (n, p_z; \mathbf{k}) \, [\delta (\varepsilon' - \varepsilon - \hbar\omega) - \delta (\varepsilon'' - \varepsilon + \hbar\omega)] + [\Phi_s (n, p_z; \mathbf{k}) - \Phi_s (n - s, p_z - \hbar k_z; \mathbf{k})] \, \delta (\varepsilon'' - \varepsilon + \hbar\omega)\} V (2\pi)^{-3}, \quad (1')$$

$$\Phi_s (n, p_z; \mathbf{k}) = W_s (n, p_z; \mathbf{k}) \, [f_{\kappa'} (1 + N_\mathbf{k}) - f_\kappa N_\mathbf{k}] \quad (1'')$$

and expand the differences in the square brackets in powers of $\hbar$, keeping only the first terms. As a result the expression in the curly brackets in (1′) assumes the form

$$\frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} \left\{\frac{\hbar\omega}{c}\left(\frac{\varepsilon}{c} - p_z \cos\vartheta\right) \varphi_s (\mathbf{p}, \mathbf{k}) \, \delta (\varepsilon' - \varepsilon - \hbar\omega)\right\} + \frac{\partial}{\partial p_z} \{\hbar k_z \varphi_s (\mathbf{p}, \mathbf{k}) \, \delta (\varepsilon' - \varepsilon - \hbar\omega)\},$$

where $\varphi_s(\mathbf{p}, \mathbf{k})$ is the classical limit of $\Phi_s(n, p_z; \mathbf{k})$ corresponding to substitution of $n \to p_\perp^2 / 2\hbar eH$ ($\hbar \to 0$); $p_\perp$ is the transverse electron momentum with respect to the magnetic field and ϑ is the angle between **k** and **H**. Substitution of this expression in (1′) yields

$$\dot{f}_\mathbf{p}^{(r)} = \frac{1}{p_\perp} \frac{\partial}{\partial p_\perp} (p_\perp j_\perp^{(r)}) + \frac{\partial}{\partial p_z} j_z^{(r)}; \quad (2)$$

$$j_\perp^{(r)} = \int \Phi (\mathbf{p}, \mathbf{k}) \frac{\hbar\omega}{c p_\perp} \left(\frac{\varepsilon}{c} - p_z \cos\vartheta\right) \frac{d\mathbf{k}}{(2\pi)^3},$$

$$j_z^{(r)} = \frac{1}{c} \int \Phi (\mathbf{p}, \mathbf{k}) \hbar\omega \cos\vartheta \frac{d\mathbf{k}}{(2\pi)^3},$$

$$\Phi (\mathbf{p}, \mathbf{k}) = \sum_{s=1}^{\infty} \varphi_s (\mathbf{p}, \mathbf{k}) \, \delta (\varepsilon' - \varepsilon - \hbar\omega) V. \quad (2')$$

In accordance with (1″) Φ can be written in the form

$$\Phi (\mathbf{p}, \mathbf{k}) = S (\mathbf{p}, \mathbf{k}) \{f_\mathbf{p} + N_\mathbf{k} (f_{\mathbf{p}'} - f_\mathbf{p})\} / \hbar\omega,$$

where the function $S(\mathbf{p}, \mathbf{k})$ gives the classical radiation intensity $dJ = S(\mathbf{p}, \mathbf{k}) d\mathbf{k} (2\pi)^{-3}$ and the difference $f_{\mathbf{p}'} - f_\mathbf{p}$ is

$$f_{\mathbf{p}'} - f_\mathbf{p} = \frac{\hbar\omega}{c} \left\{\frac{\partial f_\mathbf{p}}{\partial p_z} \cos\vartheta + \frac{1}{p_\perp} \left(\frac{\varepsilon}{c} - p_z \cos\vartheta\right) \frac{\partial f_\mathbf{p}}{\partial p_\perp}\right\}. \quad (3)$$

In what follows we assume that the electron radiates as though it were moving in vacuum. This means that the dielectric permittivity of the plasma can be taken as approximately unity. This assumption holds in the nonrelativistic case if the following inequality is satisfied:

$$(\omega_H / \Omega)^2 \, (T / m_0 c^2)^{1/2} \gg 1, \qquad \Omega = \sqrt{4\pi \mathfrak{N} e^2 / m_0}. \quad (4)$$

Under these conditions the frequencies associated with the photons radiated by the electron are

$$\omega_s = \frac{s\omega_H}{\varepsilon / m_0 c^2 - (p_z / m_0 c) \cos\vartheta}, \qquad s = 1, 2, \ldots \quad (5)$$

and $S(\mathbf{p}, \mathbf{k})$ is given by*

*ctg = cot

$$S(\mathbf{p},\mathbf{k})=\sum_{s=1}^{\infty} w_s(\mathbf{p},\vartheta)\,\delta(\omega-\omega_s)\,\hbar\omega;$$

$$w_s(\mathbf{p},\vartheta)=\frac{\pi e^2c^2}{\hbar\omega_s(1-v_z\cos\vartheta/c)}\left\{\left(1-\frac{v_z}{c\cos\vartheta}\right)^2\right.$$

$$\left.\times\,\mathrm{ctg}^2\vartheta J_s^2\left(\frac{sv_\perp\sin\vartheta}{c-v_z\cos\vartheta}\right)+\left(\frac{v_\perp}{c}\right)^2J_s'^2\left(\frac{sv_\perp\sin\vartheta}{c-v_z\cos\vartheta}\right)\right\} \quad (6)$$

(v_z and $v_\perp$ are the longitudinal and transverse components of the electron velocity $\mathbf{v}$).

