RELATIVISTIC CORRECTIONS TO THE MAGNETIC MOMENTS OF H^3 *AND* He^3

N. S. ZELENSKAYA and Yu. M. SHIROKOV

Moscow State University

Submitted to JETP editor July 12, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1934-1937 (December, 1961)

A general expression is obtained for the relativistic corrections to the magnetic moment of a nucleus which arise on account of non-Galilean relativistic corrections to the Hamiltonian of the nucleon-nucleon interaction. The corrections to the magnetic moments of the nuclei H^3 and He^3 are calculated numerically. It turns out that the corrections are of the right sign and order of magnitude, but do not explain more than 30 percent of the existing discrepancy between theory and experiment.

1. In nonrelativistic approximation the magnetic moment of a nucleus is determined by the intrinsic magnetic moments of the nucleons and the orbital magnetic moments of the protons. Inclusion of relativistic effects leads to corrections to the nucleon-nucleon interaction Hamiltonian of the order $(p/M)^2$. Because of gauge invariance the appearance in the Hamiltonian of terms that depend on the momentum is necessarily accompanied by the appearance of an additional interaction of the nucleons in the nucleus with the electromagnetic field, which in turn leads to corrections to the magnetic moment of the nucleus. We emphasize that the corrections in question occur only for pairs of nucleons and are absent for isolated nucleons.

The relativistic corrections to the interaction energy of two nucleons can be divided into two groups.

a) Corrections which satisfy the condition of Galilean invariance and consequently do not depend on the total momentum of the nucleons. Corrections to the spin-orbit interaction are corrections of this type.

b) Corrections which do not satisfy the requirement of Galilean invariance and therefore depend on the total momentum of the nucleons. Corrections of this type to the interaction Hamiltonian of two particles of arbitrary masses and spins have been derived by one of the writers.[1]

Both these groups of corrections to the interaction energy lead to corrections to the magnetic moments of nuclei. The relativistic corrections to the magnetic moments of nuclei are on the average of the order of 10^{-3} nuclear magneton. Unfortunately, the wave functions of nuclei of any complexity are known to very small accuracy, and as a rule in the calculation of the nonrelativistic values of the magnetic moments this leads to uncertainties much larger than the relativistic corrections. Therefore at present it makes sense to examine the relativistic corrections only for the simplest nuclei. For the deuteron there are no non-Galilean corrections.[1] Therefore in the present paper we analyze the corrections to the magnetic moments of the nuclei H^3 and He^3.

The values of the magnetic moments of H^3 and He^3 differ considerably from the sum of the magnetic moments of the constituent nucleons (cf. e. g.,[2]). In principle there can be nonrelativistic corrections to the magnetic moments of these nuclei owing to admixtures of states with nonzero orbital angular momenta. The total angular momentum of the ground state of these mirror nuclei is $J = 1/2$. Possible states are $^2S_{1/2}$, $^2P_{1/2}$, $^4P_{1/2}$, $^4D_{1/2}$. Mixings of these states, however, cannot completely remove the discrepancy between theory and experiment. In fact, an admixture of P states in amounts which do not disagree with the value of the spin-orbit coupling leaves the magnetic moments practically unchanged,[3] and Sachs and Schwinger[4] have shown that a 4-percent admixture of the $^4D_{1/2}$ state makes it possible to get agreement between theory and experiment only for the sum of the magnetic moments of H^3 and He^3. For the magnetic moments of the individual nuclei there remain discrepancies, which amount to +0.27 nuclear magnetons for H^3 and −0.27 nuclear magnetons for He^3.

The Galilean-invariant corrections to the magnetic moments of these nuclei on account of spin-orbit forces have been calculated by Berger.[5] He found that the correction to the difference of the magnetic moments on account of the spin-orbit interaction is of the order of only 10^{-4} nuclear magnetons, i.e., negligibly small.

In the present paper we calculate the non-Galilean relativistic corrections to the magnetic moments of H^3 and He^3.

