SOVIET PHYSICS JETP VOLUME 14, NUMBER 5 MAY, 1962

MATRIX ELEMENTS FOR BETA TRANSITIONS

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Submitted to JETP editor June 20, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1669-1672 (November, 1961)

Nuclear matrix elements for allowed β transitions of a number of strongly-deformed nuclei are calculated. The matrix elements were computed according to the independent-particle model, and residual interactions between the nucleons were then taken into account. In the independent-particle model the reduced β-decay probabilities ft differ from the experimental values by two or more orders of magnitude. Better agreement between the theoretical values of ft and those observed experimentally can be obtained by taking pair correlations into account.

ONE of the main problems in nuclear theory is the calculation of the nuclear matrix elements. The difficulty of this problem is that a many-body problem must be solved. The first step in this direction was the development of the independent-particle models (i.p.m.). These models, however, provide only a qualitative description of the different properties such as the level sequence, the spins and parities, and certain selection rules for the nuclear processes. The calculation of the matrix elements is thus far quite unreliable.

The interactions not included in the self-consistent field can be taken into account with the aid of the recently developed "configuration mixing" procedure. The corresponding calculations, however, are very cumbersome if the interaction of a large number of particles is to be taken into account. A method that is quite fruitful when applied to nuclear theory is the method of accounting for pair correlations between particles, developed by Bogolyubov.

The purpose of the present work was to calculate the matrix elements of β transitions on the basis of the i.p.m. with account of the residual pair interactions between nucleons (nn and pp interactions of nucleons with equal and opposite momentum projections on the nuclear symmetry axis). For simplicity we confine ourselves to allowed β transitions in strongly-deformed nuclei within the framework of the Nilsson model.[1]

A characteristic of β transitions is the product f(Z, E)t, which in our case has the form[2]

$$f(Z, E)\,t = \frac{D}{|\langle 1\rangle|^2 + R\,|\langle s\rangle|^2}, \qquad (1)$$

where f(Z, E) is the Fermi integral function, t the half-life of the nucleus, and D a constant that depends on the coupling constant g:

$$D = 2\pi^3\hbar^7 \ln 2/m^5c^4g^2 = 6550 \text{ sec};$$

R = 1.7 is the ratio of axial to vector interaction constants

$$|\langle 1\rangle|^2 = \sum_{\mu_f}\left|\left\langle f\left|\sum_p \tau_\pm^p\right|i\right\rangle\right|^2,$$

$$|\langle s\rangle|^2 = 4\sum_{\mu_f}\left|\left\langle f\left|\sum_p s_p\tau_\pm^p\right|i\right\rangle\right|^2. \qquad (2)$$

The last expressions represent the squares of the nuclear matrix elements of the β transitions, satisfying the Fermi and Gamow-Teller[3] selection rules, respectively. Here $\tau_\pm$ — isotopic spin operator with integral eigenvalues, **s** — usual nucleon spin operator, $|i\rangle$ and $\langle f|$ — the initial and final states of the nuclei, respectively. The summation is over all the nucleons that participate in the transition. Final summation is over the projections μ_f of the momentum of the nucleus in the final state.

In the β transitions considered by us the elements $\langle 1\rangle$ vanish because of the orthogonality of the initial and final states (there are no mirror transitions). Substituting the Nilsson wave functions[1] in the matrix element $\langle \mathbf{s}\rangle$ and going over to a system of coordinates connected with the nucleus, in analogy with [4], we obtain

$$|\langle s\rangle|^2 = 4\,|\langle I_i 1 K_i K_f - K_i | I_f K_f\rangle \sum a^f_{l\Lambda} a^i_{l\Lambda} \times (\chi_{s,\Sigma_f}, s_{K_f-K_i}\chi_{s,\Sigma_i}) + \langle I_i 1 K_i - K_f - K_i | I_f - K_f\rangle \times (-1)^{I_f - i_f} \sum a^f_{l\Lambda} a^i_{l\Lambda} (\chi_{s,-\Sigma_f}, s_{-K_f-K_i}\chi_{s,\Sigma_i})|^2 \delta_{N_i N_f}, \qquad (3)$$

where I and K — total momentum of the nucleus and its projection on the symmetry axis, $a_{l\Lambda}$ — coefficients tabulated by Nilsson,[1] $\chi_{s,\Sigma}$ — spin nucleon functions, $\Sigma = \pm 1/2$ — projection of the spin