As expected, $f_\mathbf{p}^{(r)}$, which we will call the electron radiation collision integral, has the form of a divergence of some vector $\mathbf{j}^{(r)}$ in the momentum representation. This vector may be called the electron radiation flux. Using (2′) we can write the components of the radiation flux in the following general form:

$$j_\perp^{(r)}=\frac{\varepsilon}{cp_\perp}\left(D_0-\frac{cp_z}{\varepsilon}D_1\right)f_\mathbf{p}+\frac{\varepsilon^2}{c^3p_\perp^2}\left(G_0-\frac{cp_z}{\varepsilon}G_1\right)\frac{\partial f_\mathbf{p}}{\partial p_\perp}$$

$$+\frac{\varepsilon}{c^2p_\perp}\left(G_1-\frac{cp_z}{\varepsilon}G_2\right)\left(\frac{\partial f_\mathbf{p}}{\partial p_z}-\frac{p_z}{p_\perp}\frac{\partial f_\mathbf{p}}{\partial p_\perp}\right),$$

$$j_z^{(r)}=D_1f_\mathbf{p}+\frac{1}{c}G_2\frac{\partial f_\mathbf{p}}{\partial p_z}+\frac{\varepsilon}{c^2p_\perp}\left(G_1-\frac{cp_z}{\varepsilon}G_2\right)\frac{\partial f_\mathbf{p}}{\partial p_\perp}, \quad (7)$$

where the functions D_n and G_n are given by

$$D_n(\mathbf{p})=\frac{1}{c}\int S(\mathbf{p},\mathbf{k})\cos^n\vartheta\frac{d\mathbf{k}}{(2\pi)^3},$$

$$G_n(\mathbf{p})=\frac{1}{c}\int\hbar\omega N_\mathbf{k}S(\mathbf{p},\mathbf{k})\cos^n\vartheta\frac{d\mathbf{k}}{(2\pi)^3}. \quad (8)$$

We note that $G_n = TD_n$ for an equilibrium photon distribution $N_\mathbf{k}^0 = T/\hbar\omega$.

2. The change in the photon distribution function $N_\mathbf{k}$ brought about by radiation processes is given by

$$\dot N_\mathbf{k}^{(r)}=\int\frac{S(\mathbf{p},\mathbf{k})}{\hbar\omega}\{(N_\mathbf{k}+1)f_{\mathbf{p}'}-N_\mathbf{k}f_\mathbf{p}\}\frac{d\mathbf{p}}{(2\pi\hbar)^3};$$

$$p_z'=p_z+\hbar k_z,\quad p_\perp'=p_\perp+(\hbar\omega/cp_\perp)(\varepsilon/c-p_z\cos\vartheta).$$

The quantity $\dot N_\mathbf{k}^{(r)}$, which we will call the photon radiation collision integral, can be written in the following form in the quasi-classical approximation:

$$\dot N_\mathbf{k}^{(r)}=-N_\mathbf{k}/\tau_p^{(r)}(\mathbf{k})+\nu(\mathbf{k});$$

$$\frac{1}{\tau_p^{(r)}(\mathbf{k})}=-\int\frac{S(\mathbf{p},\mathbf{k})}{\hbar\omega}(f_{\mathbf{p}'}-f_\mathbf{p})\frac{d\mathbf{p}}{(2\pi\hbar)^3},$$

$$\nu(\mathbf{k})=\int\frac{S(\mathbf{p},\mathbf{k})}{\hbar\omega}f_\mathbf{p}\frac{d\mathbf{p}}{(2\pi\hbar)^3} \quad (9)$$

where $f_{\mathbf{p}'}-f_\mathbf{p}$ is given by (3).

3. PHOTON PATH LENGTH FOR RADIATION PROCESSES

1. The quantity $l_\mathbf{p}^{(r)} = c\tau_\mathbf{p}^{(r)}(\mathbf{k})$ represents the mean path length for a photon $(\mathbf{k},\omega)$ with respect to radiation processes. This quantity must be appreciably greater than the photon wavelength if the notion of a photon is to be used at all, i.e., $\tau_\mathbf{p}^{(r)}(\mathbf{k})\,\omega \gg 1$. The path length $l_\mathbf{p}^{(r)}(\mathbf{k})$ can be easily computed in the nonrelativistic $T \ll m_0c^2$ and the extreme relativistic $T \gg m_0c^2$ cases if equilibrium electron distributions are assumed.

In the first case the radiation spectrum is essentially discrete and the quantity $\tau_\mathbf{p}^{(r)}(\mathbf{k})$ can be computed under the assumption that $\omega \sim s\omega_H$, $s = 1, 2, \ldots$. Using (9) and assuming that $s(T/m_0c^2)^{1/2} \ll 1$, $(s\omega_H/\omega-1)^2 \lesssim \cos^2\vartheta$ we have

$$\left(\frac{1}{\tau_p^{(r)}(\mathbf{k})}\right)_{\omega\sim s\omega_H}=\frac{1}{4\sqrt{2\pi}}\frac{\Omega^2}{s\omega_H}\left(\frac{m_0c^2}{T}\right)^{3/2}\frac{1+\cos^2\vartheta}{|\cos\vartheta|\sin^2\vartheta}$$

$$\times\frac{1}{2s!!}\left(s^2\frac{T}{m_0c^2}\sin^2\vartheta\right)^s\exp\left\{-\frac{m_0c^2}{2T}\frac{(\omega-s\omega_H)^2}{\omega^2\cos^2\vartheta}\right\}. \quad (10)$$

This quantity decreases with harmonic number s as $(T/m_0c^2)^s$.

When $T \ll m_0c^2$, the smallest free paths are those for photons whose frequencies lie in an interval of order $\omega_H(T/m_0c^2)^{1/2}$ about the resonance frequency $\omega = \omega_H$.

In the nonrelativistic case the strongest electron interaction is with these photons. The photon mean free time is of order

$$(\tau_p^{(r)}(\mathbf{k}))_{\omega\sim\omega_H}=(\omega_H/\Omega^2)\,(T/m_0c^2)^{1/2},\qquad T\ll mc^2. \quad (11)$$

The path length for these photons is $l_\mathbf{p}^{(r)} \approx 2$ cm with $H = 10^5$ G, $\mathfrak{N} = 10^{13}\ \mathrm{cm}^{-3}$ and $T = 10^{-2}m_0c^2$. We may note that $\tau_\mathbf{p}^{(r)}(\mathbf{k})$ is the same as the damping time for free electromagnetic oscillations in a rarefied plasma (Stepanov [3]).