2. With relativistic corrections included to order $(p/M)^2$ the Hamiltonian of the nucleon-nucleon interaction is of the form [6]

$$H = \sum_n T_n + \sum_{m>n} H_{mn} + \sum_n T'_n + \sum_{m>n} H'_{mn} + \sum_{m>n} H''_{mn}, \quad (1)$$

where T_n is the kinetic energy of the n-th nucleon, H_{mn} is the interaction energy of the m-th and n-th nucleons, T'_n is the relativistic correction to the kinetic energy of the n-th nucleon, H'_{mn} is the non-Galilean correction to the interaction energy of the m-th and n-th nucleons, and H''_{mn} is the non-Galilean correction associated with spin-orbit forces for this pair of nucleons.

For the nucleon-nucleon interaction the relativistic non-Galilean corrections are of the form[6]

$$H_{mn} = \frac{1}{8M^2}\Big\{- H_{mn}\mathbf{P}^2 + i\left(\mathbf{P}\frac{\partial H_{mn}}{\partial \mathbf{x}}\right)\left(\mathbf{P}\frac{\partial}{\partial \mathbf{p}}\right) - (\boldsymbol{\sigma}_m - \boldsymbol{\sigma}_n)\left[\mathbf{P}\frac{\partial H_{mn}}{\partial \mathbf{x}}\right] - i(\boldsymbol{\sigma}_m - \boldsymbol{\sigma}_n) H_{mn}[\mathbf{pP}] + iH_{mn}(\boldsymbol{\sigma}_m - \boldsymbol{\sigma}_n)[\mathbf{pP}] - \left(\mathbf{P}\frac{\partial H_{mn}}{\partial \mathbf{p}}\right)(\mathbf{pP}) + iP_iP_j\frac{\partial^2 H_{mn}}{\partial x_i \partial p_j}\Big\} \quad (2)^*$$

Here

$$\mathbf{P} = \mathbf{p}_m + \mathbf{p}_n, \qquad \mathbf{p} = \tfrac{1}{2}(\mathbf{p}_m - \mathbf{p}_n), \qquad \mathbf{x} = \mathbf{x}_m - \mathbf{x}_n$$

By means of the Hamiltonian (2) one can obtain the operator for the non-Galilean relativistic corrections to the magnetic moment of the nucleus. The interaction with the electromagnetic field is introduced in a gauge-invariant way [7] by the replacement:

$$\mathbf{p}_n \rightarrow \mathbf{p}_n - e_n\mathbf{A}.$$

For central forces not depending on the velocity the operator for the relativistic non-Galilean corrections to the magnetic-moment vector of the nucleus is obtained from Eq. (2) in the form

$$\Delta\boldsymbol{\mu} = \frac{1}{16M^2}\Big\{2H_{mn}(e_m + e_n)[\mathbf{n}(\mathbf{r} - \mathbf{R})]\mathbf{P} - (e_m + e_n)(\boldsymbol{\sigma}_m - \boldsymbol{\sigma}_n)\left[\mathbf{n}\left((\mathbf{r} - \mathbf{R})\frac{\partial H_{mn}}{\partial \mathbf{r}}\right) - (\mathbf{r} - \mathbf{R})\left(\mathbf{n}\frac{\partial H_{mn}}{\partial \mathbf{r}}\right)\right]\Big\} \quad (3)$$

The corrections (3) have been calculated for the $^2S_{1/2}$ state of the mirror nuclei. Harmonic-oscillator wave functions were used for the calculations. The energy H_{mn} was taken to be of the form

$$H_{mn} = (W + MP_x + BP_\sigma + \mathcal{H}P_xP_\sigma)\,V(r).$$

The calculations were made for Gaussian and Yukawa forms of the radial factor V(r), with various parameters. Two forms were used for the exchange part of the potential. The results of the calculations are shown in the table.

