Letters to the Editor

ANGULAR DISTRIBUTION OF MUONS FROM THE π-μ DECAY

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RECENTLY Vaĭsenberg, Kolganova, and Minervina (VKM) [1] have published an article dealing with the controversial subject of anisotropy in the angular distribution of muons arising from the decay of stopped pions.

The purpose of the present letter is to present certain data, up to now only incompletely published, [2] in support of the reality of this effect and to point out that some of the conclusions of the above mentioned authors are not statistically reliable.

In the same emulsion that was used in our previous experiment on the π-μ decay [3] (area scanning) we have accumulated 1734 events of π-μ-e decays under circumstances guaranteeing absence of psychological effects in the selection. The scanning was by following the tracks. Gray parallel tracks were selected from the π^+ meson beam in a narrow zone, perpendicular to the beam. Tracks for which the decay occurred in the emulsion layer in which the selection was made were discarded so that the external appearance of the decay had no influence on the observer. Other observers followed the track through the successive emulsion layers until it stopped or left the emulsion.

We give the distribution of the angles between the projection of the initial muon momentum and the direction of the beam (which coincides with the x axis of the coordinate system):

Angle interval	0—45°	45—90°	90—135°	135—180°
Number of muons	393	412	493	436

The coefficient of forward-backward asymmetry $b = -0.143 \pm 0.048$ shows a deviation from isotropy by 2.98 standard errors; that is, the probability of getting this value for b if the distribution were in reality symmetric is less than 3×10^{-3}. The total deviation from isotropy of the observed distribution is measured by Pearson's probability $P(\chi^2) = 4.6 \times 10^{-3}$. It is important to stress that the conditions under which this result was obtained (which is incidentally in good agreement with the result of the previous experiment by area [3]) fully coincide with the tracking of pions from their place of production in τ decay, which is believed to be free of psychological effects by VKM too.

Regarding the work of VKM we should like to make the following comments.

The total statistics is insufficient to establish differences between various partial distributions. Thus the ratio R/L (in the notation of [1]) is equal to 0.958 ± 0.061 in the non-dense region (all observers) and to 0.855 ± 0.052 in the dense region (observers E, F, G); the difference in these values 0.103 ± 0.080 can arise accidentally once in five experiments with identical true R/L. Clearly, the difference would be even more insignificant if one were to take the data of all observers in the dense region as well.

Generally speaking, such a division of the data by the size of the asymmetry obtained by different observers is statistically inadmissible. Indeed if one divides a certain asymmetric distribution into subdistributions and extracts those that, by accident, are characterized by a higher asymmetry, than the remainder will, obviously, be less asymmetric.

For illustration we have carried out the following calculation by the Monte Carlo method. From the total of events with the value of R/L equal to 0.905 (corresponding for example, to the result of all the observers together, in the dense region [1]) we have extracted seven pairs of values of R and L (seven "observers") and noted those three "observers" who obtained the highest asymmetry. The size of the entire sample was approximately equal to the number of muons in the dense region of the VKM work (1778 and 1774 respectively). This was repeated 10 times. The results for the ratio R/L were as follows:

All "observers"	0.907 ± 0.015
Four "observers" with smallest asymmetry	0.982 ± 0.019
Difference	0.075 ± 0.024
Three "observers" with largest asymmetry	0.826 ± 0.019

As can be seen, the procedure employed by VKM leads to the false conclusion that the distribution was symmetric while in fact the totality of events was asymmetric. What is more, the L-R asymmetry of observers E, F, G in the dense region cannot be explained by omissions because the scanning efficiencies in the L and R quadrants are indistinguishable (0.7 statistical deviation).

By the way, the existence of a correlation between the asymmetry and scanning efficiency can in principle be tested by repeating the scanning of observers E, F, G by the observers who obtained a "good" result. In the present case even such a test cannot lead to reliable results in view of the insufficient statistics.

It is not without interest to note that the forward-backward asymmetry of slow pions from τ decay in the experiment of Garwin et al [4] is, contrary to the assertion of VKM, fully significant according to the usual statistical criteria: $\chi^2 = 9.3$ with one degree of freedom, i.e., $P(\chi^2) \approx 2 \times 10^{-3}$.

We are preparing for publication a thorough analysis of the entire problem of π-μ asymmetry, including other experimental data and other considerations not touched upon here.

[1] Vaĭsenberg, Kolganova, and Minervina, JETP **41**, 106 (1961), Soviet Phys. JETP **14**, 79 (1962).