2. Because of the Doppler effect, in the relativistic case the spectrum becomes essentially continuous. Hence, when $T \gtrsim m_0c^2$ the quantity $\tau_\mathbf{p}^{(r)}(\mathbf{k})$ becomes a smooth function of frequency. In the extreme relativistic case the chief radiation effect is due to harmonics for which $s \gg 1$. In this case we can replace the summation in (6) by integration over s and use the asymptotic expression [4] of the Bessel function $J_s(sz)$ for $s \gg 1$, $1-z \ll 1$, thereby obtaining an expression for $S(\mathbf{p},\mathbf{k})$ that applies when $\epsilon \gg m_0c^2$ and $\vartheta \gg m_0c^2/\epsilon$:

$$S(\mathbf{p},\mathbf{k})=2^{2/3}\frac{e^2m_0c^2}{\omega\varepsilon\sin\vartheta}y^{1/3}\left\{\left(\frac{\varepsilon}{m_0c^2}(\theta-\vartheta)\right)^2\Phi^2\left[\left(\frac{y}{2}\right)^{2/3}\right.\right.$$

$$\left.\times\left(1+\left(\frac{\varepsilon}{m_0c^2}(\theta-\vartheta)\right)^2\right)\right]$$

$$\left.+\left(\frac{2}{y}\right)^{2/3}\Phi'^2\left[\left(\frac{y}{2}\right)^{2/3}\left\{1+\left(\frac{\varepsilon}{m_0c^2}(\theta-\vartheta)\right)^2\right\}\right]\right\},$$

$$y=\frac{1}{\sin\vartheta}\frac{\omega}{\omega_H}\left(\frac{m_0c^2}{\varepsilon}\right)^2, \quad (12)$$

where $\Phi(z)$ is the Airy function. This expression shows that when $T \gg m_0c^2$ most of the photons are radiated at frequencies $\omega \sim \omega_H(\epsilon/m_0c^2)^2$ in a range of angles ϑ of order $(m_0c^2/\epsilon)^2$ about the angle θ.

Substituting (12) in (9) we have

$$\frac{1}{\tau_p^{(r)}(\mathbf{k})}=\frac{3}{32\pi\sqrt{2}}\frac{\Omega^2}{\omega_H\sqrt{\sin\vartheta}}\sqrt{\frac{\omega_H}{\omega}}\left(\frac{m_0c^2}{T}\right)^4 \times F\left(\frac{m_0c^2}{T}\sqrt{\frac{\omega}{\omega_H\sin\vartheta}}\right); \qquad (13)$$

$$F(x)=-\sqrt{\frac{\pi}{2}}\int_0^\infty \exp(-xz^{-3/4})z^{-11/4}U(z)\,dz,$$

$$U(z)=2\Phi'(z)+z\int_z^\infty \Phi(t)\,dt. \qquad (13')$$

The order of magnitude of the time $\tau_p^{(r)}(\mathbf{k})$ is given by the following expression when $\omega\sim\omega_H(T/m_0c^2)^2$ and $T\gg m_0c^2$:

$$\tau_p^{(r)}(\mathbf{k})\sim\frac{\omega_H}{\Omega^2}\left(\frac{T}{m_0c^2}\right)^2\frac{1}{\sin\vartheta},\qquad \vartheta\gg\frac{m_0c^2}{T}. \qquad (14)$$

Using (11) and (14) we can show that the condition $\omega\tau_p^{(r)}(\mathbf{k})\gg 1$ is well satisfied over a large range of $\mathfrak{N}$, ω_H and T; when $T\ll m_0c^2$ this condition is the same as (4).

4. ELECTRON RADIATION FLUX

1. The expressions for the components of the electron radiation flux contain the functions $D_n(\mathbf{p})$ and $G_n(\mathbf{p})$, for which simple asymptotic expressions can be obtained in the nonrelativistic and extreme relativistic cases. We first consider $D_n(\mathbf{p})$. Using the relations

$$\sum_{s=1}^{\infty}s^2J_s^2(sz)=\frac{z^2(4+z^2)}{16(1-z^2)^{7/2}},$$

$$\sum_{s=1}^{\infty}s^2J_s'^2(sz)=\frac{4+3z^2}{16(1-z^2)^{5/2}},\qquad z<1,$$

after integration over ω and summation over s we have

$$D_n(\mathbf{p})=\frac{e^2(eH)^2}{64\pi\varepsilon^2c^2}\int_{-1}^{1}x^nv_\perp^2\left[\left(\frac{v_z}{v}-\frac{v}{c}x\right)^2+\left(1-\frac{v^2}{c^2}\right)\left(1-\frac{v_\perp^2}{v^2}\right)\right]^{-7/2}\left\{\left(x-\frac{v_z}{c}\right)^2\left[4\left(1-\frac{v_z}{c}x\right)^2+\frac{v_\perp^2}{c^2}(1-x)^2\right]+\left[\left(1-x\frac{v_z}{c}\right)^2-\frac{v_\perp^2}{c^2}(1-x^2)\right]\cdot\times\left[4\left(1-x\frac{v_z}{c}\right)^2+3\frac{v_\perp^2}{c^2}(1-x^2)\right]\right\}dx. \qquad (15)$$

The following expressions are obtained in the nonrelativistic case:

$$D_n(\mathbf{p})=\frac{e^2(eH)^2}{4\pi(m_0c^2)^2}\left(\frac{v_\perp}{c}\right)^2\begin{cases}\dfrac{n+2}{(n+1)(n+3)}, & n=0,2,\ldots\\[2mm] \dfrac{v_z}{c}\dfrac{3n+7}{(n+2)(n+4)}, & n=1,3,\ldots\end{cases} \qquad (16)$$

In the extreme relativistic case we have

$$D_n(\mathbf{p})=D_0(\mathbf{p})\cos^2\theta,\qquad D_0(\mathbf{p})=\frac{e^2\omega_H^2}{6\pi c^2}\left(\frac{\varepsilon}{m_0c^2}\right)^2\sin^2\theta, \qquad (17)$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{H}$. We see that when $v\sim c$ the radiation is primarily at an angle θ with respect to the magnetic field.

