WHICH IS RESPONSIBLE FOR THE DESTRUCTION OF SUPERFLUIDITY, v_s *OR* $v_s - v_n$

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Submitted to JETP editor June 10, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 1443-1448 (November, 1961).

The critical velocities for the flow of the superfluid component alone, within a capillary 0.385 cm in diameter, and for movement in opposite directions of the superfluid and normal components, have been determined from the attenuation of second sound. It is shown that the determining factor in the destruction of superfluidity is the motion of the superfluid component relative to the wall.

THE concept of a critical velocity in flowing helium II was originally introduced by Kapitza, on the basis of his experiments in 1941.[1] Since that time, many experiments have been designed for the observation of critical phenomena in helium. Up to the present, however, it has remained unclear whether the destruction of superfluidity in helium II is governed by its internal properties—i.e., the relative motion of the superfluid (v_s) and normal (v_n) components—or by the movement of the superfluid component relative to the walls. The experiments of Hung, Hunt, and Winkel[2] with a slit of width $d \sim 1\,\mu$ indicate that the critical quantity is the velocity v_s relative to the walls. However, a number of authors, such as Vinen[3] and Kramers,[4] point out that for wide channels the question of whether $v_s - v_n$ or v_s is the critical factor is not clear, since the destruction of superfluidity in wide channels can take place in a different manner than in narrow ones. No specific experiments have been performed in this field, a fact explained by the difficulty of measuring the temperature differences prevailing under slightly supercritical conditions (10^{-7} to 10^{-8} °K).

The experiment described below was proposed as a means of resolving the question of whether the destruction of superfluidity arises from the interaction of the superfluid current with the walls, or from an internal interaction between the normal and superfluid currents. Critical velocities were observed both for motion of the superfluid component alone and for the same superfluid component velocity with the normal component flowing in the opposite direction at a velocity several times higher (the counterflow case).

For the measurements we made use of a second-sound attenuation method similar to that of Vinen.[5] The apparatus in which the measurements were carried out is illustrated in Fig. 1. The entire system is of glass. Critical velocities were observed in the flow of helium within a tube 1 of diameter 0.385 ± 0.005 cm and 10 cm long. The superfluid and normal components were set into motion in opposite directions with the aid of a heater 2 having the form of a flat spiral with wide openings, made of 30 μ diameter constantan. The tube 1 was separated from the upper portion of the apparatus by means of a silk screen with openings of ~ 50 μ, above which was a densely-packed column of rouge 3, consisting of particles of Fe_2O_3 a few microns in size.

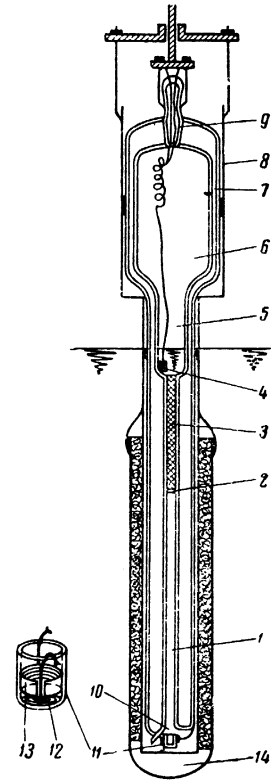

FIG. 1. Apparatus in which the measurements were performed. Scale 1 : 3. Cap shown in enlarged view.

Motion of the superfluid component alone was produced by means of the thermo-mechanical effect through the rouge, using the heater 4. The superfluid component velocity v_s was determined to an accuracy of 3% from measurements of the rise in the liquid level in the tubes 5 and 6, using a cathetometer and a stopwatch. To insure adiabatic conditions, the apparatus was enclosed in a vacuum jacket 7, and was surrounded by a copper screen 8. The plug 9, which can be lowered or raised, controlled the thermal contact between the helium within the apparatus and the external helium bath. The tube 1 and the cap 11, separated from the former by a broad ($h \approx 1.2$ mm) gap 10 formed a second sound resonator. The cap had a length $h' \approx 4.5$ mm. To insure maximum Q for the resonator, its length was determined from the condition

$$\lambda/4 = h' + h/2.$$

The second sound wavelength is $\lambda \approx 2.3$ cm; the width h of the gap was so chosen that the area of the gap was somewhat greater than the cross-section of the tube 1.

