RADIATION OF PLASMA WAVES BY A CHARGE MOVING IN A MAGNETOACTIVE PLASMA

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The spectral distribution and the angle-energy distribution of plasma waves radiated by a charge moving in a magnetoactive plasma are investigated with spatial dispersion taken into account.

THE radiation of electromagnetic waves by a charge moving in a magnetoactive plasma has been considered by a number of authors (cf. [1,2]). As a rule, however, these authors have not taken account of spatial dispersion, that is to say, thermal motion of the electrons has been neglected in these analyses. In many cases, however, spatial dispersion can be extremely important. For instance, there is the possibility of generating new kinds of characteristic waves that do not exist in a cold plasma. Shafranov[3] has obtained an expression for the energy loss of a charge moving in a magnetoactive plasma. However, his expression applies for a cold plasma only ($v_T = 0$, v_T is the mean thermal velocity of the plasma electrons).

In the present paper, using a method different from that used earlier,[2,3] we have obtained an expression for the total energy loss of a charge moving in a magnetoactive plasma with spatial dispersion taken into account. This general expression is used to compute the energy of the plasma wave radiated by the charge. Inasmuch as this energy can be appreciable, the formulas obtained can in certain cases be important in the analysis of radio emission from the radiation belts around the earth,[4] sporadic radio emission from the sun and so on.

Starting with Maxwell's equations for the medium we obtain the following equation for the electric field:

$$-\Delta E + \mathbf{e}_\alpha \frac{\partial^2 E_\beta}{\partial x_\alpha \partial x_\beta} + \frac{1}{c^2}\mathbf{e}_\alpha \varepsilon_{\alpha\beta}\frac{\partial^2 E_\beta}{\partial t^2} = -\frac{4\pi e}{c^2}\frac{\partial}{\partial t}\mathbf{v}\delta(\mathbf{r}-\mathbf{r}_e),\qquad \mathbf{D} = \mathbf{e}_\alpha \varepsilon_{\alpha\beta} E_{\alpha\beta}. \tag{1}$$

Here, indices that appear twice indicate summation, $\mathbf{e}_\alpha$ is a unit vector along the coordinate axis denoted by α and $\mathbf{r}_e$ is the radius vector to the charge moving in a magnetic field $\mathbf{H}_0$ parallel to the z axis, i.e.,

$$\mathbf{r}_e\{r_0\cos\Omega t;\quad r_0 \sin\Omega t;\quad v_0 t\},$$
$$\mathbf{v}\{-v_1\sin\Omega t;\ v_1\cos\Omega t;\ v_0\},$$
$$v_1 = r_0\Omega,\qquad \Omega = \omega_H\sqrt{1-\beta^2},$$
$$\beta^2 = c^{-2}(v_1^2+v_0^2),\quad \omega_H = eH_0/mc. \tag{2}$$

We expand (1) in a Fourier integral:

$$\mathbf{E}(\mathbf{r}, t) = \int \mathbf{E}_{\omega,\mathbf{k}} e^{i(\mathbf{kr}-\omega t)}\,d\omega\,d\mathbf{k} + \text{c.c.} \tag{3}$$

Substituting (3) in (1) and using (2) and the relations

$$\delta(\mathbf{r}-\mathbf{r}_e) = \frac{1}{(2\pi)^3}\int e^{i(\mathbf{r}-\mathbf{r}_e)\mathbf{k}}\,d\mathbf{k} = \frac{1}{(2\pi)^3}\sum_{s=-\infty}^{\infty}\int J_s(\xi)\, e^{i[\mathbf{kr}-t(s\Omega+kv_0\cos\theta)]}\,d\mathbf{k},$$

we obtain the following equations for the Fourier components $E_{\omega\mathbf{k}\beta}$ (cf. [1]):

$$T_{\alpha\beta}E_{\omega\mathbf{k}\beta} = -\frac{ei}{2\pi^2}\sum_{s=-\infty}^{\infty} j_\alpha\frac{\delta(\omega-kv_0\cos\theta-s\Omega)}{\omega};$$
$$j_x = -v_1 i J_s'(\xi),\qquad j_y = \xi^{-1}v_1 s J_s(\xi),\qquad j_z = v_0 J_s(\xi), \tag{4}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$, $J_s(\xi)$ is the Bessel function and $\xi = kr_0\sin\theta$. The components of $T_{\alpha\beta}$ are:

$$T_{\alpha\beta} = n^2(\varkappa_\alpha\varkappa_\beta - \delta_{\alpha\beta} + \varepsilon_{\alpha\beta},$$
$$n^2 = k^2c^2/\omega^2,\qquad \varkappa_\alpha = k_\alpha/k. \tag{5}$$

Using the symmetry of the problem we take $\kappa_x = 0$, $\kappa_y = \sin\theta$, $\kappa_z = \cos\theta$, i.e., the vector $\mathbf{k}$ is assumed to lie in the yz plane.