2. In the nonrelativistic case the G_n are given by

$$G_n(\mathbf{p})=\frac{1}{c}\int(\hbar\omega_1)^2w_1(\mathbf{p},\vartheta)\cos^n\vartheta N_{\mathbf{k}}\delta(\omega-\omega_1)\frac{d\mathbf{k}}{(2\pi)^3},$$

$$w_1(\mathbf{p},\vartheta)=\frac{\pi e^2c^2}{4\hbar\omega_1}\left(\frac{v_\perp}{c}\right)^2(1+\cos^2\vartheta),$$

$$\omega_1=\omega_H\left(1-\frac{v_z}{c}\cos\vartheta\right)^{-1}.$$

(In deriving this expression we have kept only the s = 1 term in $S(\mathbf{p},\mathbf{k})$, in which we have limited ourselves to the first term in the expansion in 1/c; wherever possible we have also replaced ω_1 by ω_H. We may note that this substitution can not be made in the expressions for $N_{\mathbf{k}}$ because $\tau_p^{(r)}(\mathbf{k})$ varies rapidly near $\omega=\omega_H$.) Carrying out the integration over ω in the expression for G_n we have

$$G_n(\mathbf{p})=\frac{e^2\hbar\omega_H^3}{32\pi^2c^4}v_\perp^2\int(1+\cos^2\vartheta)\cos^n\vartheta\,N(p_z,\vartheta)\,do_{\mathbf{k}}, \qquad (18)$$

where $N(p_z,\vartheta)$ is the value of $N_{\mathbf{k}}$ at $\omega=\omega_1$.

The function $S(\mathbf{p},\mathbf{k})$ has a sharp maximum at $\vartheta=\theta$ in the extreme relativistic case. Hence $G_n(\mathbf{p})$ can be written in the form

$$G_n(\mathbf{p})=\frac{\hbar}{(2\pi c)^4}\cos^n\theta\int\omega^3N(\omega,\theta,\varphi)\,d\omega\,d\varphi\int_0^\pi S(\mathbf{p},\mathbf{k})\,do_{\mathbf{k}}$$

(φ is the azimuth of the vector $\mathbf{k}$). Using (12) and (13), we have

$$\int S(\mathbf{p},\mathbf{k})\,do_{\mathbf{k}}=2\pi^{3/2}\left(\frac{\omega_H}{\omega}\right)^{2/3}\frac{e^2}{\omega}\sin^{-1/3}\theta\left(\frac{m_0c^2}{\varepsilon}\right)^{2/3}U(y^{2/3}).$$

Finally,

$$G_n(\mathbf{p})=G_0(\mathbf{p})\cos^n\theta,$$

$$G_0(\mathbf{p})=\frac{e^2\hbar}{8\pi^{5/2}c^4}\left(\omega_H\frac{m_0c^2}{\varepsilon}\right)^{2/3}\times\sin^{-1/3}\theta\int_0^\infty\omega^{4/3}U(y^{2/3})\,d\omega\int_0^{2\pi}N(\omega,\theta,\varphi)\,d\varphi. \qquad (19)$$

3. We now simplify the expressions for the components of the electron radiation flux. It is evident that in the nonrelativistic case $j_z^{(r)}$ contains an additional factor of 1/c compared with $j_\perp^{(r)}$. Hence we need only calculate $j_\perp^{(r)}$. Keeping the principal terms in 1/c in (7) we have ($\epsilon_\perp=p_\perp^2/2m_0$)

$$j_\perp^{(r)}\approx\frac{m_0c}{p_\perp}\left\{D_0f_{\mathbf{p}}+G_0\frac{\partial f_{\mathbf{p}}}{\partial\varepsilon_\perp}\right\},\qquad j_z^{(r)}\approx 0,$$

$$\dot{f}_{\mathbf{p}}^{(r)}=\frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}(p_\perp j_\perp^{(r)}). \qquad (20)$$

Then, using (18) and (20) we can write $j_\perp^{(r)}$ in the form

$$j_\perp^{(r)} \approx \frac{1}{2}\frac{p_\perp}{\tau_e^{(r)}}\left(f_{\mathbf{p}} + \zeta(t)\frac{\partial f_{\mathbf{p}}}{\partial \varepsilon_\perp}\right);$$

$$\frac{1}{\tau_e^{(r)}} = \frac{r_0\omega_H^2}{3\pi c}, \qquad \zeta(t) = \frac{3}{16\pi}\hbar\omega_H\int(1+\cos^2\vartheta)\,N(p_z,\vartheta)\,do_{\mathbf{k}}, \tag{21}$$

where $r_0 = e^2/m_0c^2$.

We note that $l_e^{(r)} = \bar{v}\tau_e^{(r)}$ represents the electron mean free path with respect to photon emission and absorption.

In the extreme relativistic case we have from (17) and (19)

$$j_z^{(r)} = j^{(r)}\cos\theta, \qquad j_\perp^{(r)} = j^{(r)}\sin\theta,$$

$$j^{(r)} = \frac{1}{2\tau_e^{(r)}}\frac{\varepsilon^2}{cT}\sin^2\theta\left(f_{\mathbf{p}} + \frac{1}{c}\frac{G_0}{D_0}\frac{\partial f_{\mathbf{p}}}{\partial p}\right), \qquad \frac{1}{\tau_e^{(r)}} = \frac{r_0\omega_H^2}{3\pi c}\frac{T}{m_0c^2} \tag{22}$$

$$\dot{f}_{\mathbf{p}}^{(r)} = \frac{1}{p^2}\frac{\partial}{\partial p}(p^2 j^{(r)}). \tag{23}$$

5. THOMSON SCATTERING IN A MAGNETIC FIELD

1. In analyzing electron and photon relaxation processes we must take account of Coulomb collisions, bremsstrahlung, pair production, and the Compton effect, in addition to radiation effects. When $T \ll m_0c^2$ the branching ratio for bremsstrahlung and synchrotron radiation is of order

$$\frac{w^{(B)}}{w^{(M)}} \approx \frac{1}{137}Z^2\left(\frac{\Omega}{\omega_H}\right)^2\left(\frac{T}{m_0c^2}\right)^{1/2}$$

(Ze is the nuclear charge) and is much smaller than unity when $Z \sim 1$ and $\omega_H \gtrsim \Omega$. The probability of pair production is also much smaller than for synchrotron radiation when $T \lesssim m_0c^2$. Hence we neglect these effects below. The role of Coulomb collisions is analyzed later; here we evaluate the role of the Compton effect.