A second sound radiator 12 in the form of a flat spiral of $30\,\mu$ diameter constantan was situated at the bottom of the cap. The second sound receiver 13, of $40\,\mu$ diameter phosphor bronze, was cemented to a strip of thin paper and attached to the side wall of the cap, as close to the bottom as possible. The Q of the resonator was found to be of the order of 140, determined to an accuracy of 8%.

To reduce contact with the external volume, we made use of a container 14 filled with cotton. The temperature of the helium bath was automatically held constant to 10^{-5} °K. The second sound measuring apparatus was essentially the same as that used in the work of Peshkov.[6] The noise level was $\sim 0.3 \times 10^{-8}$ v, which is equivalent to reliable detection of a temperature difference of $\sim 3.5 \times 10^{-7}$ °K. The signal from the second sound receiver was applied, after amplification, to an ÉNO-1 oscilloscope.

In order to determine the critical velocity in the case of motion of the superfluid component alone, we carried out the following operations.

1. Flow of the superfluid component at velocities v_s' close to the critical value was induced in the quiescent helium above the rouge with the aid of the heater 4.

2. The heater 2, below the rouge, was turned on, producing a counter current of density W_2 which was known to be well into the supercritical range (by a factor of 7—8). Simultaneously with switching on the heater 2, a sweep was started across the oscilloscope, whose screen was of the long-persistence type. The time τ required for the second sound amplitude to fall half-way to its final value was measured on the oscilloscope. We then switched off both heaters and waited a sufficient time for the turbulence in the capillary to die down and the helium to come to rest.

The following experiment was conducted to determine the time required for the helium to come to rest. As in the preceding case, we switched on the heater 2, producing a certainly supercritical flux W_2. After equilibrium had become established, we turned off the heater; switching it on again after a time t, we determined τ as before.

The dependence of τ upon t at $T = 1.32$°K and $W = 6.5 \times 10^{-2}$ w/cm^2 is presented in Figure 2. As is evident from this figure, the quiescence time of the helium was ~ 240 sec. During the experimental runs we allowed at least 300 sec for the helium to become quiet.

After the helium had come to rest, we again switched on heater 4, with a power input corresponding to another value of v_s and repeated the whole procedure for determining τ. Data on the variation of τ with v_s for motion of the superfluid alone are presented in Fig. 3. As is evident from the graph, a sharp decrease in τ is observed at $v_s \approx 3 \times 10^{-2}$ cm/sec, indicating that the supercritical mode has set in.

To determine the critical value v_s'' for the counterflow case, we carried out the following operations.

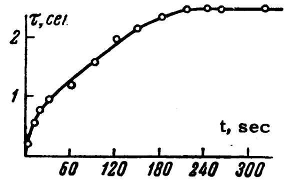

FIG. 2. Variation of τ with t. $T = 1.32$°K, $W_2 = 6.5 \times 10^{-2}$ w/cm^2.

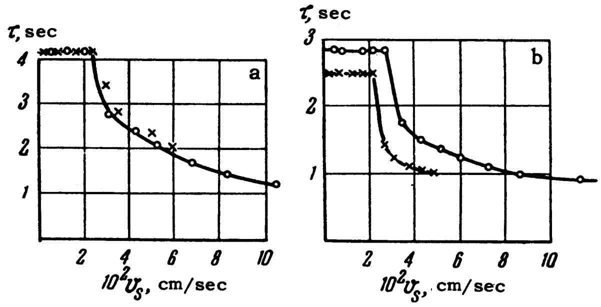

FIG. 3. Dependence of τ upon the velocity v_s: × — counterflow case, o — case of flow only of superfluid component. a — $T = 1.44$°K: for both cases $W_2 = 5.5 \times 10^{-2}$ w/cm^2, b — $T = 1,32$°K; $W_2 = 6.5 \times 10^{-2}$ w/cm^2 for counterflow and $W_2 = 5.5 \times 10^{-2}$ w/cm^2 for motion of the superfluid component alone.

