EFFECT OF THE PION MASS DIFFERENCE ON THE $K_{\pi 3}$ DECAY PROBABILITIES

V. B. MANDEL'TSVEĬG and V. V. SOLOV'EV

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We calculate the effect of the π-meson mass difference on the relations between the probabilities for the different $K_{\pi 3}$ decays. It is found that when the $\pi^{\pm}$ is replaced by the π^0, the phase volume is increased by about 12%. The ratios of the statistical weights are given for these decays.

It is known[1-4] that the four τ decays of the K mesons*

$$K^+ \to \pi^+ + \pi^- + \pi^+ \quad (\tau_0),$$
$$K_2^0 \to \pi^0 + \pi^- + \pi^+ \quad (\tau_1),$$
$$K^+ \to \pi^0 + \pi^0 + \pi^+ \quad (\tau_2),$$
$$K_2^0 \to \pi^0 + \pi^0 + \pi^0 \quad (\tau_3)$$

satisfy the isotopic-spin relations

$$W(\tau_2)/W(\tau_0) = \tfrac{1}{4}, \qquad (1)$$

$$W(\tau_3)/W(\tau_1) = \tfrac{3}{2}, \qquad (2)$$

$$W(K_2^0 \to 3\pi)/W(K^+ \to 3\pi) = 1. \qquad (3)$$

Relations (1) and (2) are easily obtained if one assumes that the π mesons produced are in the pure T = 1 state. For this it is sufficient to assume the selection rule $\Delta T \leq 3/2$ (we assume the π-meson state to be completely symmetric, since the energies liberated in the τ decays are small). As for (3), it is a direct consequence of the $\Delta T = 1/2$ rule; therefore an experimental measurement of this ratio is a verification of this rule.

The $\Delta T = 1/2$ rule is not exact, as follows, for instance, from the existence of the $K^+ \to \pi^0 + \pi^+$ decay, which this rule would forbid. Nevertheless, we shall consider the relations obtained with its aid strictly correct, and shall calculate the effect of the mass difference between the $\pi^{\pm}$ and the π^0 only on the volume in phase space. Because of the low energies liberated in the $K_{\pi 3}$ decays, this will have a relatively large effect.

The matrix element for a $K_{\pi 3}$ decay is

$$M_\tau = f_\tau \varphi_K \varphi_1 \varphi_2 \varphi_3,$$

where φ_K is the K-meson wave function, φ_1, φ_2, and φ_3 are the meson wave functions, and f_τ is a function depending on the energies of the π's and including the effect of isotopic spin on the branching ratios for the possible reactions. (Since the energy liberated is small, one may consider f_τ a constant.) For the decay probability we obtain

$$W(\tau) = \frac{f_\tau^2}{(2\pi)^5 2m_K} \int \frac{dk_1\, dk_2\, dk_3}{2E_1 2E_2 2E_3} \delta^4(k_1 + k_2 + k_3 - k).$$

By simple algebraic operations, this expression can be transformed to the form

$$W(\tau) = \frac{f_\tau^2}{64\pi^3} \frac{\sqrt{\beta}\, t_m^2}{m_K} \int_0^1 \sqrt{y(1-y)} \left\{\frac{1+\alpha y}{1-\beta\alpha y}\right\}^{1/2};$$

$$\beta = \left(\frac{m_K + \mu}{m_K - \mu}\right)^2 - 1, \quad \alpha = \frac{t_m}{2\mu},$$

$$t_m = (m_K + 2m - \mu)(m_K - 2m - \mu)/2m_K,$$

where t_m is the maximum kinetic energy of a π meson (μ is the mass of the unpaired meson, while m is the mass of the paired mesons). The integral can be calculated by expanding $[(1+\alpha y)/(1-\beta\alpha y)]^{1/2}$ as a power series in α (note that $\alpha \approx 1/6$). We have carried this expansion out to terms in α^3. (Carrying it out to terms in α^4 would give a correction in the fourth decimal place.)

The final form for the probability then becomes

$$W(\tau) = \frac{f_\tau^2}{512\pi^2} \frac{\sqrt{4\gamma - 1}}{m_K} t_m^2 \{1 + \gamma\alpha + \tfrac{5}{8}\gamma(3\gamma - 1)\alpha^2 + \tfrac{7}{16}(10\gamma^2 - 6\gamma + 1)\alpha^3\},$$

$$\gamma = \tfrac{1}{4}(\beta + 1).$$

The following values were used for the masses[5] (given in Mev):

$$m_{\pi^\pm} = 139.59 \pm 0.05, \quad m_{K^+} = 493.9 \pm 0.2,$$

$$m_{\pi^0} = 135.00 \pm 0.05, \quad m_{K^0} = 497.8 \pm 0.6.$$

The uncertainties in the masses will lead to errors no greater than 0.002 in the ratios of the volumes in phase space. We give below the results of the calculations for $W(\tau_i)/W(\tau_k)$ (the numbers are the ratio of the statistical weight of the reaction designated in the column to the statistical weight of the reaction designated in the row):

*The τ_0 and τ_2 decays are often denoted by τ and τ'.

	τ_0	τ_1	τ_2	τ_3
τ_0	1	0.822	0.803	0.675
τ_1	1.217	1	0.977	0.822
τ_2	1.245	1.024	1	0.841
τ_3	1.481	1.217	1.189	1

It is easily seen that replacing a $\pi^{\pm}$ by the π^0 meson increases the volume in phase space by about 12%. Thus corrections for the mass differences change the decay ratios for $K_{\pi 3}$ decays in the following way:

$$W(\tau_2)/W(\tau_0) = \frac{1}{4} \rightarrow 0.311^*,$$

$$W(\tau_1)/W(\tau_3) = \frac{2}{3} \rightarrow 0.547,$$

$$W(K_2^0 \rightarrow 3\pi)/W(K^+ \rightarrow 3\pi) = 1 \rightarrow 1.311.$$

The relations between the decay probabilities were calculated also for the case in which the energies do not appear in the denominator of the integrand (noninvariant volume in phase space), i.e., for

$$W_\tau = g_\tau \int d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 \delta^4 (k_1 + k_2 + k_3 - k).$$

The calculations are carried through similarly as above, and the results obtained are very close to those for the invariant volume in phase space (as might have been expected in view of the low energy liberated). For this case one obtains

$$\rho(\tau_0) : \rho(\tau_1) : \rho(\tau_2) : \rho(\tau_3) = 1 : 1.248 : 1.240 : 1.526,$$

where ρ is the statistical weight.

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*Dalitz' calculation of this ratio[2] gave 0.325.

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268