In the nonrelativistic case the Thomson scattering is of resonance nature because of the magnetic field and can become appreciable when $\omega \sim \omega_H$. The cross section for Thomson scattering in the magnetic field is given by:*

$$d\sigma^{(c)} = \sigma^{(c)}(p_z, \mathbf{k}, \mathbf{k}')\,do_{\mathbf{k}'}$$
$$= \frac{1}{2}r_0^2\frac{\omega^4 F}{[\omega^2(1-v_z\cos\vartheta/c)^2 - \omega_H^2]^2 + \gamma^2\omega^2}do_{\mathbf{k}'}, \tag{24}$$

where $\gamma = 2r_0^2\omega_H^2/3c$ is the damping due to synchrotron radiation

$$F = (1-\omega_H^2/\omega^2)(1+\cos^2\chi) + 2(\omega_H/\omega)^2(\cos^2\vartheta + \cos^2\vartheta') + (\omega_H/\omega)^4\sin^2\vartheta\sin^2\vartheta',$$

ϑ and ϑ' are the angles between **H** and the wave vectors for the incident (**k**) and scattered (**k'**) photons and χ is the angle between **k** and **k'**. (The factor $1 - v_z\cos\vartheta/c$ is important near resonance and takes account of the motion of the scattering electron.)

Using the expression for $\sigma^{(c)}$ when $T \ll m_0c^2$ we can estimate the mean time between electron Compton collisions in which $p_\perp$ is not changed:

$$1/\tau_e^{(c)} \sim (r_0\omega_H^2/c)\,T/m_0c^2. \tag{25}$$

The quantity $\tau_e^{(c)}$ is appreciably greater than $\tau_e^{(r)}$:

$$\tau_e^{(r)}/\tau_e^{(c)} \sim T/m_0c^2 \ll 1.$$

This estimate shows that Compton scattering can only affect the p_z relaxation in the nonrelativistic case; the effect of Compton scattering is then the same as that of the radiation processes.

2. In the nonrelativistic case the change in the photon distribution function due to Thomson scattering is given by

$$\dot{N}_{\mathbf{k}}^{(c)} = \int(N_{\mathbf{k}'} - N_{\mathbf{k}})\,c\,\sigma^{(c)}(p_z;\mathbf{k},\mathbf{k}')\,\delta(\varepsilon + \hbar\omega - \varepsilon' - \hbar\omega')\,d(\hbar\omega')\frac{do_{\mathbf{k}'}d\mathbf{p}'}{(2\pi\hbar)^3}. \tag{26}$$

The total change in the photon distribution function is

$$\dot{N}_{\mathbf{k}} = \dot{N}_{\mathbf{k}}^{(r)} + \dot{N}_{\mathbf{k}}^{(c)}.$$

Equation (21), which gives the electron radiation flux, contains the function $N(p_z,\vartheta) = (N_{\mathbf{k}})_{\omega=\omega_1}$, where $\omega_1 = \omega_H(1 - v_z\cos\vartheta/c)^{-1}$. The function $N(p_z,\vartheta)$ can be given by a simple integral equation if we take account of the fact that the cross section for Thomson scattering has a sharp maximum at γ ($\gamma = \omega_H\sqrt{T/m_0c^2}$). This equation is of the form

$$N(p_z,\vartheta) = -\frac{1}{\tau_p}\frac{1+\cos^2\vartheta}{|\cos\vartheta|}\left\{N(p_z,\vartheta) - \frac{\bar{\varepsilon}_\perp}{\hbar\omega_H} + 4\pi\left(N(p_z,\vartheta) - \frac{\zeta}{\hbar\omega_H}\right)\right\},$$

$$\frac{1}{\tau_p} = \frac{e^2c}{16\pi^2\hbar^3}\omega_H^{-1}\int f_{\mathbf{p}}\,d\mathbf{p}_\perp, \qquad \bar{\varepsilon}_\perp = \int\varepsilon_\perp f_{\mathbf{p}}\,d\mathbf{p}_\perp \Big/ \int f_{\mathbf{p}}\,d\mathbf{p}_\perp. \tag{27}$$

The quantity τ_p is appreciably smaller than $\tau_e^{(r)}$ over a wide range of variation of H, $\mathfrak{N}$, and T. Under these conditions τ_p has a simple physical meaning: it determines the time in which a quasi-equilibrium photon distribution, corresponding to a given energy for the electron distribution function $f_{\mathbf{p}}$, is established.

The quasi-equilibrium distribution, which is found from the equation $\dot{N}(p_z,\vartheta) = 0$, is of the form

$$\tilde{N}(p_z,\vartheta) = \bar{\varepsilon}_\perp/\hbar\omega_H. \tag{27'}$$

*The form of the function F has been given by Gurevich and Pavlov[5].

This quasi-equilibrium distribution and the electron distribution slowly approach the Rayleigh-Jeans equilibrium distribution in a frequency interval of order $\omega_H(T/m_0c^2)^{1/2}$ about the frequency $\omega = \omega_H$.

Comparison of the quantities τ_p and $\tau_p^{(r)}$ in the nonrelativistic case shows that the photon mean path with respect to radiation processes is of the same order as the photon Thomson mean free path in the magnetic field. In the relativistic case the Compton effect is no longer a resonance effect and thus has no effect on the relaxation of the photon gas.

