RELATION BETWEEN THE COLLECTIVE AND SHELL DESCRIPTIONS OF DIPOLE EXCITATIONS OF ATOMIC NUCLEI

V. V. BALASHOV

Institute of Nuclear Physics, Moscow State University

Submitted to JETP editor August 19, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **42**, 275-281 (January, 1962)

The position and width of giant resonances and the total photoabsorption cross section are calculated in a simple way with a model based on the shell interpretation of collective dipole excitation of nuclei. The model also yields a qualitative explanation of the relation between different photoabsorption branches and the character of the energy spectra of the photoproducts. Calculations of photodisintegration of C^{12}, O^{16}, Ca^{40} and Pb^{208} nuclei are carried out.

THE main difficulty of the single-particle model of photonuclear reactions [1] lies in the explanation of the position of the maximum energy of dipole photon absorption. The "giant resonance" energy calculated with this model is 1.5 or 2 times lower than obtained by experiment, because correlations between nucleons are neglected in the single-particle model. Detailed calculations of the photodisintegration of several nuclei (O^{16}, Ca^{40}, Pb^{208}) were undertaken to determine the influence of residual interaction on the formation of giant resonance. [2-4] These yielded good agreement with experiment, indicating that the shell model of the nucleus, with account of the residual interactions between the nucleons, permits a detailed description of nuclear photodisintegration in the giant-resonance region. These calculations have shown that the residual interactions between nucleons play an important collectivizing role in the formation of the dipole excitation of the nucleus, namely that the dipole excitations are characterized by strong mixing of a large number of single-particle states.

Unfortunately these detailed calculations, which call for the diagonalization of the energy matrix of the dipole states, have been carried out only for the doubly-magic nuclei. To extend this procedure to non-magic nuclei, with the exception of a few special cases, tremendous technical difficulties must be overcome. It is therefore desirable to have a simpler description of the dipole states, one that does not require a complex diagonalization procedure. One such attempt is well known—the collective model proposed in 1945 by Migdal. [5] The collective dipole excitations of the nucleus were treated in this model hydrodynamically, and therein lies the reason for its failure. While yielding the true position of the giant resonance, the hydrodynamic model cannot explain the character of the decay of the dipole state, and particularly the large yield of fast photoprotons from the nucleus. This raises the question whether it is possible to borrow from this model the idea of collective dipole excitation and to develop the idea on the basis of a shell model rather than a hydrodynamic model.

We propose here such a generalized model. The model claims a collective description of the energy position of the giant resonance, the total absorption cross sections, and a qualitative explanation of the spectra of the photoproducts.

1. WAVE FUNCTION OF THE DIPOLE STATE

The starting premise of the model is the experimental fact that all nuclei have a narrow energy band of dipole absorption (giant resonance). The exact expression for the nuclear dipole excitation operator is proportional to the total nuclear electric dipole moment:

$$\hat{D} = eZNA^{-1}(\mathbf{R}_Z - \mathbf{R}_N). \qquad (1)$$

Here $\mathbf{R}_Z$ and $\mathbf{R}_N$ are the positions of the centroids of all the protons and neutrons of the nucleus.

The wave function $\Psi_{\rm dip}$ of the dipole state of the nucleus is constructed by having the operator D act on the wave function Ψ_0 of the ground state of the nucleus:

$$\Psi_{dip} = (N[\Psi_0])^{-1/2}\hat{D}\Psi_0, \qquad (2)$$

where $N[\Psi_0]$ is a normalization factor that depends on the form of Ψ_0.

The wave function Ψ_{dip} is not an eigenfunction of the Hamiltonian $\hat{H}$ of the nucleus. (Brink has shown[6] that Ψ_{dip} is an eigenfunction of $\hat{H}$ for a system of noninteracting protons and neutrons in the central potential of a harmonic oscillator.) However, the relatively small width of the giant resonance enables us to treat Ψ_{dip} as an eigenfunction of some approximate Hamiltonian $\hat{H}'$, which is close to $\hat{H}$. The deviation of $\hat{H}'$ from $\hat{H}$ determines the width of the collective state and the width of the giant resonance. The ground state wave function Ψ_0 is chosen in accord with the shell model.