The solution of (4) is elementary:

$$E_{\omega\mathbf{k}\beta} = -\frac{ei}{2\pi^2\omega}T_{\beta\alpha}^{-1}j_\alpha\delta(\omega-kv_0\cos\theta-s\Omega); \tag{6}$$

where $T_{\beta\alpha}^{-1}$ is the reciprocal of $T_{\alpha\beta}$.

The effect of spatial dispersion on the dielectric tensor of a magnetoactive plasma $\epsilon_{\alpha\beta}(\omega, \mathbf{k})$ has been considered by a number of authors (cf. [3,5,6]). Hence we shall not stop here to give the values of

$\epsilon_{\alpha\beta}(\omega, \mathbf{k})$, which are taken from the cited papers,* but immediately write an expression for the components of $T^{-1}_{\alpha\beta}$ [cf. (5)]:

$$T^{-1}_{xx} = \Delta^{-1}\{-n^2(\varepsilon_1 \sin^2\theta + \varepsilon_3 \cos^2\theta) + \varepsilon_1\varepsilon_3 - \varepsilon_5^2\},$$
$$T^{-1}_{xy} = \Delta^{-1}\{i\varepsilon_2(-n^2\sin^2\theta + \varepsilon_3) + \varepsilon_4(n^2\sin\theta\cos\theta - i\varepsilon_5)\} = T^{-1*}_{yx},$$
$$T^{-1}_{xz} = \Delta^{-1}\{-i\varepsilon_2(n^2\sin\theta\cos\theta + i\varepsilon_5) - \varepsilon_4(-n^2\cos^2\theta + \varepsilon_1) = T^{-1*}_{zx}$$
$$T^{-1}_{yy} = \Delta^{-1}\{(-n^2 + \varepsilon_0)(-n^2\sin^2\theta + \varepsilon_3) - \varepsilon_4^2\},$$
$$T^{-1}_{yz} = -\Delta^{-1}\{(-n^2 + \varepsilon_0)(n^2\sin\theta\cos\theta + i\varepsilon_5) - i\varepsilon_4\varepsilon_2\} = T^{-1*}_{zy},$$
$$T^{-1}_{zz} = \Delta^{-1}\{(-n^2 + \varepsilon_0)(-n^2\cos^2\theta + \varepsilon_1) - \varepsilon_2^2\}. \quad (7)$$

Here

$$\varepsilon_{xx} = \varepsilon_0, \quad \varepsilon_{yy} = \varepsilon_1, \quad \varepsilon_{xy} = -i\varepsilon_2, \quad \varepsilon_{zz} = \varepsilon_3, \quad \varepsilon_{xz} = \varepsilon_4, \quad \varepsilon_{yz} = i\varepsilon_5$$

(cf. [3,5,6]) while $\Delta = |T_{\alpha\beta}|$ is the determinant made up of the components of $T_{\alpha\beta}$. We note that the dispersion relation is obtained from the condition $\Delta(\omega, \mathbf{k}) = 0$.

Now, using (2), (3), and (6) it is easy to compute the energy loss of a charge moving in the plasma ($\tau = 2\pi/\Omega$):

$$A = e\int_0^{\tau} \mathbf{vE}(\mathbf{r}=\mathbf{r}_e)dt = -\frac{e^2\tau i}{2\pi^2}\sum_{s=-\infty}^{\infty}\int \frac{d\omega}{\omega} j_\alpha j^*_\beta T^{-1}_{\beta\alpha}\delta(\omega - kv_0\cos\theta - s\Omega)\,d\mathbf{k} + \text{c.c.} \quad (8)$$