6. RELAXATION OF THE ELECTRON GAS

1. We now write the electronic kinetic equation in the presence of a magnetic field

$$\frac{\partial f_p}{\partial t} + \omega_H \frac{\partial f_p}{\partial \varphi} = \dot{f}_p^{(s)} + \dot{f}_p^{(r)}, \qquad (28)$$

where $\dot{f}_p^{(s)}$ and $\dot{f}_p^{(r)}$ are the changes in the electron distribution function due to Coulomb collisions and radiation effects. An expression for the Coulomb collision integral has been given by Landau.[6] In what follows we shall be interested in determining the conditions for which radiation effects are more important than Coulomb collisions; for this reason we evaluate the latter using a very approximate expression for $\dot{f}_p^{(s)}$:

$$\dot{f}_p^{(s)} = (f_p^0 - f_p)/\tau_e^{(s)}, \qquad (29)$$

where f_p^0 is the equilibrium Maxwellian distribution and $\tau_e^{(s)}$ is the mean time between collisions. In the nonrelativistic[6] and the extreme relativistic[7] cases this quantity is given by

$$\tau_e^{(s)} = \begin{cases} (4\pi c/r_0\Omega^2 L)(T/m_0c^2)^{3/2}, & T \ll m_0c^2 \\ (4\pi c/r_0\Omega^2 L)(T/m_0c)^2, & T \gg m_0c^2 \end{cases}, \qquad (29')$$

where L is the Coulomb logarithm which, in general, depends on H.

Using (21), (27′), and (29) in the nonrelativistic case with $t \gg \tau_p$ ($\tau_p \ll \tau_e^{(r)}$) we write (28) in the form

$$\frac{\partial f_p}{\partial t} + \omega_H \frac{\partial f_p}{\partial \varphi} = \frac{1}{\tau_e^{(r)}} \frac{\partial}{\partial \varepsilon_\perp}\left(\varepsilon_\perp \left(f_p + \bar{\varepsilon}_\perp \frac{\partial f_p}{\partial \varepsilon_\perp}\right)\right) + \frac{f_p^0 - f_p}{\tau_e^{(s)}}. \qquad (30)$$

It follows from this expression that $\bar{\epsilon}_\perp$ is independent of t when $\tau_e^{(r)} \ll \tau_e^{(s)}$. This limiting value of $\bar{\epsilon}_\perp$ for $t \gg t_p$ is denoted by T: $\bar{\epsilon}_\perp(\infty) = T$.*

Taking

$$f_p = f_p^0(1+\eta), \qquad f_p^0 = e^{-\xi}, \qquad \xi = \varepsilon_\perp/T,$$

when $\tau_e^{(r)} \ll \tau_e^{(s)}$ we can write (30) in the form

$$\frac{\partial \eta}{\partial t} + \omega_H \frac{\partial \eta}{\partial \varphi} = \frac{1}{\tau_e^{(r)}}\left\{\xi \frac{\partial^2 \eta}{\partial \xi^2} + (1-\xi)\frac{\partial \eta}{\partial \xi}\right\} - \frac{\eta}{\tau_e^{(s)}}. \qquad (31)$$

The solution of this equation is

$$\eta = \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} a_{nm}(t) L_n(\xi) e^{im\varphi}, \qquad (32)$$

where $L_n(\xi)$ is the Laguerre polynomial. It follows from the normalization condition that $a_{00} = 0$. Further, since

$$\bar{\varepsilon}_\perp = T = \int \varepsilon_\perp f_p d\mathbf{p}_\perp \Big/ \int f_p d\mathbf{p}_\perp$$

it follows that $a_{10} = 0$.

The coefficients $a_{nm}(t)$ satisfy the equation

$$\dot{a}_{nm} + (n/\tau_e^{(r)} + 1/\tau_e^{(s)} + im\omega_H)\, a_{nm} = 0,$$

whence

$$a_{nm}(t) = a_{nm}(0)\exp\{-t(n/\tau_e^{(r)} + 1/\tau_e^{(s)} + im\omega_H)\}. \qquad (33)$$

These expressions show that the relaxation of the n-th harmonic in the expansion in (32) is determined by the quantity $n/\tau_e^{(r)} + 1/\tau_e^{(s)}$. The fact that $a_{0m}(t)$ approaches zero when $t \to \infty$ is due to Coulomb collisions only. However, the quantities $a_{nm}(t)$ ($n \neq 0$) approach zero when $t \to \infty$ because of radiation effects in addition to Coulomb collisions.

If $1/\tau_e^{(r)} \gg 1/\tau_e^{(s)}$, the quantities $p_\perp$, p_z, and φ do not reach Maxwellian distributions at the same rate. Equilibrium is established first in the distribution over $p_\perp$ (in a time $\sim \tau_e^{(r)}$). Equilibrium is established more slowly for p_z and φ (in a time $\tau_e^{(s)}$). The quantity $1/\tau_e^{(r)}$ has a simple physical meaning. It obviously represents the ratio of mean energy radiated by the electron per unit time to the mean energy of the electron. From (21) and (29′)

$$\tau_e^{(r)}/\tau_e^{(s)} \approx (\Omega/\omega_H)^2 (m_0c^2/T)^{3/2}, \qquad T \ll m_0c^2.$$

This ratio diminishes as the density or temperature decrease and as the magnetic field increases, and is of order unity when $T \sim 10^{-2} m_0c^2$, $\mathfrak{N} \sim 10^{13}$ cm^{-3}, $H \sim 2 \times 10^5$ G and of order 10^{-2} when $T \sim 10^{-2} m_0c^2$, $\mathfrak{N} \sim 10^{11}$ cm^{-3} and $H \sim 2 \times 10^5$ G.

The ratio of the photon relaxation time to the electron relaxation time in the nonrelativistic case is, from (21) and (11):

$$\tau_p^{(r)}/\tau_e^{(r)} \sim (r_0\omega_H/c)(\omega_H/\Omega)^2 (T/m_0c^2)^{1/2}.$$

This quantity must be small if the present analysis is to apply. When $T \sim 10^{-2} m_0c^2$, $\mathfrak{N} \sim 10^{13}$ cm^{-3} and $H \sim 2 \times 10^5$ G the ratio $\tau_p^{(r)}/\tau_e^{(r)}$ is 10^{-9}. (We note that under these conditions $l_p^{(r)} \sim 10$ cm where $l_p^{(r)}$ is the photon mean free path.)

*In other words, when $\tau_e^{(s)} \gg \tau_e^{(r)}$ the distribution in (27′) is an equilibrium distribution.

