POLARIZATION OF HIGH-ENERGY COSMIC-RAY MUONS

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The polarization of high-energy (2×10^9—10^{13} eV) cosmic-ray μ mesons is calculated by employing a new solution of the diffusion equation for π mesons and assuming that all μ mesons are produced in the decay of π mesons. The energy dependence of the polarization is obtained.

THE polarization of cosmic-ray μ mesons was calculated by Hayakawa,[1] who, by means of relativistic transformations of the spin 4-pseudovector, found the polarization of μ mesons with energy E produced in the decay of π mesons with energy ϵ in the laboratory system of coordinates (l.s.):

$$\cos\theta = EE^*/pp^* - \varepsilon m_\mu^2/pp^* m_\pi,$$
$$E^* = (m_\pi^2 + m_\mu^2)/2m_\pi, \qquad p^* = (m_\pi^2 - m_\mu^2)/2m_\pi, \tag{1}$$

where θ is the angle between the μ-meson spin and its momentum, E and p are the energy and momentum of the μ meson in the l.s., E^* and p^* are the energy and momentum of the μ meson in the π-meson rest system, m_μ is the μ-meson mass, and m_π is the π-meson mass; c = 1 throughout. To find the mean polarization, Eq. (1) was then averaged over the π-meson spectrum, which was assumed to be the π-meson production spectrum. This assumption, however, is not correct. In fact, in order to find the mean polarization of the μ mesons produced at a given altitude, it is necessary to average Eq. (1) over a function representing the dependence of the number of μ mesons with energy E produced at this altitude per unit time on the energy ϵ of the π mesons during whose decay the μ mesons are produced. This function should be calculated with sufficient accuracy, since the polarization in Eq. (1) is a difference of two nearly equal values. This function is calculated below, and is used to find the mean value of polarization at sea level. The exact knowledge of the polarization as a function of energy and its comparison with experimental data may enable us to obtain information on the contribution of the K_{μ_2} decay to the μ-meson component, on the regeneration of π mesons, etc.

Let us consider a beam of μ mesons with energy E incident vertically downwards, and produced in the decay of π mesons with energy ϵ. For the energy range of the μ mesons under consideration (2×10^9—10^{13} eV), we can consider the problem as a one-dimensional one. This means that the μ mesons were produced in the decay of π mesons propagating vertically. The latter, in turn, were produced by a vertical beam of primary-component nucleons.

The diffusion equation for the vertical π-meson beam, neglecting ionization losses,[2] is

$$\frac{\partial\pi(\varepsilon, x)}{\partial x} = -\varkappa(\varepsilon)\pi(\varepsilon, x) + B_0 e^{-\mu x}\varepsilon^{-\gamma} - \frac{E_\pi}{x\varepsilon}\pi(\varepsilon, x), \tag{2}$$

where x is the atmospheric depth in nuclear units (taken as 80 g/cm^2) and $\pi(\epsilon, x)$ is the number of π mesons with energy ϵ at the depth x.

The first term in the right-hand side of Eq. (2) describes the absorption of π mesons in the atmosphere as a result of nuclear interactions; $\kappa(\epsilon)$ is the inverse of the mean effective range for inelastic collisions of π mesons with air nuclei. Zatsepin[2] sets this mean free path equal to one nuclear length, which corresponds to $\kappa(\epsilon) = 1$. Introducing $\kappa(\epsilon) < 1$ is equivalent to taking into account a possible regeneration of π mesons and also using a larger pion mean free path, in agreement with the experimental data.

The second term in Eq. (2) represents the production of π mesons by the nucleonic component, which is absorbed in the atmosphere exponentially. $\mu = 0.65$ is the inverse of the mean free path of the primary protons, measured in nuclear lengths. The energy spectrum of the π-meson production in the energy range under consideration can be assumed to follow a power law with exponent $\gamma = 2.65$. The form of the term describing the π-meson production renders the equation invalid for energies $\epsilon < 2$ BeV, since the problem cannot be considered as one-dimensional for these energies, and the π meson production spectrum cannot be represented by a power law.