1. A small thermal flux W_1, of the same order as the critical flux, was applied to the quiescent helium, using the heater 2.

2. After waiting for a time $t_2 = 60$ sec, sufficient for propagation of the turbulence along the length of the tube for supercritical values of v_s'', we applied via heater 2 a heat flux W_2 which was known to exceed the critical value. The time τ was then determined as before. Subsequently, heater 2 was turned off, the helium was allowed to come to rest (by waiting at least 300 sec), and the operation was repeated for a new value of W_1.

In the counterflow case, the mean mass current density is zero; i.e.,

$$\bar{j} = \rho_s \bar{v}_s + \rho_n \bar{v}_n = 0, \qquad \bar{v}_s = -\rho_n \bar{v}_n / \rho_s, \tag{1}$$

where ρ_s and ρ_n are the superfluid and normal component densities, respectively, and $\bar{v}_s$ and $\bar{v}_n$ are the corresponding velocities, averaged over the cross section of the tube. Since the thermal current density in superfluid helium is $W = \rho Q \bar{v}_n$, where Q is the heat content of the helium, we have, using (1)

$$W = \rho \frac{\rho_s}{\rho_n} Q \bar{v}_s = \rho_s Q (\bar{v}_n - \bar{v}_s). \tag{2}$$

In the subcritical regime $\bar{v}_s = v_s$. Knowing W_1, one can, with the aid of (2) find v_s''. The dependence of τ upon v_s'', with the latter determined in this manner, is also shown in Fig. 3. At $T = 1.44°$ K, the flux $W_2 = 5.5 \times 10^{-2}$ w/cm^2, while at $T = 1.32°$ K, $W_2 = 6.5 \times 10^{-2}$ w/cm^2. Different values for W_2 were used in the counterflow case and in the case of motion of the superfluid component alone in order to demonstrate that the values obtained for $v_{s\,cr}$ are independent of W_2.

It must be mentioned that a parasitic thermal flux from the second sound radiator and receiver, amounting to approximately half the value of the critical flux $W_{1\,cr}$, introduces an uncertainty in the numerical value of $v_{s\,cr}$ as determined from Fig. 3. The effect of this parasitic flux is small, however, since due to the laminar character of the flow, the presence of the broad gap 10 prevents the propagation of the disturbances into the body of the tube. In control experiments, we turned on the second sound receiver and radiator and then turned them off a few seconds before applying the flux W_2. This had no effect; i.e., the influence of the parasitic flux is small. Moreover, in determining the critical velocity for the case of motion of the superfluid alone, we waited for a longer time than usual (up to 150 sec) after turning heater 4 on, and then switched it off at the same time heater 2 was turned on. This did not affect the data on $v_{s\,cr}$.

Let us analyze our experiments on the basis of the hypothesis that excitations arise during the destruction of superfluidity by vortex rings. This viewpoint makes it possible to explain a whole series of experiments (Atkins[7], Peshkov and Tkachenko[8]). If one assumes that the quantity $v_s - v_n$ is the determining factor in the onset of criticality, then it is natural to consider that the vortex ring arises somewhere within the tube. We then have, according to Landau's relation

$$\varepsilon / p < |v_s - v_n|.$$

Using the relationship between ε and p

$$\frac{\varepsilon}{p} = \frac{\hbar}{mr'} \ln \frac{r'}{a}$$

for a vortex ring of unknown radius r' (Atkins[7]), and taking into account the parabolic profile of the normal component velocity, one can readily demonstrate that the formation of a vortex ring of radius $r' = R/\sqrt{3}$ is the most profitable, where R is the radius of the tube. Then

$$(v_s - v_n)_{cr} = (\varepsilon/p)_{r' = R/\sqrt{3}} \approx 3 \cdot 10^{-2} \text{ cm/sec}.$$

Experimentally, in the counterflow case at $T = 1.32°$ K, the value of $(v_s - v_n)_{cr}$ at $r = R/\sqrt{3}$ is 0.6 cm/sec. This is higher by a factor of 20 than the theoretical value, which indicates that the assumption that $v_n - v_s$ is the critical velocity, and, consequently, that the vortices are formed within the tube, is incorrect.

