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QUANTUM-MECHANICAL DIELECTRIC TENSOR FOR AN ELECTRON PLASMA IN A MAGNETIC FIELD

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A quantum mechanical analysis is given for the complex dielectric tensor of an electron plasma in a magnetic field; the orbital motion of the electrons in the magnetic field is quantized in this treatment. The transition to the classical and quasi-classical cases is studied and the relation to well-known results is established.[1,3,4] The limiting quantum case is investigated. An expression is derived for the anti-Hermitian part of the tensor, which is responsible for dissipation. The magnetic-field dependence of the interaction screening range is discussed.

1. The dielectric tensor of an electron plasma with a Maxwellian distribution function in a uniform magnetic field has been given by Sitenko and Stepanov[1] on the basis of classical kinetic theory. In strong magnetic fields, however, the energy of a Larmor quantum is of the same order as or larger than that of the random motion of the particles and the classical theory no longer applies because the orbital electron motion must be quantized.

In the present paper we derive the quantum-mechanical dielectric tensor $\epsilon_{ij}(\omega, \mathbf{q})$, which plays a central role in investigations of the electromagnetic properties of plasma; the correlation energy of the particles is expressed in terms of this tensor. As has recently been shown by Silin,[2] this tensor also plays an important role in shielding of the Coulomb field of particles in the collision integral, so that the formulation of the quantum collision integral for Coulomb particles in a magnetic field also depends on the quantum-mechanical tensor $\epsilon_{ij}(\omega, \mathbf{q})$.

In computing $\epsilon_{ij}(\omega, \mathbf{q})$ we shall use the linear, self-consistent field approximation. The orbital electron motion in the magnetic field is quantized by describing the particle motion with a single-particle statistical operator $\hat{\rho}$, whose equation of motion assumes the following form in the present approximation:

$$i\hbar\partial\hat{\rho}'/\partial t = [\hat{\mathcal{H}}_0, \hat{\rho}']_- + [\hat{\mathcal{H}}', \hat{\rho}_0]_+, \quad (1)$$

where $[A, B]_-$ and $[A, B]_+$ are the commutators and anticommutators for the operators A and B; ρ_0 is the ground state operator, $\hat{\rho}'$ is the perturbed value of $\hat{\rho}$ ($\hat{\rho} = \hat{\rho}_0 + \hat{\rho}'$), which is a linear functional of the self-consistent field

$$\hat{\mathcal{H}}_0 = (\hat{\mathbf{p}} - e\mathbf{A}_0/c)^2/2\mu, \quad (2)$$

$\mathbf{A}_0 \equiv \{-Hy, 0, 0\}$ is the vector potential of the uniform magnetic field

$$\hat{\mathcal{H}}' = -\frac{e}{2\mu c}\left[\mathbf{A}', \left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}_0\right)\right]_+, \quad (3)^*$$

and $\mathbf{A}'$ is the vector potential of the self-consistent field. We shall use the gauge:

$$\varphi \equiv 0, \qquad \operatorname{div}(-\dot{\mathbf{A}}') = 4\pi c \operatorname{Sp}\{\hat{\rho}'(\mathbf{r}')\,\delta(\mathbf{r} - \mathbf{r}')\}. \quad (4)$$

The vector $\mathbf{E} = -(1/c)\dot{\mathbf{A}}'$ obeys the equation

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial}{\partial t}\left(\mathbf{j} + \frac{1}{4\pi}\frac{\partial}{\partial t}\mathbf{E}\right), \quad (5)^\dagger$$

where

$$\mathbf{j} = \frac{e}{2\mu}\operatorname{Sp}\left\{\hat{\rho}(\mathbf{r}')\left[\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}_0 - \frac{e}{c}\mathbf{A}'\right), \delta(\mathbf{r} - \mathbf{r}')\right]_+\right\}. \quad (6)$$

2. To form $\epsilon_{ij}(\omega, \mathbf{q})$ we use Eq. (1) to express $\hat{\rho}'$ in terms of $\mathbf{A}$ and then find the current-density vector (6); Eq. (5) can then be used to find $\epsilon_{ij}(\omega, \mathbf{q})$ directly. This calculation is carried out conveniently in the representation based on the eigenfunctions of the operator $\hat{\mathcal{H}}_0$ (Landau representation):

$$\hat{\mathcal{H}}_0|k_x, k_z, n\rangle = E_{k_z,n}|k_x, k_z, n\rangle,$$
$$E_{k_z,n} \equiv E_\nu = \hbar\Omega(n + 1/2) + \hbar^2k_z^2/2\mu, \quad (7)$$

$$|\nu\rangle \equiv |k_x, k_z, n\rangle$$
$$= (4\pi^2\alpha)^{-1/2}\exp(ik_x x + ik_z z)\,\Phi_n[(y + \alpha^2 k_x)/\alpha)],$$

$$\alpha^2 = \hbar/\mu\Omega, \qquad \Omega = |e|H/\mu c. \quad (8)$$

$^*[\mathbf{A}', (\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}_0)] = \mathbf{A}' \times (\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}_0)$.