The operator $\hat{D}$ which generates the dipole state can be represented as a sum of proton and neutron single-particle operators

$$\hat{D} = \sum_{j_p, j_{p'}} \left(e\frac{N}{A}\right) a^+_{j_p} \langle j_p | \mathbf{r} | j_{p'} \rangle a_{j_{p'}} + \sum_{j_n, j_{n'}} \left(-e\frac{Z}{A}\right) a^+_{j_n} \langle j_n | \mathbf{r} | j_{n'} \rangle a_{j_{n'}}. \quad (3)$$

Here a^+_{jp} and $a_{jp'}$ are the operators of creation and annihilation of the proton single-particle state, while a^+_{jn} and $a_{jn'}$ are the creation and annihilation operators for the neutron single-particle state ($j_{p,n}$ denote arbitrarily the entire set of individual quantum numbers of the nucleon). From (3) we readily obtain an expansion of Ψ_{dip} in terms of the single-particle states Φ_λ:

$$\Psi_{dip} = \sum_\lambda \alpha_\lambda \Phi_\lambda. \quad (4)$$

(The coefficients α_λ are proportional to the single-particle matrix elements of the electric dipole moment $\langle \hat{d} \rangle_\lambda$.) Thus, the procedure (2) gives rise to a dipole state with strong mixing of shell configurations. It is clear therefore that the position of this state depends appreciably on the nondiagonal (in the single-particle states) residual interaction between the nucleons.

2. ENERGY OF DIPOLE EXCITATION, TOTAL PHOTODISINTEGRATION CROSS SECTION

The average dipole state energy $\bar{E}$ is given by

$$\bar{E} = \langle \Psi_{dip} | \hat{H} - E_0 | \Psi_{dip} \rangle \qquad (E_0 = \langle \Psi_0 | \hat{H} | \Psi_0 \rangle). \quad (5)$$

Let the Hamiltonian $\hat{H}$ of the nucleus be given as the sum of the single-particle Hamiltonian $\hat{H}_0$ and the potential $\hat{V}$ of the residual nucleon pair interaction:

$$\hat{H} = \hat{H}_0 + \hat{V},$$

$$\hat{H}_0 = \sum_{i=1}^{A} H_i = \sum_{i=1}^{A} \left(\frac{p_i^2}{2M} + V_i\right); \qquad \hat{V} = \sum_{i<j} V_{ij}. \quad (6)$$

We substitute $\hat{H} = \hat{H}_0 + \hat{V}$ in (5):

$$\langle \Psi_{dip} | \hat{H} | \Psi_{dip} \rangle = (N[\Psi_0])^{-1/2} \langle \hat{D}\Psi_0 | H_0 | \hat{D}\Psi_0 \rangle + (N[\Psi_0])^{-1/2} \left\langle \hat{D}\Psi_0 \left| \sum_{i<j} V_{ij} \right| \hat{D}\Psi_0 \right\rangle. \quad (7)$$

In the calculation of the first term in (7) it is convenient to use the expansion of Ψ_{dip} in terms of the single-particle states (4), where Φ_λ are the eigenfunctions of the Hamiltonian $\hat{H}_0$:

$$\hat{H}_0 \Phi_\lambda = E_{0\lambda} \Phi_\lambda. \quad (8)$$

Therefore

$$(N[\Psi_0])^{-1/2} \langle \hat{D}\Psi_0 | \hat{H}_0 | \hat{D}\Psi_0 \rangle = \sum_\lambda |\alpha_\lambda|^2 E_{0\lambda}. \quad (9)$$

Expression (9) enables us to get around the difficulties connected with the uncertainty in the single-particle potential, since the values $E_{0\lambda}$ can be obtained directly from the experimental data on the levels of the neighboring nuclei (see [3]).