Allowed β transitions in nuclei with odd A

Parent nucleus	$I\pi\,[Nn_Z\Lambda]$	Daughter nucleus	$I'\pi'[N'n'_Z\Lambda']$	Type of transition	Energy of state of daughter nucleus, kev	$\log_{10} ft$ (i.p.m.)	R_β	$\log_{10}[R_\beta^{-1} ft]$, theory	$\log_{10} ft$, experiment	Reference
					Type au					
$_{67}Ho^{159}$	$7/2-$ [523]	$_{66}Dy^{159}$	$5/2-$ [523]	ec	?	3.55	0.38	3.97	$\lesssim 5$	[5]
$_{64}Gd^{161}$	$5/2-$ [523]	$_{65}Tb^{161}$	$7/2-$ [523]	β^-	418	3.42	0.26	4,00	~ 4.8	[5]
$_{67}Ho^{161}$	$7/2-$ [523]	$_{66}Dy^{161}$	$5/2-$ [523]	ec	26	3.55	0.31	4.06	$\lesssim 4.5$	[5]
$_{68}Er^{163}$	$5/2-$ [523]	$_{67}Ho^{163}$	$7/2-$ [523]	ec	0	3.55	0.36	3.99	$\lesssim 5$	[5]
$_{68}Er^{165}$	$5/2-$ [523]	$_{67}Ho^{165}$	$7/2-$ [523]	ec	0	3.55	0.41	3.94	$\lesssim 5$	[5]
$_{67}Ho^{167}$	$7/2-$ [523]	$_{68}Er^{167}$	$5/2-$ [523]	β^-	700	∞	0.52	3.70	~ 4.8	[5]
$_{70}Yb^{167}$	$5/2-$ [523]	$_{69}Tm^{167}$	$7/2-$ [523]	ec	293	∞	0.39	3.96	$\lesssim 5$	[5]
$_{71}Lu^{173}$	$9/2-$ [514]	$_{70}Yb^{173}$	$7/2-$ [514]	ec	637	3.46	0.45	3.81	5	[7]
$_{70}Yb^{175}$	$7/2-$ [514]	$_{71}Lu^{175}$	$9/2-$ [514]	β^-	396	3.36	0.35	3.82	4.5	[5]
$_{74}W^{179}$	$7/2-$ [514]	$_{73}Ta^{179}$	$9/2-$ [514]	ec	30	3.46	0.12	4.38	~ 4.6	[8]
$_{94}Pu^{235}$	$5/2+$ [633]	$_{93}Np^{235}$	$7/2+$ [633]	ec	?	3.56	~ 0.03	~ 5.00	$\lesssim 5$	[9]
					Type ah					
$_{62}Sm^{155}$	$3/2-$ [521]	$_{63}Eu^{155}$	$5/2-$ [523]	β^-	105	∞	0.10	5.38	5.7	[10]
$_{66}Dy^{157}$	$3/2-$ [521]	$_{65}Tb^{157}$	$5/2-$ [532]	ec	327	∞	0.42	4.92	?	[11]
$_{64}Gd^{159}$	$3/2-$ [521]	$_{65}Tb^{159}$	$5/2-$ [532]	β^-	364	∞	0.08	5.48	6,4—6,7	[5,12]
$_{68}Er^{161}$	$3/2-$ [521]	$_{67}Ho^{161}$	$5/2-$ [532]	ec	826	∞	0.46	4.90	5.4—5.7	[13,14]
$_{68}Er^{171}$	$5/2-$ [512]	$_{69}Tm^{171}$	$7/2-$ [523]	β^-	425	∞	0.15	5.20	6.3	[5,15]
$_{89}Ac^{231}$	$3/2+$ [651]	$_{90}Th^{231}$	$5/2+$ [633]	β^-	0	4.78	~ 0.40	~ 5.20	6.6	[9]
$_{90}Th^{231}$	$5/2+$ [633]	$_{91}Pa^{231}$	$3/2+$ [651]	β^-	166	∞	~ 0.15	~ 5.60	5.8	[9]
$_{93}Np^{239}$	$5/2+$ [642]	$_{94}Pu^{239}$	$7/2+$ [624]	β^-	512	5.00	0.07	6.15	6.8	[9]
$_{94}Pu^{243}$	$7/2+$ [624]	$_{95}Am^{243}$	$5/2+$ [642]	β^-	84	∞	0.24	5.62	5.9—6.2	[5,9,12]
$_{90}Th^{231}$	$5/2+$ [633]	$_{91}Pa^{231}$	$5/2+$ [642]	β^-	84	4.00	~ 0.30	~ 4.50	5.7	[9]
$_{94}Pu^{243}$	$7/2+$ [624]	$_{95}Am^{243}$	$7/2+$ [633]	β^-	465	4.10	0.62	4.31	5.5—6.0	[5,9]
$_{93}Np^{239}$	$5/2+$ [642]	$_{94}Pu^{239}$	$5/2+$ [622]	β^-	286	4.71	0.12	5.63	6.9—7.0	[5,9]

on the symmetry axis of the nucleus, N — principal quantum number; the summation in (3) is over l, Λ, Σ_i, and Σ_f.