In what follows we shall not be interested in the total energy loss of the moving particle, but only that part associated with the radiation of weakly damped electromagnetic waves, particularly plasma waves. Thus we can limit ourselves to the case $\beta_T^2 \ll 1$, i.e., the case where the velocities of the plasma electrons are small. Then, if the following conditions are satisfied[7]:

$$(v_T k/\omega_H)^2\sin^2\theta \ll 1, \quad (\beta_T n\cos\theta)^2 \ll 1;$$
$$(1 - \omega_H^2/\omega^2)^3 \gg \beta_T^2, \quad 1 - 4\omega_H^2/\omega^2 \gg \beta_T^2, \quad (9)$$

the left-hand side of the dispersion relation can be written in the form† (cf. [7])

$$\Delta = |T_{\alpha\beta}| = \frac{1}{1-u}\{\beta_T^2 VRn^6 - [1 - u - V + uV\cos^2\theta]\,n^4 + [2(1-V)^2 + uV\cos^2\theta - u(2-V)]n^2 + (1-V)[u - (1-V)^2]\}; \quad (10)$$

where

$$u = \frac{\omega_H^2}{\omega^2}, \quad V = \frac{\omega_0^2}{\omega^2} = \frac{4\pi e^2 N}{m\omega^2},$$
$$R = \frac{3\sin^4\theta}{1-4u} + \left[1 + \frac{5-u}{(1-u)^2}\right]\sin^2\theta\cos^2\theta + 3(1-u)\cos^4\theta$$

(N is the number density of the plasma electrons). We need retain only terms with β_T^0 in the numerators of the expressions in (7) so that

$$\varepsilon_0 = \varepsilon_1 = 1 - V/(1-u), \quad \varepsilon_2 = -V\sqrt{u}/(1-u),$$
$$\varepsilon_3 = 1 - V, \quad \varepsilon_4 = \varepsilon_5 = 0. \quad (11)$$

The equation $\Delta(\omega, \mathbf{k}) = 0$ is cubic in n^2 indicating the possibility that three characteristic waves can be radiated. To calculate the radiated energy associated with each wave, we substitute the values of $T^{-1}_{\alpha\beta}$ [cf. (7)] in (8), taking account of (10) and (11) and carrying out the appropriate integration. The difference between the present case and the case of an electron moving in a cold plasma appears at values of the variables ω and θ for which $n^2 \gg 1$.

A particularly simple result is obtained if the following condition is satisfied[7]:

$$F = 1 - u - V + uV\cos^2\theta \gg \beta_T. \quad (12)$$

Taking account of (9) and (12) we can write (10) in the form

$$\Delta = RV\beta_T^2(n^2 - F/VR\beta_T^2)(n^4 + Bn^2 + C)/(1-u),$$
$$B = -F^{-1}[2(1-V)^2 + uV\cos^2\theta - u(1-V)], \quad (13)$$
$$C = -F^{-1}(1-V)[u - (1-V)^2].$$

The second bracket in (13) corresponds to the radiation of ordinary and extraordinary waves in a cold plasma. The first bracket, however, which is important only when $n^2 \gg 1$, corresponds to the radiation of a longitudinal wave. The electric field of this wave forms a very small angle with the vector $\mathbf{k}$.

Using (7), (11) and (13) we write (8) in the form ($\tilde{T}^{-1}_{\beta\alpha} = T^{-1}_{\beta\alpha}\Delta$):*

$$A = -\frac{e^2\tau i}{2\pi^2\beta_T^2}\sum_{s=-\infty}^{\infty}\int \frac{\delta(\omega - kv_0\cos\theta - s\Omega)\,j_\alpha j^*_\beta \tilde{T}^{-1}_{\beta\alpha}(1-u)\,d\omega d\mathbf{k}}{\omega RV(n^2 - F/RV\beta_T^2)(n^4 + Bn^2 + C)} + \text{c.c.} = A_{1,2} + A_3. \quad (14)$$

Here, the term A_3 is associated with the longitudinal wave

$$n^2 = F/RV\beta_T^2 = n_3^2, \quad (15)$$

while $A_{1,2}$ correspond to ordinary and extraordinary waves ($n^2 = n_{1,2}^2$). For example, the Cerenkov

*Everywhere below it is assumed that the plasma electrons have a Maxwellian distribution in momentum space in the zeroth approximation. Then, for example, $v_T^2 = T/m$ when $\beta_T^2 = v_T^2/c^2 \ll 1$.