The third term describes the π-meson decay. $E_\pi/x\epsilon$ is the decay probability of a π meson with energy ϵ traversing one nuclear length; $E_\pi = 10^{11}$ eV.

Considering the solution of Eq. (2) as a continuous function of the parameter κ, we expand $\pi(\epsilon, x, \kappa)$ in powers of the difference $a = \kappa - \mu$, which depends on the π-meson energy. Equating the coefficients of like powers of a, we obtain from (2) the system of equations

$$\frac{\partial \pi (\varepsilon, x, \mu)}{\partial x} + \left(\mu + \frac{E_\pi}{x\varepsilon}\right) \pi (\varepsilon, x, \mu) = B_0 e^{-\mu x} \varepsilon^{-\gamma},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \pi}{\partial \varkappa}\bigg|_{\varkappa=\mu}\right) + \left(\mu + \frac{E_\pi}{x\varepsilon}\right)\frac{\partial \pi}{\partial \varkappa}\bigg|_{\varkappa=\mu} = -1!\, \pi (\varepsilon, x, \mu) \ldots$$

The solution under the boundary condition $\pi(\epsilon, x) = 0$ when $x = 0$ is

$$\pi (\varepsilon, x) = B_0 e^{-\mu x} \varepsilon^{-\gamma} \frac{x}{1 + E_\pi/\varepsilon} \times \left(1 - \frac{ax}{2 + E_\pi/\varepsilon} + \frac{a^2x^2}{(2 + E_\pi/\varepsilon)(3 + E_\pi/\varepsilon)} - \ldots\right). \tag{3}$$

At an altitude x, $(m_\pi/\epsilon\tau_0)\pi(\epsilon, x)\,d\epsilon\,dx$ pions with energy ϵ decay per unit time; τ_0 is the π-meson lifetime in its rest system.

In the decay of π mesons with energy ϵ, μ mesons in an energy range from E_{min} to E_{max} are produced. Multiplying the number of decaying π mesons of energy ϵ by the probability of production in such a decay of a μ meson with energy E

$$W(\varepsilon, E)\, dE \sim dE/\varepsilon \sqrt{1 - (m_\pi/\varepsilon)^2},$$

we find the number of μ mesons with energy E produced in the decay of π mesons with energy ϵ at the altitude x, which we shall denote by $N(E, \epsilon, x)$:

$$N(E, \varepsilon, x) = Bxe^{-\mu x} f(E, \varepsilon, x),$$

$$f(E, \varepsilon, x) = \frac{\varepsilon^{-(2+\gamma)}}{(1 + E_\pi/\varepsilon)\sqrt{1 - (m_\pi/\varepsilon)^2}} \times \left[1 - \frac{ax}{2 + E_\pi/\varepsilon} + \frac{a^2x^2}{(2 + E_\pi/\varepsilon)(3 + E_\pi/\varepsilon)} - \ldots\right]. \tag{4}$$

where B is a new constant. μ mesons with energy E are produced from π mesons with energy ϵ in the range $\epsilon_- \le \epsilon \le \epsilon_+$, where

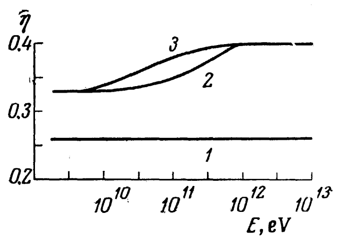
$$\varepsilon_+ = (EE^* + pp^*)\, m_\pi/m_\mu^2, \quad \varepsilon_- = (EE^* - pp^*)\, m_\pi/m_\mu^2.$$