The quantity $\tau_e^{(r)}/\tau_p^{(r)}$ has a simple physical meaning when $T \ll m_0c^2$: it represents the ratio of the equilibrium density of electron energy $\mathscr{E}_e \sim T\mathfrak{N}$ to the equilibrium density of photon energy $\mathscr{E}_p \sim (T/\hbar\omega_H)(\omega_H^2/c^3)\Delta$ for photons in the range $(\omega_H - \Delta, \omega_H + \Delta)$ where $\Delta \sim \omega_H(T/m_0c^2)^{1/2}$ is the width of the radiation line for $s = 1$:

$$\mathscr{E}_e / \mathscr{E}_p \sim \tau_e^{(r)} / \tau_p^{(r)}.$$

This quantity is appreciably greater than unity. In other words, even though the photon reservoir contains a small fraction of the total energy it can have an important effect on the relaxation of the electron gas.

2. The coefficients $a_{nm} = 0$ are easily related to the values of η at the initial time:

$$a_{nm}(0) = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi}d\varphi \int_0^{\infty} e^{-\xi}L_n(\xi)\,\eta(0, \xi, \varphi)\,d\xi.$$

Substituting this expression in (33) and noting that

$$\sum_{n=0}^{\infty} L_n(\xi)L_n(\xi')\,z^n = \frac{1}{1-z}\exp\left(-\frac{z}{1-z}(\xi+\xi')\right)I_0\left(\frac{2\sqrt{\xi\xi' z}}{1-z}\right) \equiv K(\xi, \xi'; t),$$

where I_0 is the Bessel function of imaginary argument, $z = \exp(-t/\tau_e^{(r)})$, we have

$$\eta(t; \xi, \varphi) = \int_0^{\infty} e^{-\xi'}K(\xi, \xi'; t)\,\eta(0; \xi', \varphi + \omega_H t)\,d\xi'.$$

If $\tau_e^{(r)} \ll \tau_e^{(s)}$, $t \gtrsim \tau_e^{(s)}$ then $K \sim 1$ and

$$\eta(t; \xi, \varphi) \approx \int_0^{\infty} e^{-\xi'}\eta(0; \xi', \varphi + \omega_H t)\,d\xi'.$$

This formula shows clearly that the radiation collisions lead to relaxation in $p_\perp$ but do not affect relaxation in the variables p_z and φ.

3. The analysis of electron relaxation is much more complicated in the relativistic case. It can be shown, however, that when $T \gtrsim m_0c^2$ radiation collisions cause relaxation in $p_\perp$ as well as p_z. This result follows because when $T \lesssim m_0c^2$ the radiation flux $j_z^{(r)}$, which we have neglected for $T \lesssim m_0c^2$, will be of the same order as $j_\perp^{(r)}$. Relaxation in φ, however, occurs only by virtue of Coulomb collisions.

The order of magnitude of the relaxation time $\tau_e^{(r)}$ can be estimated from (21) when $T \sim m_0c^2$. The relativistic radiation relaxation time gets smaller when T increases whereas $T_e^{(s)}$ gets bigger when T increases. Thus the relative importance of radiation effects is increased markedly under these conditions.

4. In the extreme relativistic case the radiation collision integral is given by (22). The expression for $j^{(r)}$ contains the function G_0 which can be found explicitly from (19) for $t \gg \tau_p^{(r)}$. In this case N_k can be replaced by the quasi-equilibrium distribution $\tilde{N}_k = \tau_p^{(r)}(k)\,\nu(k)$ where $\nu(k)$ and $\tau_p^{(r)}(k)$ are determined from (9). Taking $f_p = f_p^0(1+\eta)$ and assuming that $\eta \ll 1$ we have finally

$$\dot{\eta}^{(r)} = \frac{1}{\tau_e^{(r)}}\frac{e^{\xi}}{\xi^2}\sin^2\theta\frac{\partial}{\partial\xi}\int_0^{\infty} g(\xi, \xi')\left\{\frac{\partial\eta(\xi')}{\partial\xi'} - \frac{\partial\eta(\xi)}{\partial\xi}\right\}d\xi',$$

$$g(\xi, \xi') = 2^{1/2}(\xi\xi')^{1/2}e^{-\xi-\xi'}\int_0^{\infty} x^{-2/3}F^{-1}(x) \times U\left[\left(\frac{x}{\xi}\right)^{2/3}\right]U\left[\left(\frac{x}{\xi'}\right)^{2/3}\right]dx; \qquad (34)$$

$$1/\tau_e^{(r)} = (r_0\omega_H^2/3\pi c)\,T/m_0c^2, \qquad \xi = cp/T, \qquad \xi' = cp'/T.$$

In this expression we have not written out terms that contain the derivatives of the distribution function with respect to the angle θ which differ by the factor m_0c^2/T from the term above.

It is evident that θ appears only in the form $\sin^2\theta$. It follows that if the initial electron distribution is independent of θ it will then relax to a Maxwellian distribution in a time of order $\tau_e^{(r)}$. If the initial distribution depends on θ, however, it cannot relax to a Maxwell distribution by virtue of radiation processes alone.

7. EFFECT OF RADIATION ON PLASMA ELECTRICAL CONDUCTIVITY

1. Having an expression for the radiation collision integral, in principle we can calculate the effect of radiation on various transport processes in the plasma. Here we investigate the effect on plasma electrical conductivity in a strong magnetic field in the nonrelativistic case.* Obviously the problem reduces to the solution of the kinetic equation

$$-\omega_H\frac{\partial f_p}{\partial\varphi} + eE_z\frac{\partial f_p}{\partial p_z} + e(E_x\cos\varphi + E_y\sin\varphi)\frac{\partial f_p}{\partial p_\perp} + e(E_y\cos\varphi - E_x\sin\varphi)\frac{1}{p_\perp}\frac{\partial f_p}{\partial\varphi} = \frac{1}{p_\perp}\frac{\partial}{\partial p_\perp}(p_\perp j_\perp^{(r)}) + \frac{1}{\tau_e^{(s)}}(f_p^0 - f_p), \qquad (35)$$

where $\mathbf{E}$ is the electric field. This equation can be solved when $E_\perp \neq 0$, $E_z = 0$ or $E_\perp = 0$, $E_z \neq 0$.