[2] Hulubei, Ausländer, Friedländer, and Titeica, Int. Working Meeting on Cosmic Rays, Bucharest (1959), Acad. RPR, Inst. de Fizica Atomica, Bucuresti, p. 130 (1960). J. Ausländer, Ninth Int. Ann. Conf. on High Energy Physics, Kiev (1959), Acad. of Science USSR and IUPAP, Plenary Ses. VI-IX, p. 239, Moscow (1960).

[3] Hulubei, Ausländer, Balea, Friedländer, and Titeica, Proc. of the 2-nd Int. Conf. on Peaceful Uses of Atomic Energy, p. 1283, Geneva (1958).

[4] Garwin, Gidal, Lederman, and Weinrich, Phys. Rev. **108**, 1589 (1957).

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LOW-LYING NEGATIVE-PARITY LEVELS IN NONSPHERICAL NUCLEI

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LOW-LYING 1^- levels with energy 0.3–1.3 MeV have recently been observed in deformed even-even nuclei. [1] These levels correspond to internal excitations with K = 0 and negative parity. States of this type are treated in the hydrodynamic model as octupole oscillations of the nucleus relative to the equilibrium form, with energy ~ 3 MeV for the actinides. [2] The hydrodynamic model can thus explain neither the absolute positions of these levels nor their connection with the shell structure.

We propose an interpretation of the 1^- levels on the basis of the superfluid model. The Hamiltonian of the system includes, in addition to the pair interaction, an octupole-octupole interaction which has the same nature as the quadrupole-quadrupole used to describe collective oscillations of the quadrupole type in spherical [3] and nonspherical [4] nuclei. Using the method of approximate second quantization we can obtain, in the same manner as in [4], an equation for the excitation energy ($\hbar = 1$)

$$1 = \varkappa \sum_{\lambda\lambda'} \sum_{n,\, p} |(q_{30})_{\lambda\lambda'}|^2 \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'}} \frac{E_\lambda + E_{\lambda'}}{(E_\lambda + E_{\lambda'})^2 - \omega^2}. \qquad (1)$$

Here $q_{30} = r^3 Y_{30}(\theta)$ —single-particle octupole moment operator; the summation is over both the neutron (λ) and proton (λ') states; the single-particle state energies ϵ_λ are reckoned from the Fermi surface; $E = \sqrt{\epsilon_\lambda^2 + \Delta^2}$. In the summations over the neutron (proton) states in (1) we use the values $\Delta = \Delta_n(\Delta_p)$ and $\epsilon_0^n = \epsilon_0^n(\epsilon_0^p)$, where $\Delta_n(\Delta_p)$ is a constant characterizing the neutron (proton) pair correlation energy; $\epsilon_0^n(\epsilon_0^p)$ is the Fermi energy boundary for the neutrons (protons); κ —octupole-octupole interaction constant ($\kappa \sim \epsilon_0/AR^6$). This constant is assumed to be the same for nn, pp, and np interactions.

From the microscopic point of view, the foregoing excitations are bound states of quasi-particles with energy ω, projection of the total momentum on the nuclear symmetry axis K = 0, and negative parity. Equation (1) has, generally speaking, two solutions, which correspond at $\kappa \to 0$ to the breakup of a neutron or proton pair. As can be seen from (1), the excitation energies can be found if the scheme of the single-particle levels (say, the Nilsson scheme [5]) $\Delta_{n,p}$, and κ are specified. For a given group of nuclei, κ can be regarded constant. Therefore, by determining κ from the position of the 1^- level for one nucleus, we can determine from (1) the value of ω for the other nuclei of the given group.

It is of interest to find the probabilities of the electric dipole transition from the 1^- state to the ground state. In the model considered here the reduced probability of this transition is

$$B(E1;\, 1^- \to 0^+) = \frac{1}{6\omega} \left\{ \sum_{\lambda\lambda'} \sum_{n,\, p} e_{n,\, p} (q_{10})_{\lambda\lambda'} (q_{30})_{\lambda\lambda'} \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'}} \times \frac{E_\lambda + E_{\lambda'}}{(E_\lambda + E_{\lambda'})^2 - \omega^2} \right\}^2 \left\{ \sum_{\lambda,\lambda'} \sum_{n,\, p} |(q_{30})_{\lambda\lambda'}|^2 \frac{E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} + \Delta^2}{2E_\lambda E_{\lambda'}} \times \frac{E_\lambda + E_{\lambda'}}{[(E_\lambda + E_{\lambda'})^2 - \omega^2]^2} \right\}^{-1}, \qquad (2)$$