†Equation (10) is easily obtained by expanding in powers of $\varepsilon_{\alpha\beta}$ in (7) [5,6] keeping terms of order β_T^2 where necessary (cf. also [8]).

*We note that the integration in (14) extends over values of ω and θ that satisfy (9) and (12).

losses due to radiation of the ordinary and extraordinary waves by a charge moving along the magnetic field $W_{1,2}$ can be obtained from (7), (11) and (13):

$$W_{1,2} = -A_{1,2}$$
$$= -\frac{e^2\tau v_0^2 i}{2\pi^2}\int \frac{\delta(\omega - kv_0\cos\theta)\,[n^4\cos^2\theta - n^2\varepsilon_1(1+\cos^2\theta) + \varepsilon_1^2 - \varepsilon_2^2]\,d\omega dk}{\omega(\varepsilon_1\sin^2\theta + \varepsilon_3\cos^2\theta)(n^2 - n_1^2)(n^2 - n_2^2)}$$
$$+ \text{c.c.},$$

where ϵ_1, ϵ_2, and ϵ_3 are determined by (11). Our expression for $W_{1,2}$ agrees with the corresponding expression given by Sitenko and Kolomenskiĭ[1].

Nothing new is learned from studying the term $W_{1,2} = -A_{1,2}$ [cf. (14)], which is essentially the radiation in a cold plasma (cf. [2]). For this reason, we discuss below only the plasma radiation (A_3). Because $n_3^2 \gg 1$ the expression for A_3 can be simplified considerably. Keeping only the highest power of n in $\tilde{T}^{-1}_{\alpha\beta}$ we have

$$\tilde{T}^{-1}_{yy} \approx n^4\sin^2\theta, \qquad \tilde{T}^{-1}_{yz} \approx n^4\sin\theta\cos\theta, \qquad \tilde{T}^{-1}_{zz} \approx n^4\cos^2\theta. \tag{16}$$

The other components of $\tilde{T}^{-1}_{\alpha\beta}$ can be neglected. Thus

$$G = j_\alpha j_\beta^* \tilde{T}^{-1}_{\beta\alpha} = n_3^4\Big(\frac{v_{1s}}{\xi}\sin\theta + v_0\cos\theta\Big)^2 J_s^2(\xi).$$

Since (14) contains a δ-function of argument $y = \omega - kv_0\cos\theta - s\Omega$, we have $G(y=0) = c^2 n_3^2 J_s^2(\xi)$. Thus, the final expression is ($x = \cos\theta$)

$$A_3 = \sum_{s=-\infty}^{\infty} A_3^{(s)} = -\frac{e^2\tau}{c\beta_T^2}\sum_{s=-\infty}^{\infty}\int \frac{J_s^2(\xi)\,|\omega^2 - \omega_H^2|\,dx}{Vn_3\,|R\,d[\omega(1-\beta_0 n_3 x)]/d\omega|}$$
$$= -\frac{e^2\tau}{v_0\omega_0^2\beta_T^2}\sum_{s=-\infty}^{\infty}\int \frac{J_s^2(\xi)\,|\omega^2 - \omega_H^2|\,\omega d\omega}{n_3\,|R\,d(n_3 x)/dx|}. \tag{17}$$

In order to go over from (14) to (17) it is necessary to carry out the integration over k, using the pole of the integrand at $n^2 = n_3^2$. The integration over $d\omega$ or dx is carried out with the δ-function $\delta(\omega - kv_0\cos\theta - s\Omega)$, i.e.,

$$\omega(1 - \beta_0 n_3\cos\theta) = s\Omega, \qquad \omega > 0, \qquad \beta_0 = v_0/c. \tag{18}$$

In view of the above the limits of integration in (17) are determined from the requirement that the function $x(\omega)$ or $\omega(x)$, given by (18), must be real.

