

*INVESTIGATION OF HIGH-ENERGY MUONS IN EXTENSIVE AIR SHOWERS*

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Results of an experimental study of the following average characteristics of the  $\mu$ -meson flux in extensive air showers at sea level are presented: the lateral distribution, the dependence of the  $\mu$ -meson flux on shower-size, the energy spectrum in the 5–10 Bev range, and the  $\mu$ -meson density spectrum.

A study of the high-energy  $\mu$ -meson flux in extensive air showers (EAS) was carried out from 1958 to 1960 using the Moscow State University array for the comprehensive study of EAS. Results were obtained concerning the irregularities in the lateral distribution of the  $\mu$  mesons and the fluctuations in the  $\mu$ -meson flux in individual showers.<sup>[1-3]</sup> Using the same array, we have obtained detailed data concerning the following average characteristics of the  $\mu$ -meson flux in EAS: the lateral distribution of the  $\mu$  mesons, the dependence of the  $\mu$ -meson flux on the shower size, the energy spectrum of  $\mu$  mesons in the 5–10 Bev range, and the density spectrum of  $\mu$  mesons. The large array has enabled us to carry out a more thorough study of the above characteristics of  $\mu$  mesons than had been possible in a number of earlier experiments.<sup>[4,5]</sup>

**METHOD AND EXPERIMENTAL SETUP**

The study of average characteristics of  $\mu$  mesons in EAS requires the measurement of three quantities: the  $\mu$ -meson density, the total number of particles  $N$  in the shower, and the distance  $R$  from the shower axis to the  $\mu$ -meson detector. In addition, the density of  $\mu$  mesons with various energies can be measured, so that data on the energy spectrum of  $\mu$  mesons can also be obtained.

The array which was used to measure these quantities was situated partly on the surface of the earth (that part of the array which served to determine the shower size and axis location), and partly underground ( $\mu$ -meson detectors) at the depths of 20 and 40 m water equivalent. The minimum  $\mu$ -meson energy necessary for penetrating to these depths amounted to 5 and 10 Bev respectively. Both underground laboratories were situated on one vertical line, below the center of the array detecting the EAS on the surface of the earth.

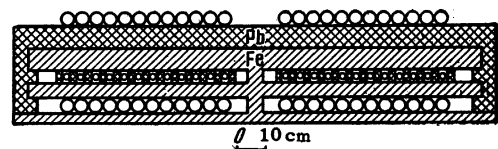


FIG. 1.  $\mu$ -meson detector.

The underground laboratories have been described in detail in<sup>[1]</sup>.

The array for the detection of EAS consists of a large number of hodoscoped Geiger-Müller counters placed in groups at points forming the geometric array described in detail in<sup>[2]</sup>. The method of shower-size determination and axis location using such an array has already been described.<sup>[6,7]</sup>

Hodoscoped Geiger-Müller counters shielded by lead and iron served as the  $\mu$ -meson detectors in the underground laboratories. At the depth of 20 m w.e., we used the detector described in<sup>[2]</sup>. At the depth of 40 m w.e., the same detector was used from 1958 to 1959, while, in 1960, the detector shown in Fig. 1 was substituted. The hodoscope used has been described by Korablev.<sup>[8]</sup>

The study of the  $\mu$ -meson flux density at different distances from the shower axis and in showers of different size made it necessary to trigger the array by different selection systems. Table I presents general data on the triggering systems of the array that were used in different variants of the experiment. The results obtained using systems 2 and 3 have been published earlier.<sup>[3]</sup>

**RESULTS**

For showers selected by means of triggering systems 1 and 2, the shower size and the axis location on the surface of the earth were determined for each shower. The detected showers were divided into the following groups with respect to

Table I.