In the latter case a solution of (35) that is linear in E_z is

*Actually we determine the limiting value of the electrical conductivity of a uniform plasma in an alternating field at vanishingly low frequency.

$$f_{\mathbf{p}} = f_{\mathbf{p}}^0 (1 + \eta), \qquad \eta = (e\tau_e^{(s)}/m_0T)\, p_z E_z,$$

and the electrical conductivity is given by usual formula $\sigma_{zz} = (e^2\mathfrak{N}/m_0)\,\tau_e^{(s)}$. Thus, as expected, in the nonrelativistic case radiation does not affect the longitudinal plasma electrical conductivity.

We now investigate the transverse plasma conductivity. Taking $\eta = \eta_1(\xi)\cos\varphi + \eta_2(\xi)\sin\varphi$ and introducing the notation $w = \eta_1 + i\eta_2$, $E^+ = E_x + iE_y$ after linearizing in $\mathbf{E}$ we obtain an equation for w:

$$\xi \frac{d^2w}{d\xi^2} + (1-\xi)\frac{dw}{d\xi} - \left(\frac{\tau_e^{(r)}}{\tau_e^{(s)}} + i\omega_H\tau_e^{(r)}\right) w = -\tau_e^{(r)} e E^+ \left(\frac{2}{m_0T}\right)^{1/2} \xi^{1/2}. \qquad (36)$$

Expanding w in Laguerre polynomials $L_n(\xi)$ we have

$$w(\xi) = eE^+\tau_e^{(r)}\left(\frac{2}{m_0T}\right)^{1/2} \sum_{n=0}^{\infty} \alpha_n L_n(\xi)\left(n + \frac{\tau_e^{(r)}}{\tau_e^{(s)}} + i\omega_H\tau_e^{(r)}\right)^{-1},$$

$$\alpha_n = \int_0^{\infty} e^{-\xi'}\xi'^{1/2} L_n(\xi')\, d\xi'. \qquad (36')$$

The components of the vector density of the electrical current $s_\perp$ are related to η_1 and η_2 by

$$s_x = \frac{e}{2m_0}\int \eta_1 p_\perp f_{\mathbf{p}}^0 \frac{d\mathbf{p}}{(2\pi\hbar)^3}, \qquad s_y = \frac{e}{2m_0}\int \eta_2 p_\perp f_{\mathbf{p}}^0 \frac{d\mathbf{p}}{(2\pi\hbar)^3}.$$

Determining η_1 and η_2 from (36) we find $s_\perp = \sigma_\perp \mathbf{E} + (\mathbf{E}\times\mathbf{H})/RH$, where

$$\sigma_\perp = \frac{\mathfrak{N}e^2}{m_0}\frac{\pi}{4}\sum_{n=0}^{\infty} \frac{n/\tau_e^{(r)} + 1/\tau_e^{(s)}}{\omega_H^2 + (n/\tau_e^{(r)} + 1/\tau_e^{(s)})^2}\left(\frac{(2n-3)!!}{2n!!}\right)^2,$$

$$\frac{1}{R} = \frac{\mathfrak{N}e^2}{m_0}\frac{\pi}{4}\sum_{n=0}^{\infty} \frac{\omega_H}{\omega_H^2 + (n/\tau_e^{(r)} + 1/\tau_e^{(s)})^2}\left(\frac{(2n-3)!!}{2n!!}\right)^2. \qquad (37)$$

When $\tau_e^{(r)} \gg \tau_e^{(s)}$, since

$$\sum_{n=0}^{\infty}\left(\frac{(2n-3)!!}{2n!!}\right)^2 = \frac{4}{\pi},$$

we obtain the familiar relations

$$\sigma_\perp = \mathfrak{N}e^2 m_0^{-1}\tau_e^{(s)}\,(1 + \omega_H^2\tau_e^{(s)^2})^{-1},$$
$$R^{-1} = \mathfrak{N}e^2 m_0^{-1}\omega_H\tau_e^{(s)^2}\,(1 + \omega_H^2\tau_e^{(s)^2})^{-1}.$$

In the limit $\tau_e^{(r)} \ll \tau_e^{(s)}$, in which case radiation effects are especially marked in the electron relaxation, the expressions for $\sigma_\perp$ and R^{-1} assume the form

$$\sigma_\perp = \frac{\mathfrak{N}e^2}{m_0}\frac{\pi}{4}\sum_{n=0}^{\infty}\frac{n\tau_e^{(r)}}{n^2 + \omega_H^2\tau_e^{(r)^2}}\left(\frac{(2n-3)!!}{2n!!}\right)^2,$$

$$\frac{1}{R} = \frac{\mathfrak{N}e^2}{m_0}\frac{\pi}{4}\sum_{n=0}^{\infty}\frac{\omega_H\tau_e^{(r)^2}}{n^2 + \omega_H^2\tau_e^{(r)^2}}\left(\frac{(2n-3)!!}{2n!!}\right)^2. \qquad (37')$$

In this case $\sigma_\perp$ is of order

$$\sigma_\perp \approx \mathfrak{N}e^2/m_0\omega_H^2\tau_e^{(r)}, \qquad \omega_H\tau_e^{(r)} \gg 1.$$

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[1] B. A. Trubnikov and A. E. Bazhanova, Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsii (Plasma Physics and the Problem of a Controlled Thermonuclear Reaction), AN SSSR, Vol. 3, page 121.

[2] V. S. Kudryavtsev, ibid. page 114.

[3] K. N. Stepanov, JETP **35**, 283 (1958), Soviet Phys. JETP **8**, 195 (1959).

[4] V. A. Fock, Tablitsy funktsii Éĭri (Table of Airy Functions), M., 1946.

[5] L. É. Gurevich and S. T. Pavlov, ZhTF **30**, 41 (1960), Soviet Phys. Tech. Phys. **5**, 37 (1960).

[6] L. D. Landau, JETP **7**, 203 (1937).

[7] S. T. Belyaev and G. I. Budker, DAN SSSR **107**, 807 (1956), Soviet Phys. Doklady **1**, 218 (1956). S. T. Belyaev, Collection, Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsii (Plasma Physics and the Problem of a Controlled Thermonuclear Reaction), Acad. Sci. Press, Vol. 3, page 66.

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