We now consider (17) for a number of particular cases. The Cerenkov radiation corresponds to the $s = 0$ term in (17):

$$A_3^{(0)} = -\frac{e^2\tau}{v_0\beta_T^2\omega_0^2}\int_{\beta_0 n_3 \geqslant 1} \frac{|\omega^2 - \omega_H^2|\,J_0^2(\xi)\,\omega d\omega}{n_3\,|R\,d(n_3 x)/dx|}. \tag{19}$$

For Cerenkov radiation of the plasma wave, i.e., for $v_1 = 0$, (19) becomes

$$A_3^{(0)} = -\frac{e^2\tau}{v_0\beta_T^2\omega_0^2}\int_{\beta_0 n_3 \geqslant 1} \frac{|\omega^2 - \omega_H^2|\,\omega d\omega}{n_3\,|R\,d(n_3 x)/dx|}. \tag{20}$$

The value of $\cos\theta = x$ in the integrals (19) and (20) is determined from the equation $1 - \beta_0 n_3(\omega, x)x = 0$. The integration in (2) can be carried out easily if the charge moves in an isotropic medium ($\omega_H = 0$, $n_3^2 = (\omega^2 - \omega_0^2)/3\beta_T^2\omega_0^2$, $R = 3$). Using some simple transformations we have

$$A_3^{(0)} = -\frac{e^2\tau}{v_0}\int_{\omega^2=\bar{\omega}^2}^{\omega_m^2} \frac{d\omega\omega^3}{\omega^2 - \omega_0^2} = -\frac{e^2\tau}{v_0}\Big\{\frac{1}{2}[\omega_m^2 - \bar{\omega}^2]$$
$$+ \omega_0^2\ln\frac{k_m v_0}{\omega_m}\Big\}, \qquad \bar{\omega}^2 = \omega_0^2(1 + 3v_T^2/v_0^2), \tag{21}$$

where, in the usual way, the frequency ω_m is determined from the condition of applicability of macroscopic electrodynamics (cf. [3]), $k_m = \omega_m n_3(\omega_m)/c$. The lower limit of integration in (21) corresponds to the Cerenkov radiation threshold $\beta_0^2 n_3^2 = 1$. In the present case $\beta_T^2 \ll 1$ and $\omega_m - \omega_0 \ll \omega_0$. Hence, the radiated energy associated with Cerenkov excitation of the plasma wave $W_3^{(0)} = -A_3^{(0)}$ is characterized by the quantity

$$W_3^{(0)} \approx \frac{e^2\tau\omega_0^2}{v_0}\ln\frac{k_m v_0}{\omega_0},$$

which, as expected, coincides with the polarization losses in a cold plasma.

When $\xi \ll 1$, i.e., oscillatory motion, the summation in (17) must contain the $s = \pm 1$ terms in addition to (20). As a result we have ($\beta_1 = v_1/c$)

$$A_3^{(\pm 1)} = \frac{-p_0^2\tau}{4v_T^2 v_0\omega_0^2}\int \frac{|\omega^2 - \omega_H^2|\,n_3\sin^2\theta\,\omega^3 d\omega}{|R\,d(n_3 x)/dx|}, \qquad p_0 = er_0, \tag{22}$$

where $x = \cos\theta$ is determined from the Doppler relations (18) for $s = \pm 1$ respectively.

When the electron moves in a circle ($v_0 = 0$) the radiated energy is given by the expression [cf. (17)]

$$A_3^{(s)} = -\frac{e^2\tau}{cV\beta_T^2}\int \frac{|s^2\Omega^2 - \omega_H^2|\,J_s^2(\xi)\,dx}{n_3\,|R|}$$
$$(\omega = s\Omega,\ s = 1, 2, 3, \ldots). \tag{23}$$

The applicability of (23) is determined by (9). Strictly speaking it can only be used when the relativistic dependence of Ω on velocity ($\Omega = \omega_H\sqrt{1-\beta^2}$) is taken into account.

If the motion is of oscillatory nature, i.e., $\xi = \beta_1 n_3\sin\theta \ll 1$, the basic contribution to the radiation comes from the $s = 1$ term [cf. (23)] and we have

$$A_3^{(1)} = -\frac{p_0^2\tau\Omega^2\,|\Omega^2 - \omega_H^2|}{4\beta_T^2 c^3 V}\int \frac{n_3\sin^2\theta\,dx}{|R|}, \qquad p_0 = er_0. \tag{24}$$

We may note that the last expression is equal to twice the energy of a plasma wave radiated by an oscillator oscillating perpendicularly to the magnetic field $\mathbf{H}_0$. If the oscillator oscillates at a frequency Ω_0 in an isotropic plasma, i.e.,

$$H_0 = 0, \quad n_3^2 = (\Omega_0^2 - \omega_0^2)/3\beta_T^2\omega_0^2, \quad 1 - \omega_0^2/\Omega_0^2 \ll 1$$