System	Distance from the triggering system to the array center, m	Range of distances R, m	Range of N	Method of shower selection
1	0	> 20	$10^5 - 5 \cdot 10^7$	According to electron density. Sixfold coincidences, $\sigma = 0.132 \text{ m}^2$
2	25	22-27	$10^4 - 6 \cdot 10^5$	According to electron density. Sixfold coincidences, $\sigma = 0.132 \text{ m}^2$
3	100	70-130	$5 \cdot 10^4 - 10^6$	According to high-energy electrons. Core selector + double coincidence, $\sigma = 0.264 \text{ m}^2$
4	250	200-300	$2 \cdot 10^6 - 10^7$	According to electron density. Sixfold coincidence system, $\sigma = 330 \text{ cm}^2$
5	0	< 25	$10^4 - 10^5$	Single $\mu$ meson with energy > 10 Bev on an area of $4.75 \text{ m}^2$ + twofold coincidence in laboratory I <sup>[1]</sup> , $\sigma = 0.264 \text{ m}^2$ .

shower size N and the distance R from the intersection of the shower axis with the surface of the

earth to the vertical passing through the  $\mu$  meson detectors.

$\Delta N:$	$10^4 - 10^5$	$10^5 - 6 \cdot 10^5$	$2 \cdot 10^5 - 1 \cdot 10^6$	$1 \cdot 10^6 - 2 \cdot 10^6$	$2 \cdot 10^6 - 4 \cdot 10^6$	$4 \cdot 10^6 - 10^7$	$> 10^7$
$\Delta R, \text{m}:$	22-27	22-27	20-40	20-40	20-40	20-40	20-40
			60-100	60-100	60-100	60-100	60-100
						100-160	100-160

The  $\mu$ -meson density for each shower group was determined according to the formula

$$\rho = \sum_{i=1}^n m_i / ns, \tag{1}$$

where  $m_i$  is the number of discharged counters in the lower (or middle) counter tray of the detector in Fig. 1 in the  $i$ -th shower,  $n$  is the number of showers in the group, and  $s$  the total area of the counter of the lower (or middle) detector tray. Eq. (1) is correct if  $m_i \ll M$  (where  $M$  is the total number of counters of the lower or middle detector tray) a condition which was always satisfied during the experiments. The contribution of the electron-photon component in equilibrium with the  $\mu$ -meson flux to the number of discharged counters was not greater than 10% for the counters of the lower tray, and 1% for the counters of the middle tray.

The statistical errors  $\Delta\rho$  in the  $\mu$  meson flux density were calculated for  $m_i = 0, 1$  according to the formula

$$\Delta\rho = \sqrt{\sum m_i} / ns, \tag{2}$$

and for larger values of  $m_i$  according to the formula

$$\Delta\rho = \sqrt{D(\rho) / n}, \tag{3}$$

where  $D(\rho)$  is the dispersion of the distribution  $Q(\rho', \rho)$  of the values  $\rho'$  with respect to  $\rho$ . The distribution  $Q(\rho', \rho)$  has been studied in [2]. Using this distribution, and from the following

$$D(\rho) = \sum_j (\rho'_j - \rho)^2 Q(\rho'_j, \rho) \tag{4}$$

we find  $D(\rho) = 1.2\rho^2$ . Hence,

$$\Delta\rho = 1.1 \sum_{i=1}^n m_i / n^{1.5}s. \tag{3'}$$

The  $\mu$ -meson flux density given by Eq. (1) was referred to the mean arithmetic value of  $\bar{N}$  and  $\bar{R}$  of the corresponding shower group. To identical intervals  $\Delta N$  and  $\Delta R$  correspond average values of  $\bar{N}$  and  $\bar{R}$  differing by not more than 20%.

The densities of  $\mu$  mesons with energy  $E \geq 5$  and  $E \geq 10$  Bev obtained for the two distance ranges  $\Delta R$  from 20-40 m ( $\bar{R} = 25$  m) and from

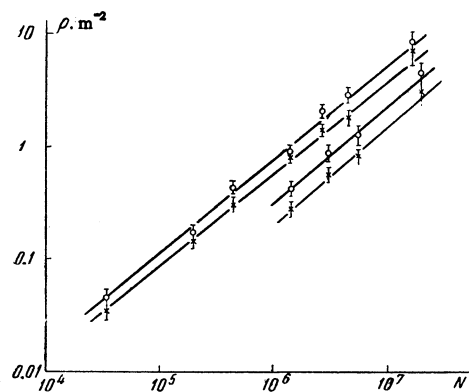


FIG. 2. Dependence of the  $\mu$ -meson flux density on the total number of particles in the shower for a given distance from the shower axis: o - for  $E \geq 5$  Bev, x - for  $E > 10$  Bev.