Form and parameters of V(r) (V_0 in Mev, a in 10^{-13} cm)	$W=M=0.5,\ B=\mathcal{H}=0$		$W=0{,}222,\ M=0{,}58,\ B=0{,}222,\ \mathcal{H}=-0.022$[9]	
	$\Delta\mu(H^3)$	$\Delta\mu(He^3)$	$\Delta\mu(H^3)$	$\Delta\mu(He^3)$
Gaussian potential				
$V_0=51.9,\ a=1.73$ [8]	0	—0,048	0.014	—0.014
$V_0=45.\ \ a=1.94$ [9]	0	—0.032	0.009	—0,009
$V_0=68.8,\ a=1.55$ [10]	0	—0,018	0,005	—0.005
Yukawa potential				
$V_0=68,\ a=1.17$ [11]	0	—0.086	0.026	—0,026
$V_0=46.48,\ a=1.184$ [12]	0	—0.056	0.017	—0.017

3. The following conclusions can be drawn from the results.

1) The non-Galilean relativistic corrections to the magnetic moments of the nuclei H^3 and He^3 depend strongly on the choice of the potential. The signs of the corrections are as required, but their size is insufficient for the complete explanation of the discrepancy between theory and experiment. The maximum size of the corrections is 0.086 nuclear magnetons, which is 30 percent of the previously mentioned discrepancy of 0.27 nuclear magnetons.

2) The relativistic non-Galilean correction considerably increases the correction for the spin-orbit interaction.

Thus the relativistic corrections cannot fully explain the discrepancy between the theoretical and experimental values of the magnetic moments of these nuclei. The remaining discrepancy is evidently to be ascribed to the effect of meson-exchange currents. Corrections of this sort are not taken into account in the phenomenological theory developed here. A rough estimate of these corrections has been made by Drell and Walecka [13]; their work shows that inclusion of meson-exchange currents leads to a decrease of the anomalous magnetic moments of the proton and the neutron. Their calculations are made for nuclei in which there is one nucleon outside an even-even core. The even-even core itself is regarded as a Fermi sphere with a definite limiting momentum and energy. Their result gives a change of the spin gyromagnetic ratio by $\Delta g_S - 0.26\ \tau_3$, where τ_3 is the isotopic spin component of the nucleus. Ac-

*$[\mathbf{pP}] = \mathbf{p}\times\mathbf{P}$, $(\mathbf{pP}) = \mathbf{p}\cdot\mathbf{P}$.

cording to this the exchange correction is −0.13 magneton for He^3 and +0.13 magneton for tritium. It must be noted that these estimates are very rough. Nevertheless, they give a direct indication that inclusion of the exchange currents will make it possible to explain the differences between the calculated magnetic moments of the mirror nuclei and the experimental values.

In fact, the total correction to the magnetic moment (relativistic correction + correction for exchange currents) is −0.23 magneton for He^3 and +0.23 magneton for H^3, which agrees very well with the observed discrepancy of ± 0.27 magneton.

[1] Yu. M. Shirokov, JETP **36**, 474 (1959), Soviet Phys. JETP **9**, 330 (1959).

[2] H. A. Bethe and Philip Morrison, Elementary Nuclear Theory, John Wiley and Sons, Inc., New York, 1956.

[3] R. Avery and R. G. Sachs, Phys. Rev. **74**, 1320 (1948).

[4] R. G. Sachs and J. Schwinger, Phys. Rev. **70**, 41 (1946).

[5] J. M. Berger, Phys. Rev. **115**, 384 (1959).

[6] Zhivopistsev, Perelomov, and Shirokov, JETP **36**, 478 (1959), Soviet Phys. JETP **9**, 333 (1959).

[7] R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951).

[8] R. A. Ferrell and W. M. Visscher, Phys. Rev. **102**, 450 (1956).

[9] A. C. Butcher and J. M. McNamee, Proc. Phys. Soc. **74**, 529 (1959).

[10] Pearlstein, Tang, and Wildermuth, Nuclear Phys. **18**, 23 (1960).

[11] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, John Wiley and Sons, Inc., New York, 1952, p. 55.

[12] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951).

[13] S. D. Drell and J. D. Walecka, Phys. Rev. **120**, 1069 (1960).

Translated by W. H. Furry
325