The calculations performed have shown that the matrix elements depend rather weakly on the deformation. The calculated values of $\log_{10} ft$ are listed in the seventh column of the table.

According to the classification of Mottelson and Nilsson [5] all the transitions are divided into type au transitions (allowed unhindered) and type ah (allowed hindered). The latter are forbidden in the asymptotic quantum numbers N, n_Z, Λ. We note also that in the Nilsson model the β transitions with $\Delta N = 2$ are forbidden.

The effect of pair correlations on the β decay was considered in many papers, for example, those by Solov'ev. [6] In terms of the i.p.m. the residual interactions lead to "smearing" of the Fermi energy surface (E_F). The levels (n) with energy $E_n > E_F$ are partially filled (upon inclusion of the pair correlations) with probability $v_n^2 \le 1$; the levels with energies $E_n < E_F$ are free, with probability $u_n^2 \ge 0$. It is obvious that the residual pair interactions will hinder the transitions that are allowed in the i.p.m. (the statistical weights of the states participating in the transitions will decrease). On the other hand, the β transitions to the filled levels with $E_n < E_F$, which are forbidden in the i.p.m. (by the Pauli principle) turn out to be allowed in the presence of pair correlations.

The correction factors R_β to the β-transition probabilities, necessitated by the smearing of the Fermi surface, have the following form [6]:

1) β decay of the type

$$R_\beta = [u_{n_i}^{(2n_N)} u_{n_f}^{(2n_Z)}]^2 \prod_{n \neq n_i} (u_n^{(2n_N)} u_n^{(2n_N+1)} + v_n^{(2n_N)} v_n^{(2n_N+1)})^2 \times \prod_{n' \neq n_f} (u_{n'}^{(2n_Z)} u_{n'}^{(2n_Z+1)} + v_{n'}^{(2n_Z)} v_{n'}^{(2n_Z+1)})^2, \tag{4}$$

where n_i and n_f are respectively the initial and final states of the odd neutron and odd proton. Here and throughout $2n_N + 1$, $2n_N$ and $2n_N - 1$ denote the number of the neutrons, and $2n_Z + 1$ etc denote the number of the protons.

2) β decay of the type

$$R_\beta = [v_{n_i}^{(2n_Z)} v_{n_f}^{(2n_N)}]^2 \prod_{n \neq n_i} (u_n^{(2n_Z-1)} u_n^{(2n_Z)} + v_n^{(2n_Z-1)} v_n^{(2n_Z)})^2 \times \prod_{n' \neq n_f} (u_{n'}^{(2n_N-1)} u_{n'}^{(2n_N)} + v_{n'}^{(2n_N-1)} v_{n'}^{(2n_N)})^2. \tag{5}$$

In the case of e-capture the corrections have a similar form. The calculated corrections R_β are listed in the eighth column of the table.

All the foregoing β transition can be divided into transitions with change in the number of paired neutrons (protons) in the shell, and transitions with decay of the odd nucleon. In the i.p.m. there is no difference between these groups of transitions, but the observed values of ft are somewhat greater for the first group than for the second. This difference is understandable from the point of view of pair correlations, and the calculated corrections R_β reflect this fact.

As can be seen from the table, the theoretical values of ft systematically approach the experimental ones. On the one hand, it is surprising that such an improvement is obtained (the difference $\log_{10}(ft)_{exp} - \log_{10}(ft)_{theor}$ has an average value 0.8), since the calculations are based on very crude premises: the Nilsson model and the inclusion, from among all the residual interactions, of only the pair correlations of nucleons with equal and opposite momentum projections. It must be noted that the characteristics of the "smearing" of the Fermi surface u_n and v_n have been obtained from spectrum calculations,[6] i.e., no additional parameters were involved in the calculation of $(ft)_{theor}$. On the other hand, the calculations did not fulfill the hopes of the greatest optimists, since the calculated values of ft are on the average seven times smaller than the experimentally observed ones. This should serve as a stimulus for further research in this direction.

In conclusion, the authors are grateful to Ya. A. Smorodinskii and V. G. Solov'ev who called their attention to this problem.

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Translated by J. G. Adashko
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