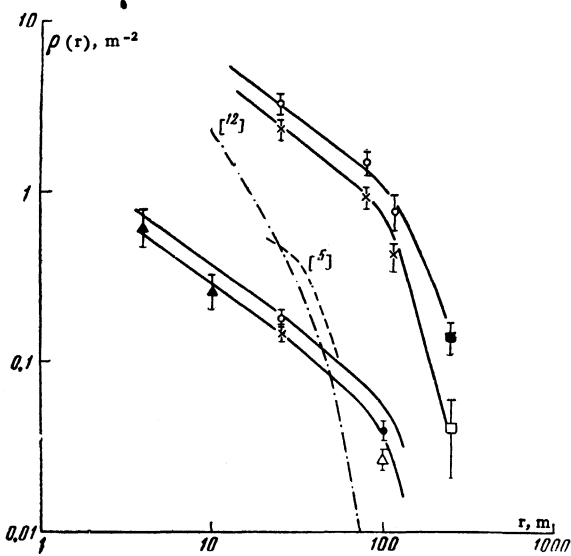


FIG. 3. Lateral distribution of  $\mu$  mesons in EAS with  $2 \times 10^5$  particles and  $6 \times 10^6$  particles.  $\circ$ ,  $\bullet$ ,  $\blacksquare$  – density of  $\mu$  mesons with  $E \geq 5$  Bev,  $\times$ ,  $\triangle$ ,  $\square$  – density of  $\mu$  mesons with  $E \geq 10$  Bev obtained with the triggering systems 1, 3, and 4 respectively,  $\blacktriangle$  – density of  $\mu$  mesons with  $E \geq 10$  Bev obtained for showers with known zenith angle.

60–100 m ( $\bar{R} = 80$  m) and for different shower sizes are shown in Fig. 2. If we take the function  $\rho(N)$  for each distance  $R$  as  $\rho(N) \sim N^\alpha$ , then we can obtain the value of  $\alpha$  for different  $E$  and  $\bar{R}$  from Fig. 2 by the least-squares method. Within the limits of experimental error, the values of  $\alpha$  were found to be the same for  $E \geq 5$  and  $E \geq 10$  Bev, and for  $\bar{R} = 25$  and 80 m and equal to  $\alpha = 0.85 \pm 0.10$ .\*

From the mean values of  $\rho$  for  $E \geq 5$  Bev and  $E \geq 10$  Bev, corresponding to the straight lines in Fig. 2, we can obtain the exponent  $\gamma$  on the integral  $\mu$ -meson spectrum in the 5–10 Bev range:  $\gamma = 0.5 \pm 0.1$  for  $\bar{R} = 25$  m, and  $\gamma = 0.67 \pm 0.15$  for  $\bar{R} = 80$  m.

The independence of  $\alpha$  from the distance  $\bar{R}$  in the range  $\Delta \bar{R} = 25$ –80 m indicates that the lateral distribution of the  $\mu$ -meson flux in that same range is independent of the shower size. From Fig. 2 we can obtain data on the function  $\rho(\bar{R})$ . The lateral distribution function of  $\mu$  mesons may, however, be different from the function  $\rho(\bar{R})$ . In fact, the spectrum  $\Phi(r, R)$  of possible distances  $r$  from the  $\mu$ -meson detector to the shower axis corresponds to a given distance  $R$  on the surface of the earth. The distance  $r$  is different for different angles  $\theta$  and  $\varphi$  of the shower axis. The  $\mu$ -meson density detected in the experiment was

$$\rho(R) = \int \Phi(r, R) f(r) dr = \bar{f}(R), \quad (5)$$

where  $f(r)$  is the  $\mu$ -meson lateral distribution function.