Expression (5) is of little value in comparison of theory with experiment. It is more convenient to introduce in place of $\bar{E}$ the quantity

$$E_{dip} = (1/\bar{E}) \langle \Psi_{dip} | (\hat{H} - E_0)^2 | \Psi_{dip} \rangle, \quad (10)$$

which, as can be readily seen, gives an exact expression for the energy position of the "centroid" of the dipole-absorption curve. With the aid of (4) we obtain

$$E_{dip} = \frac{\sum E_{0\lambda}^2 |\alpha_\lambda|^2 + 2\sum \alpha^*_\lambda \alpha_{\lambda'} E_{0\lambda} V_{\lambda\lambda'} + \sum \alpha^*_\lambda \alpha_{\lambda'} V_{\lambda\lambda''} V_{\lambda''\lambda'}}{\sum E_{0\lambda} |\alpha_\lambda|^2 + \sum \alpha^*_\lambda \alpha_{\lambda'} V_{\lambda\lambda'}}. \quad (11)$$

The radiation width of the dipole state, together with the total photoabsorption cross section, is calculated from the usual formulas with the aid of (4). The result is

$$\sigma_{\text{abs}} \sim E_{dip} \sum_\lambda |\langle \hat{d} \rangle_\lambda|^2. \quad (12)$$

Let us compare (12) with the corresponding expression for the single-particle model:

$$\sigma_{\text{abs}} \sim \sum_\lambda E_{0\lambda} |\langle \hat{d} \rangle_\lambda|^2. \quad (13)$$

It is appropriate to note that the series of integrals in (5) and (10) and later in (15) can be calculated without implying any particular nuclear model, using the well known "sum rules." For example,

$$\bar{E} = \sigma_0 / \sigma_{-1}, \quad (14)$$

where

$$\sigma_0 = \int \sigma(E_\gamma) dE_\gamma, \qquad \sigma_{-1} = \int \frac{\sigma(E_\gamma)}{E_\gamma} dE_\gamma$$

etc. Unfortunately, although relations based on the "sum rules" are most general, they nevertheless give skimpy and rather one-sided information on giant resonance. The use of nuclear models is apparently unavoidable if we want a theory that describes various aspects of photonuclear reactions (including the properties of the disintegration products), and particularly a theory in which a connection is established between photonuclear reactions with other nuclear-spectroscopy reactions. Consequently a comparison with the "sum rules" can be naturally regarded as a check on the initial premises of the model, while agreement between individual results of the model and the "sum rules" can be regarded as some indication of the correctness of the model.

3. ENERGY DISTRIBUTIONS OF PHOTODISINTEGRATION PRODUCTS

The expansion (4) of the function Ψ_{dip} in single-particle shell states enables us to use the usual shell-model technique of fractional-parentage coefficients to calculate the reduced widths of the dipole state corresponding to decay into different states of the final nucleus. We can therefore obtain a qualitative picture of the photoproton and photoneutron energy spectra corresponding to giant resonance. It is obviously clear that, generally speaking (particularly in light nuclei), the proposed model, which is based on the procedure of [2], cannot give a quantitative description of the spectra, which will be slightly impoverished owing to insufficient admixture of "weak transitions" in the giant resonance. A quantitative description of the photoproduct energy distributions can therefore be obtained at present only by detailed calculation based on diagonalization, similar to that given in [3] for the photodisintegration of Ca^{40}.

At the same time, this model, unlike the hydrodynamic model, explains the large fraction of fast photoprotons contained in the photodisintegration spectrum as being due to strong admixture to the dipole state of single-particle states above the lowest levels of the final nucleus. We note that the single-particle model also fails to give a consistent explanation of the spectra, for if we disregard the mixing of the configurations, the majority of the channels corresponding to "strong transitions" are closed.