A second viewpoint, which makes the velocity of the superfluid component relative to the walls the critical one, appears to be the correct one. In this case the formation of vortices of maximum radius, close to that of the tube, is the most favored. There must, then be agreement between the values of the critical velocity $v_{s\,cr}$ obtained for v_s and v_n oppositely directed, and for movement of the superfluid component alone, as is, indeed, observed experimentally. As can be seen from Fig. 3, these velocities do agree, and we find $v_{s\,cr} \approx 0.3 \times 10^{-1}$ cm/sec. This is close to the value $v_{s\,cr} \approx 0.35 \times 10^{-1}$ cm/sec obtained by use of the semiempirical formula derived by Peshkov[9] on the basis of certain assumptions regarding vortex rings.

In addition to finding the critical velocity, we also determined the character of the attenuation of second sound with the superfluid component alone in motion. The smallest value for the attenuation, limited by the sensitivity of the apparatus, was found at $\bar{v}_s = 0.2$ cm/sec. The attenuation coefficient was determined from

$$\gamma = \frac{\pi\nu}{Q(0)\,u_2}\left(\frac{A_0}{A_1} - 1\right),$$

where ν is the resonance frequency ($\nu \approx 850$ cps), $Q(0)$ is the quality factor of the resonator, and A_0 and A_1 are the amplitudes before and after the onset of the attenuation. The dependence of γ upon $\overline{v}_s$ was found to be quadratic, and is illustrated in Fig. 4.

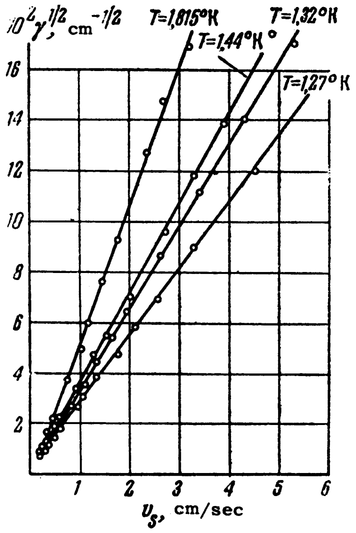

FIG. 4. Attenuation of second sound for the case of movement of superfluid component alone.

The value of γ does not change as the second sound amplitude is varied. The force $\mathbf{F}_{sn}$ developed per unit volume may be represented (Vinen,[10] Atkins[7]) in the form

$$F_{sn} = A\rho_n\rho_s\bar{v}_s^2(v_s - v_n),$$

The constant A in this formula is found from the expression $\gamma = A\rho v_s^2/2u_2$, where u_2 is the second sound velocity; it has, at various temperatures, the following values:

T, °K =	1.27	1.32	1.444	1.815
A, cm sec g^{-1} =	20	28.4	35	73

The accuracy of determination of the constant is 15–20%, due to the fact that contact between the internal resonator and the external helium bath was not completely eliminated in our system.

The existence of the force $\mathbf{F}_{sn}$, as Vinen[11] has shown, may be explained by the appearance in moving helium of well-developed turbulence. Comparison of the results of Kidder and Fairbank,[12] which cannot be explained by the existence of well-developed turbulence leading to the generation of the force $\mathbf{F}_{sn}$, with our data, indicates the presence in broad channels of a transition from "small-scale" to "large-scale" turbulence. In narrow channels (Hung, Hunt and Winkel[2]) the "small-scale" turbulence region is evidently either absent or extremely short, due to the large critical values for v_s.

The experiments which we have performed thus support the viewpoint that the destruction of superfluidity is a consequence of the formation of vortex rings of the Onsager-Feynman type. The critical velocity, moreover, appears to be v_s and not $v_s - v_n$; i.e., vortices are formed, not as a result of counterflow within the helium, but due to the interaction of the superfluid motion with the wall of the capillary.

In conclusion, the authors take the opportunity to express their gratitude to P. L. Kapitza for his unfailing interest in and attention to this work.

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Translated by S. D. Elliott
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