It can be shown that, for a zenith-angle distribution of the type  $I_0 \cos^8 \theta d \cos \theta$  and for  $f(r) \sim r^{-n}$  where  $n \leq 1$ , the values of  $\bar{f}(R)$  do not differ from the values of  $f(R)$  by more than 10% for  $R > H$ , where  $H$  is the depth of the  $\mu$ -meson detector. For an experimental distribution of  $\rho(\bar{R})$  represented by the formula  $\rho(\bar{R}) \sim \bar{R}^{-n}$ , we obtain from Fig. 2  $n \lesssim 1$ , where  $\Delta \bar{R} = 25$ –80 m. We can, therefore, assume with an accuracy of 10% that the distribution  $\rho(\bar{R})$  for  $R > 25$  m is the true lateral distribution of the  $\mu$  mesons. In order to obtain complete data on the lateral distribution function of the  $\mu$  mesons, measurements of the  $\mu$ -meson density were carried out at distances  $r < 25$  m and  $r > 100$  m.

For a small number of showers, the zenith angle of the shower axis was found in<sup>[3]</sup> from the data of the diffusion chamber placed at the surface of the earth, and the density of  $\mu$  mesons with energies  $E \geq 10$  Bev were obtained at distances  $\bar{r} = 4$  and 10 m. These data are shown in Fig. 3 for showers with  $N = 2 \times 10^5$ .

On the periphery of the shower, the  $\mu$ -meson density with  $E \geq 5$  Bev and  $E \geq 10$  Bev was obtained by means of triggering systems 3 and 4. The triggering system 3 enabled us to study  $\mu$ -meson densities at distances  $\bar{r} = 100$  m in showers with  $N = 2 \times 10^5$ , the results being shown in Fig. 3. (The reduction of the data obtained with the triggering system 3 has been described in<sup>[3]</sup>.)

The triggering system 4 enabled us to study the  $\mu$ -meson density at a distance  $\bar{r} = 250$  m in showers with  $\bar{N} = 6 \times 10^6$ . In selecting the showers using the triggering system 4, it was necessary that the density  $\rho_e$  of all charged particles on the surface of the earth above the  $\mu$ -meson detectors be within the range  $1$ – $5$   $m^{-2}$ , (which meant that 9 to 45 counters out of 264 with an area  $\sigma = 330$   $cm^2$  were discharged). The number of particles  $N$  and the distance  $r$  was not determined for individual showers. For the showers selected by the triggering system 4, the mean number of particles in a shower  $\bar{N}$  and the mean distance  $\bar{r}$  were calculated using the formulae

$$\begin{aligned} \bar{N} &= \iint N W(N, r) \rho(N, r) dN dr, \\ \bar{r} &= \iint r W(N, r) \rho(N, r) dN dr, \end{aligned} \quad (6)$$

where  $W(N, r)$  is the detection probability of a shower such that  $1 \leq \rho_e \leq 5$   $m^{-2}$  for the triggering system 4, and  $\rho(N, r)$  is the density of the  $\mu$

\*The value of  $\alpha = 0.6 \pm 0.1$  given in<sup>[3]</sup> is less reliable, since it was obtained from two points only.

mesons in a shower of size  $N$  at a distance  $r$  from the shower axis. The probability  $W(N, r)$  could be calculated knowing the lateral distribution functions of all charged particles in the shower.<sup>[9]</sup> It was assumed that  $\rho(N, r) = kN^{0.85}r^{-n}$ .

The mean values of  $\bar{N}$  and  $\bar{r}$  were calculated for various values of  $n$  ( $n = 1, 2, 3$ ). The calculation showed that the mean values of  $\bar{N}$  and  $\bar{r}$  vary little as  $n$  varies from 1 to 3, and, in particular,  $\bar{r}$  agrees to an accuracy of 15% with the distance from the triggering system 4 to the  $\mu$ -meson detectors equal to 250 m. The calculation of the contribution of showers with different  $N$  and  $r$  to the number of detected  $\mu$  mesons showed that showers with  $N$  from  $2 \times 10^6$  to  $10 \times 10^6$  and with  $r$  from 200 to 300 m contribute 70%. The density of the  $\mu$  mesons with energy  $E \geq 5$  BeV and  $\geq 10$  BeV for  $\bar{N} = 6 \times 10^6$  and  $\bar{r} = 250$  m is shown in Fig. 3.