4. WIDTH OF GIANT RESONANCE

The question of the width of giant resonance is closely connected with the question of verification of the present model. The energy spread Δ of the dipole state is a measure of the extent to which Ψ_{dip} does not satisfy the Schrödinger equation with total Hamiltonian $\hat{H}$:

$$\Delta^2 = (1/\bar{E})\, \langle \Psi_{dip} \,|\, (\hat{H} - E_0 - E_{dip})^2\, (\hat{H} - E_0) \,|\, \Psi_{dip} \rangle$$

$$= \frac{\langle \Psi_{dip} \,|\, (\hat{H} - E_0)^3 \,|\, \Psi_{dip} \rangle}{\langle \Psi_{dip} \,|\, \hat{H} - E_0 \,|\, \Psi_{dip} \rangle} - (E_{dip})^2. \qquad (15)$$

From (15) follow, in particular, the conditions for an infinitely narrow resonance, corresponding to the Brink theorem [6] and the Brown-Bolsterli scheme. [7]

When $V_{\lambda\lambda'} = 0$

$$\Delta^2 = \frac{1}{2} \frac{\Sigma E_{0\lambda} E_{0\lambda'} (E_{0\lambda} - E_{0\lambda'})^2 \,|\alpha_\lambda|^2 \,|\alpha_{\lambda'}|^2}{|\,\Sigma E_{0\lambda} \,|\alpha_\lambda|^2 \,|^2}, \qquad (16)$$

Δ vanishes with $E_{0\lambda} = E_{0\lambda'} = \text{const}$. This holds true for a harmonic oscillator (Brink's theorem).

In the case when $V_{\lambda\lambda'} \neq 0$ we get $\Delta = 0$ when

$$E_{0\lambda} = E_{0\lambda'} = \text{const}, \qquad V_{\lambda\lambda'} \sim \alpha_\lambda \alpha_{\lambda'} \qquad (17)$$

—the Brown-Bolsterli scheme. [7]

In the present article we defer the theoretical justification of the introduction of the potential $\hat{H}'$, which will be treated in greater detail later on. A similar situation occurrs in the treatment of the giant resonance of the force function that describes the scattering of slow neutrons by nuclei. For the time being we can advance only qualitative arguments in favor of the approximate diagonalization of the total proton-neutron potential: the nondiagonal interactions between the protons and the neutrons in the deep internal shells are cut off by the Pauli principle, and therefore the smearing of the dipole state is explained essentially by the interaction between the protons and neutrons on the Fermi surface.

We have not touched upon the problem of "natural" width of the dipole state, which determines the lower limit of the giant resonance width. The "natural" decay width of the dipole state is calculated in usual fashion from the reduced widths that correspond to different decay channels (see Sec. 3).

5. EXAMPLES

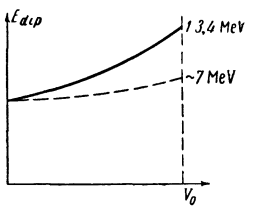
a) Energy position of the dipole states. Table I lists the calculated energy positions of the giant resonance and the total absorption cross sections in the given model, for the nuclei C^{12}, O^{16}, Ca^{40}, and Pb^{208}. The column "Exact calculation" contains the results obtained by diagonalizing the energy matrix of the dipole states. In our calcula-

Table I

		Exact calculation	Present work	Experiment
O^{16}	E_{dip}	22.6—25.2 [2]	23.7	22.0—25.5 [8]
	σ_{abs}	370	390	160
Ca^{40}	E_{dip}	19.6 [3]	19.4	19.0 [9]
	σ_{abs}	1080	1000	
Pb^{208}	E_{dip}	13.5 [4]	13.4	13.0—14.0 [8]
	σ_{abs}	3915	4000	3600
C^{12}	E_{dip}	22.2 [7]	24.0	23.0
	σ_{abs}	—	260	

tion the parameters of the single-particle states and of the residual interaction for the different nuclei are the same as in the corresponding calculations with diagonalization. The calculations for the non-magic nucleus C^{12} are given in the LS-coupling scheme, i.e., the ground-state function was chosen to be $(1s)^4(1p)^8[444]^{11}S_0$, and the 1s → 1p transitions have been left out for simplicity.