Averaging Eq. (1) over the found distribution function $N(E, \epsilon, x)$, we find the mean value of the polarization of the μ mesons with energy E produced at the depth x, which we shall denote by $\eta(E, x)$:

$$\eta (E, x) = \frac{EE^*}{pp^*} - \frac{m_\mu^2}{m_\pi pp^*} \int_{\varepsilon_-}^{\varepsilon_+} \varepsilon f (E, \varepsilon, x)\, d\varepsilon \Big/ \int_{\varepsilon_-}^{\varepsilon_+} f(E, \varepsilon, x)\, d\varepsilon. \tag{5}$$

The first approximation $\kappa(\epsilon) = \mu$ corresponds to the physical assumption that the character of the nuclear interactions and the regeneration of the π mesons lead to their absorption in the atmosphere according to the same exponential law as the absorption of nucleons. In that case the diffusion equation (2) has a solution in the form of the finite algebraic function obtained from Eq. (3), where we put $a = 0$. Moreover, the μ-meson polarization (5) is then independent of the production altitude. This is explained by the fact that in this approximation Eq. (2) can be solved by separating the variables ϵ and x, i.e., the energy spectrum of the π mesons is the same at all altitudes.

The semilogarithmic dependence of the polarization of the produced μ mesons on their energy is shown in the figure (curve 2). The figure also shows the results obtained by Hayakawa using as the exponent of the π-meson production spectrum $\gamma = 2.65$ (curve 1). The increase in the polarization with energy follows clearly from the shape of the spectrum 3. Because of the ionization losses,



the μ mesons of energy E at sea level had at the instant of production different energies, and therefore a different polarization η. However, owing to the extremely weak energy dependence of η and to the relatively narrow energy interval of the μ mesons produced at different altitudes, we can neglect this difference in polarization, so that the mean polarization at sea level is given by the same curve 2 as the energy dependence of the polarization η at the moment of production.

We have calculated the polarization of the μ mesons with energy E at different atmospheric depths x and under different assumptions of the character of the π-meson absorption (the values of κ were varied from $\mu = 0.65$ to 1). The calculations were carried out according to Eq. (5). Alternation of the signs, and the fast decrease of the terms of the series, enabled us to reach the necessary accuracy. The computation showed that the polarization increases with atmospheric depth x and with increasing absorption coefficient κ of the π mesons. This can easily be understood from physical considerations. The polarization increases with increasing steepness of the π-meson production spectrum and of the spectrum of the π mesons from high altitudes. The latter, however, is less steep because of the decay of the π mesons, and therefore the resulting spectrum becomes steeper the smaller the contribution of the π mesons arriving from high altitudes, i.e., the denser the atmosphere and the larger the π meson absorption.

The first approximation obtained by us pertains to the case where the polarization has a minimum value, since $ax = 0$. Another value of the polarization, one that is certainly overestimated, is represented by curve 3. This was calculated for $a = 0.35$ and $1.9 \leq x_1 \leq 2.8$ nuclear lengths, where x_1 is the assumed depth of μ-meson production. The μ-meson polarization measured at sea level is less than that calculated from curve 3 since $a(\epsilon) \leq 0.35$, and the effective depth of μ-meson production for the energy range under consideration is smaller than x_1. Thus, the polarization of μ mesons with energy $E \geq 2 \times 10^9$ eV at sea level is described by a curve intermediate between the curves 2 and 3.

When the experimental parameters γ and E_π vary in the intervals $\Delta\gamma = \pm 0.05$ and $\Delta E_\pi = \pm 3 \times 10^{10}$ eV, the polarization changes little: $\Delta\eta \leq 0.01$, and the equal sign corresponds to the most disadvantageous combination of errors.

In conclusion, the author wishes to express his deep gratitude to Prof. G. T. Zatsepin for constant interest in the work, and to V. I. Ritus for helpful discussion.

[1] S. I. Hayakawa, Phys. Rev. **108**, 1533 (1957).

[2] G. T. Zatsepin, Doctoral Thesis, Phys. Inst. Acad. Sci., 1952.

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