We have thus obtained the lateral distribution of the  $\mu$  mesons with energy  $E \geq 5$  and  $\geq 10$  BeV at distances from 4 to 250 m, as shown in Fig. 3. This lateral distribution can well be described by the function

$$\rho(N, r) = kN^\alpha r^{-n} \exp(-r^2/r_0^2), \quad (7)$$

where  $n = 0.7 \pm 0.1$ ,  $\alpha = 0.85 \pm 0.1$ , and

$$k = 5.8 \cdot 10^{-5}, \quad r_0 = 195 \pm 15 \text{ for } E \geq 5 \text{ BeV}, \\ k = 4.1 \cdot 10^{-5}, \quad r_0 = 155 \pm 15 \text{ for } E \geq 10 \text{ BeV}.$$

The distributions (7) for  $N = 2 \times 10^6$  and  $N = 6 \times 10^6$  are shown in Fig. 3 by solid lines.

Using distribution (7), we shall calculate the total flux of  $\mu$  mesons with energies  $E \geq 5$  and  $\geq 10$  BeV in EAS. We have

$$N_\mu = \int_0^\infty kN^\alpha r^{-n} \exp(-r^2/r_0^2) 2\pi r dr \\ = \pi k r_0^{2-n} \Gamma(1 - n/2) N^\alpha. \quad (8)$$

For  $E \geq 5$  BeV we find  $N_\mu = 0.24 N^{0.85}$  and for  $E \geq 10$  BeV we have  $N_\mu = 0.13 N^{0.85}$ .

The calculated total number of  $\mu$  mesons\* shows that the exponent  $\gamma$  of the integral  $\mu$ -meson energy spectrum over the whole shower is equal to 1 in the 5–10 BeV energy range.

We have studied the spectrum of EAS accompanied by  $m$   $\mu$  mesons traversing a detector with a given area  $s$ . This spectrum is related to the  $\mu$ -meson density spectrum  $I(\rho)d\rho = B\rho^{-K'-1}d\rho$  in the following way:

\*The distributions  $k_1 \exp(-r/r_1)$  or  $k_2 r^{-n} \exp(-r/r_2)$ , for which  $k_{1,2}$  and  $r_{1,2}$  were so chosen as to approximate the experimental lateral distribution in the range  $r > 80$  m, give a result differing from Eq. (8) by not more than 10–15%.

$$I(m, s) = \int_{\rho_m}^\infty B\rho^{-K'-1} (\rho s)^m e^{(-\rho s)} d\rho/m! \\ = Bs^{K'} \int_{\rho_m s}^\infty x^{m-K'-1} e^{-x} dx/m!. \quad (9)$$

For  $\mu$  mesons with energy  $E \geq 10$  BeV, the spectrum  $I(m, s)$  was obtained with the triggering systems 1 and 5. The experimental values of  $I(m)$  for detector areas  $s$  equal to 3.15 and 6.3 m<sup>2</sup> respectively are shown in Fig. 4. In the figure, the  $m-1$  scale is logarithmic to facilitate comparison with Eq. (11).

For  $s = 6.3$  m<sup>2</sup> and  $m > 7$ , the spectra  $I(m)$  obtained with triggering systems 1 and 5 coincide. For the same value of  $s$  but for  $m < 7$ , the spectra are different. The spectrum  $I(m < 7)$  ob-