The figure shows the change in energy of dipole states in Pb^{208} when the residual interaction is turned on. The lower dashed curve corresponds to an account of the diagonal part of the residual interaction only.



Dependence of the energy position of the dipole state in Pb^{208} on the nucleon-nucleon interaction.

b) Spectra of photodisintegration products. We give the expansion of the wave function of the dipole state of Ca^{40} in terms of the shell states:

$$\Psi_{dip} = -0.09\,|d_{3/2}^{-1}p_{3/2}\rangle + 0.21\,|d_{3/2}^{-1}p_{1/2}\rangle + 0.53\,|d_{3/2}^{-1}f_{5/2}\rangle + 0.33\,|s_{1/2}^{-1}p_{3/2}\rangle + 0.24\,|s_{1/2}^{-1}p_{1/2}\rangle + 0.63\,|d_{5/2}^{-1}f_{7/2}\rangle + 0.28\,|d_{5/2}^{-1}p_{3/2}\rangle + 0.14\,|d_{5/2}^{-1}f_{5/2}\rangle. \quad (18)$$

The use of Ca^{40} as an example is far from being the most fortunate, since giant resonance corresponds here to a small number of transitions. However, this example illustrates clearly the advantages and shortcomings of the proposed model in the description of the photoproduction spectra. In addition, a detailed calculation of the partial cross sections has been carried out for Ca^{40}, something not yet available for other nuclei.[3]

With the aid of the function (18) we calculated the widths of the decay of the dipole state of the nucleus Ca^{40} (E_{dip} = 19.4) over the different proton and neutron channels (Table II). As can be seen from Table II, there are two pronounced groups in the photoproton spectrum: $E_p \sim 11$ MeV and ~ 4 MeV. This conclusion agrees with experiment and with the results of the "Exact calculation." However, the intensity ratio of these two groups is approximately the inverse of what is obtained in experiment: in accordance with the general tendency indicated in Sec. 3, the decay to the excited state K^{39} with $J = \frac{1}{2}^+$, corresponding to the "weak transition" 2s → 2p, turns out to be somewhat suppressed in this calculation.

According to Table II, the neutrons group around an energy 3.5 MeV, which approximately agrees with the results of the "Exact calculation."[3] There are no published experimental data on the spectrum of photoneutrons from Ca^{40}.

We note that the proposed model enables us to estimate the intensity ratios of different photodisintegration branches. Experiment has shown that the photoproton yield from Ca^{40} is approximately seven times greater than the photoneutron yield. In this calculation, as can be seen from Table II, $\sigma(\gamma, p)/\sigma(\gamma, n)$ = 3.5.

Table II

		K^{39} and Ca^{39} levels		
		$3/2^+$	$1/2^+$	$5/2^+$
$K^{39} + p$	E_p	11.1	4.9	3.1
	Γ_p	1000	380	2
$Ca^{39} + n$	E_n	3.7	—	—
	Γ_n	400	—	—

6. CONCLUSIONS

1. The proposed model, based on the simple procedure (2), enables us to calculate the position of the giant resonance and the total photoabsorption cross sections, and gives a qualitative explanation of the relationships between different photodisintegration branches and the character

of the energy spectra of the photoproducts.

2. The main conclusions of the model are identical with the results of diagonalization calculations made for the magic nuclei. The simplicity of the model makes it useful in the calculation of photodisintegration of non-magic nuclei, and also for account of correlation in ground states, corrections for phonon excitations, etc.

3. Generalizing the results of the recent detailed shell calculations of nuclear photodisintegration, and taking into account the considerations and calculations given above, we can state that the collective and shell descriptions of nuclear dipole excitations are not mutually exclusive.

The collective character of the dipole excitation is successfully explained by means of a shell model that takes into account the residual interaction between nucleons in the nucleus.

The author is grateful to V. G. Neudachin, V. G. Shevchenko, and N. P. Yudin for discussions.

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Translated by J. G. Adashko
41