(cf. [9]), we have from (24)

$$A_3^{(1)} = -p_0^2\tau\Omega_0^4 n_3/9\beta_T^2 c^3. \tag{25}$$

Proceeding as in the case of radiation in a cold plasma[2], we can obtain from (17) a formula that applies for high harmonic numbers:

$$|s| = \omega\,|\,1 - \beta_0 n_3 \cos\theta\,|\,/\,\Omega \gg 1.$$

As a result, for example, when $\beta_1 n_3 \sin\theta\,/|\,1 - \beta_0 n_3 \sin\theta\,| \gtrsim 1$, we have

$$A_3^{|s|\gg 1} = -\frac{2^{2/3} e^2 \tau}{\pi c \Omega^{1/3} \beta_1^{2/3} \beta_T^2} \int \frac{d\omega\, dx \Phi^2(z)\,|\,\omega^2 - \omega_H^2\,|}{V(\omega n_3 \sin\theta)^{2/3} n_3\,|\,R\,|}, \tag{26}$$

where $\Phi(z)$ is the Airy function [10]:

$$\Phi(z) = \frac{1}{\sqrt{\pi}} \int_0^\infty \cos\left(\frac{t^3}{3} + zt\right) dt,$$

$$z = \left(\frac{2\omega^2}{\Omega^2 n \beta_1 \sin\theta}\right)^{1/3} [1 - \beta_0 n_3 \cos\theta - \beta_1 n_3 \sin\theta].$$

A characteristic feature of all the expressions we have obtained describing energy radiated by plasma waves is the fact that A_3 is proportional to a large quantity $1/\beta_T^2$. For this reason, in certain cases the energy radiated by the plasma wave can be quite appreciable and can even be greater than the energy associated with the transverse waves. Thus, in the case of an oscillator in an isotropic medium the energy ratio for the longitudinal wave [cf. (25)] and the transverse wave ($A_{1,2}^{(1)} = -2p_0^2\Omega_0^4\tau n_1/3c^3$, $n_1^2 = 1 - V$) is $A_3^{(1)}/A_{1,2}^{(1)} = 1/6\beta_T^3 \gg 1$, (cf. [9]). A similar relation will obviously hold for an oscillator in a magnetoactive plasma.

In conclusion we direct attention to the following situation. As we have indicated above, the Cerenkov radiation in a plasma is characterized by the quantity

$$A_3^{(0)} \approx (-e^2\tau\omega_0^2/v_0)\ln(k_m v_0/\omega_0).$$

Thus, when longitudinal waves are excited by an oscillator (with peak velocity v_0) the energy radiated by the plasma wave is appreciably greater than that due to Cerenkov radiation:

$$A_3^{(1)}/A_3^{(0)} \approx \beta^3 n_3/9\beta_T^2 \gg 1.$$

In the final analysis this situation comes about because the frequency spectrum of the excitation current that excites the Cerenkov radiation $[\mathbf{j} = \mathbf{v}\delta(\mathbf{r} - \mathbf{v}_0 t)]$ is continuous, in contrast with the oscillator, which is a monochromatic source. The Fourier component of the longitudinal field in an isotropic plasma (6) is given by $E_{\omega\mathbf{k}}^{\|} \approx 1/\epsilon_{\|}(\omega, \mathbf{k})$ [cf. [3], Eq. (6.38)] where $\epsilon_{\|}(\omega, \mathbf{k})$ is the dielectric constant for the longitudinal wave; this is a small quantity if the spatial dispersion is small:

$$\varepsilon_{\|} = 1 - \omega_0^2/\omega^2 - 3k^2 v_T^2/\omega^2, \qquad \omega - \omega_0 \ll \omega_0.$$

The order of the singularity $1/\epsilon(\omega, \mathbf{k})$ is reduced by integration over frequency. This means that the radiated energy due to the excitation of plasma oscillations by a monochromatic external force $\omega \approx \omega_0$ is greater than that due to excitation by a force with a continuous frequency spectrum.* It will be evident that the above considerations also apply to the excitation of a longitudinal wave in a magnetoactive plasma.

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*The situation here is, to some extent, similar to that which obtains when a harmonic oscillator is driven by an external force.