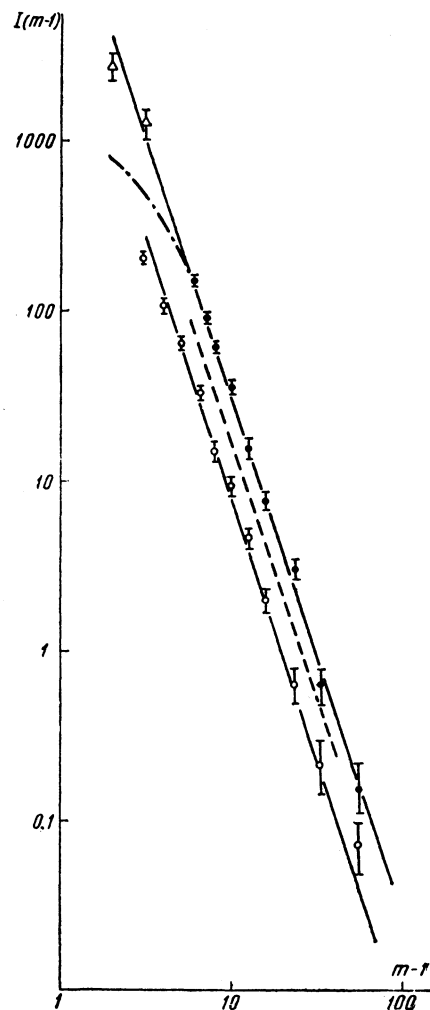


FIG. 4. Spectra  $I(m, s)$  for different  $E$  and  $s$ : o –  $E \geq 10$  BeV,  $s = 3.15$  m<sup>2</sup>, triggering system 1; ● –  $E \geq 10$  BeV,  $s = 6.3$  m<sup>2</sup>, triggering system 1; Δ –  $E \geq 10$  BeV,  $s = 6.3$  m<sup>2</sup>, triggering system 5; dots –  $E \geq 5$  BeV,  $s = 3.15$  m<sup>2</sup>; dot-dash –  $E \geq 10$  BeV,  $s = 6.3$  m<sup>2</sup>, triggering system 1. The y axis represents the number of events in 1000 hours.

Table II.

m	I(> m)		$\kappa'$	m	I(> m)		$\kappa'$
	s = 6.3 m <sup>2</sup>	s = 3.15 m <sup>2</sup>			s = 6.3 m <sup>2</sup>	s = 3.15 m <sup>2</sup>	
7	539	146	1.9±0.18	15	80	21	1.93±0.48
8	390	107	1.86±0.2	20	40	12	—
9	297	77	1.95±0.25	30	11	5	—
10	238	60	2.0±0.3	40	6	3	—
13	116	30	2.0±0.4	50	3	1	—

tained with system 1 is shown in Fig. 4 by the dashed line. The values of  $I(m < 7)$  obtained with system 5 are shown in Fig. 4 by triangles.

The dependence of the selection system on the spectrum  $I(m)$  can be described qualitatively by introducing the minimum  $\mu$ -meson density  $\rho_m$  which will still be detected by the triggering system. The triggering systems 1 and 5 have different  $\rho_m$ . However, for  $m > 7$  ( $s = 6.3 \text{ m}^2$ ), both triggering systems give identical results. This means that the contribution of densities  $\rho \sim \rho_m$  to the number of events with  $m > 7$  is not important. In such a case, we can set  $\rho_m = 0$ . Equation (9) then simplifies to

$$I(m, s) = Bs^{\kappa'}(m - \kappa' - 1)/m!,$$

$$\kappa' = \log [I_1(m, s_1)/I_2(m, s_2)] / \log (s_1/s_2). \quad (10)$$

Table II presents the integral spectra  $I(> m)$  for  $\mu$  mesons with  $E \geq 10$  Bev and for two values of  $s$ , obtained during 1000 hours of operation. In the same table, the values of  $\kappa'$  calculated according to Eq. (10) are given. The mean value of  $\kappa'$  for the range  $7 \leq m \leq 15$  is  $\kappa' = 1.95 \pm 0.1$ . Assuming  $\kappa' = 2$ , we obtain from Eq. (10)

$$I(m, s) \approx Bs^2(m - 1)^{-3}. \quad (11)$$

The straight lines of Fig. 4, with exponent equal to 3, are in good agreement with the experimental spectrum in the range  $3 \leq m \leq 50$  for  $s = 6.3 \text{ m}^2$ .

The shape of the spectrum  $I(m)$  for  $\mu$  mesons with energy  $E \geq 5$  Bev coincides, within the limits of error, with the spectrum  $I(m)$  for  $E \geq 10$  Bev. In Fig. 4, the spectrum  $I(m)$  for  $s = 3.15 \text{ m}^2$ ,  $E \geq 5$  Bev is denoted by a dashed line. Thus, the exponent of the density spectrum of  $\mu$  mesons with energy  $E \geq 10$  Bev and  $E \geq 5$  Bev has the value  $\kappa' = 1.95 \pm 0.1 \approx 2$ . The coefficient  $B$  is  $B_1 = 0.8 \text{ hour}^{-1}\text{m}^{-4}$  for  $E \geq 10$  Bev and  $B_2 = 1.3 \text{ hour}^{-1}\text{m}^{-4}$  for  $E \geq 5$  Bev.

The value of  $\kappa'$  obtained in studying the spectrum  $I(m)$  is simply related to the exponent  $\kappa$  of the size spectrum of showers and to the exponent  $\alpha$  in Eq. (7). If the size spectrum is  $F(N) dN = AN^{-\kappa-1} dN$ , and the  $\mu$ -meson density is connected with  $r$  and  $N$  by Eq. (7), then the  $\mu$ -meson density spectrum is

$$I(\rho) d\rho = B\rho^{-\kappa/\alpha-1} d\rho,$$

$$B = A\alpha^{-1}k^{\kappa/\alpha}2\pi \int_0^{\infty} r [r^n \exp(r^2/r_0^2)]^{-\kappa/\alpha} dr. \quad (12)$$

The experimentally obtained value  $\kappa' = \kappa/\alpha$  is in good agreement with the value  $\alpha = 0.85$  for  $\kappa = 1.65$ .

The factor  $A$  can be determined for  $\kappa = 1.65$  from the intensity of EAS of a given size given in<sup>[10]</sup>. We find  $A = 1.25 \times 10^6 \text{ hour}^{-1}\text{m}^{-2}$ . Substituting the values  $n$ ,  $\alpha$ , and  $k$  from (7) into Eq. (12), we find  $B_1 = 0.72 \text{ hour}^{-1}\text{m}^{-4}$ , which agrees with the experimental value of  $B_1$ .

## DISCUSSION OF RESULTS

We compare the results obtained with the data of Andronikashvili and Bibilashvili<sup>[5]</sup> and Fukui et al<sup>[11]</sup> on the  $\mu$ -meson flux with energy 5–10 Bev in EAS at sea level. Data of the Tokyo group on the lateral distribution of  $\mu$  mesons with  $E \geq 5$  Bev at distances  $r < 100$  m from the shower axis are in good agreement with the data of the present experiment, both with respect to the absolute number of the  $\mu$  mesons and with respect to the shape of the lateral distribution.

The results of Andronikashvili and Bibilashvili contradict the present data. In Fig. 3, the lateral distribution of  $\mu$  mesons from<sup>[5]</sup> is shown by a dashed line. Analytically, this distribution can be written in the form

$$\rho(r) = 0.6 \exp(-r^2/r_0^2), \quad r_0 = 41.7 \text{ m}.$$

As can be seen from Fig. 3, this distribution is much narrower than the lateral distribution obtained in the present experiment. The difference in the energy of the  $\mu$  mesons investigated (corresponding to a depth of observation of 65 and 40 m w.e.) cannot explain the discrepancy. It is possible that the results of<sup>[5]</sup> are due to an inaccurate determination of the shower size at different distances from the shower axis because of the small size of the array used at sea level for detecting the EAS.

We shall, moreover, discuss which factor should play the main role in producing the observed

difference in the lateral distribution of high-energy  $\mu$  mesons. Khristiansen<sup>[12]</sup> has calculated the lateral distribution of  $\mu$  mesons with energy  $E \geq 1.5 \times 10^{10}$  ev due to multiple scattering of particles in the atmosphere and their deflection in the earth's magnetic field. This distribution is shown in Fig. 3 by the dot-dash line. As can be seen from Fig. 3, the Coulomb scattering and the deflections in the earth's magnetic field cannot play a determining role in producing the discrepancy. A more probable main reason for the observed difference is the angular spread of  $\pi$  mesons in the nuclear interactions.

Let us estimate what transverse momentum must be gained by the  $\pi$  mesons in the nuclear interactions in order to explain the effect. For this we use the following approximate relation between the effective height  $h$  of  $\pi$ -meson production, the radius  $r$  of the  $\mu$ -meson shower, the momentum  $p$  of the detected  $\mu$  mesons, and the effective value of the transverse momentum  $p_{\perp}$ :

$$p_{\perp} = rp/h. \quad (13)$$

We assume that the height  $h$  can vary from 5 to 10 km. Then, substituting the values of  $r$  and  $p$  from (7) into (13), we find  $p_{\perp} = 1-2 \times 10^8$  ev/c. It is interesting to note that the lateral distribution of  $\mu$  mesons with energy  $E \geq 6 \times 10^{11}$  ev obtained by Barrett et al,<sup>[13]</sup> characterized by a radius  $r = 10$  m, leads to the value  $p_{\perp} \approx 6 \times 10^8$  ev/c.

The mean energy of nuclear interactions in which  $\mu$  mesons of  $E \sim 10^{12}$  ev are produced is about 100 times higher than the energy of interactions producing  $\mu$  mesons with  $E \sim 10^{10}$  ev. Thus, the experimental data on the lateral distribution of  $\mu$  mesons of different energies show that the mean transverse momentum gained by the  $\pi$  mesons increases with increasing energy of the nuclear interaction. This experimental prediction is in good agreement with the predictions of the hydrodynamical theory of the interaction of ultra-high-energy particles.<sup>[14]</sup>

Comparison of the data of the present experiment with those of Barrett et al.<sup>[13]</sup> also enables us to find a more exact energy spectrum of  $\mu$  mesons in EAS. The spectrum of events involving  $\mu$  mesons with energy  $E \geq 6 \times 10^2$  Bev was found to be of the form  $bN_{\mu}^{-3.4}dN_{\mu}$ . Assuming that the relation between the number of  $\mu$  mesons  $N_{\mu}$  and the total number of particles in the shower  $N$  is  $N_{\mu} \sim kN^{\alpha}$ , we can relate the spectrum of events with a given size with the spectrum of events with a given number of  $\mu$  mesons, and obtain the values of  $k$  and  $\alpha$  (see the article of Greisen in<sup>[15]</sup>).

However, it has been shown<sup>[2,11]</sup> that the relation between  $N_{\mu}$  and  $N$  is correct only on the average, and that the number of  $\mu$  mesons in showers with a given  $N$  fluctuates. If these fluctuations are taken into account in estimating  $N_{\mu}$ , the number of  $\mu$  mesons in EAS with given  $N$  decreases. Therefore, the estimate of the number of  $\mu$  mesons with  $E \geq 6 \times 10^2$  Bev given in<sup>[15]</sup> (60  $\mu$  mesons per shower with  $N = 10^6$ ) should be regarded as the upper limit for  $N_{\mu}$ . Using the number of  $\mu$  mesons with energy  $E \geq 10$  Bev found in present experiment, and the estimate of the number of  $\mu$  mesons with energy  $E \geq 6 \times 10^2$  Bev from<sup>[15]</sup>, we find that the exponent of the integral energy spectrum of  $\mu$  mesons in the range of  $E = 10-600$  Bev should be  $\gamma \geq 1.4$ .

Assuming the spectrum of  $\mu$  mesons to be of the form

$$f_{\mu}(E) dE = \begin{cases} k_1 E^{-2} dE & \text{for } E = 5 \div 10 \text{ Bev} \\ k_2 E^{-2.4} dE & \text{for } E \geq 10 \text{ Bev} \end{cases},$$

we obtain an energy of the  $\mu$  mesons with  $E \geq 5$  Bev equal to  $7.2 \times 10^{14}$  ev in a shower with  $N = 10^6$ . The energy of the particle producing a shower with  $N = 10^6$  amounts to  $E = 10^{16}$  ev, according to the usual estimate. Consequently, the energy of the  $\mu$ -meson flux with  $E \geq 5$  Bev amounts to  $\sim 7\%$  of the primary particle energy.\*

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\*It should be noted that the energy of the  $\mu$  meson flux detected in the present experiment with  $E$  in the 5-10 Bev range amounts to  $1.5 \times 10^{14}$  ev for  $N = 10^6$ , which is roughly equal to the energy of the electron-photon component in EAS at sea level.

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