†rot = curl.

Assuming that $\hat{\rho}' \sim E \sim \exp(i\omega t - i\mathbf{q}\cdot\mathbf{r})$, using (1) we find the following expression for the matrix elements of the operator $\hat{\rho}$

$$\langle \nu' | \hat{\rho} | \nu \rangle = f_0(\nu)\,\delta_{\nu\nu'} + \frac{f_0(\nu') - f_0(\nu)}{E_{\nu'} - E_\nu - \hbar\omega - i\Delta}\langle \nu' | \hat{\mathcal{H}}' | \nu \rangle. \quad (9)$$

From (3) and (8) we have

$$\langle k_x, k_z, n | \hat{\mathcal{H}}' | k_x + q_x, k_z + q_z, n' \rangle$$
$$= (e/\mu c)\,\mathbf{K}_{nn'}(q_x, q_z)\,\mathbf{A}'(q_x, q_z);$$
$$\mathbf{K}_{nn'}(q_x, q_z) \equiv {}^1/_2\,\{\langle n | \mu\Omega\,[\hat{y}, J]_+ | n' \rangle,$$
$$\langle n | [\hat{p}_y, \hat{J}]_+ | n' \rangle,\ \langle n | (2\hbar k_z + \hbar q_z)\,\hat{J} | n' \rangle\}. \quad (10)$$

Here, $\hat{J} = \exp\{\alpha^2 q_x^2 \partial/\partial y\}$ and $\mathbf{A}'(q_x, q_z)$ is the Fourier transform of the self-consistent vector potential.

The operator $\hat{J}$ is unitary:

$$\hat{J}\hat{J}^+ = \hat{I}, \quad (11)$$

where $\hat{I}$ is the unit operator.

To compute the current density we must write the matrix elements for the kinetic momentum operator

$$\langle k_x', k_z', n' | \hat{\mathbf{p}} - e\mathbf{A}_0/c | k_x, k_z, n \rangle = \delta(k_z - k_z' + q_z)$$
$$\times\,\delta(k_x - k_x' + q_x)$$
$$\{\langle n' | \mu\Omega y | n \rangle,\ \langle n' | \hat{p}_y | n \rangle,\ \langle n' | \hbar k_z | n \rangle\}. \quad (12)$$

Using (6) and (9) — (12) we obtain from (5)

$$\sum_{i=1}^{3} \{n^2(\varkappa_i\varkappa_j - \delta_{ij}) + \varepsilon_{ij}(\omega, \mathbf{q})\}E_i = 0, \quad (13)$$
$$n = cq/\omega, \qquad \varkappa_i = q_i/q;$$
$$\varepsilon_{ij}(\omega, \mathbf{q}) = \delta_{ij} + 4\pi\sigma_{ij}(\omega, \mathbf{q})/i\omega, \quad (14)$$

$$\sigma_{ij}(\omega, \mathbf{q}) = \frac{e^2 N_0}{\mu i\omega}\delta_{ij} + \frac{e^2}{\mu^2 i\omega}\frac{1}{2\pi^2\alpha^2}$$
$$\times \lim_{\Delta\to 0}\sum_{n, n'}\int dk_z \frac{f_0(k_z + q_z, n') - f_0(k_z, n)}{E_{k_z+q_z, n'} - E_{k_z, n} - \hbar\omega - i\Delta}(\mathbf{K}^*_{nn'})_i(\mathbf{K}_{n'n})_j, \quad (15)$$

where N_0 is the mean particle-number density:

$$N_0 = (2\pi^2\alpha^2)\sum_n \int dk_z f_0(E_{k_z, n}).$$

The tensor $\epsilon_{ij}(\omega, \mathbf{q})$ [and consequently $\sigma_{ij}(\omega, \mathbf{q})$] satisfies the well-known Onsager symmetry relation

$$\varepsilon_{ij} = (-1)^a \varepsilon_{ji}, \qquad \varepsilon_{ij}(\mathbf{q}, \mathbf{H}, \omega) = \varepsilon_{ji}(-\mathbf{q}, -\mathbf{H}, \omega) \quad (16)$$

(the number a indicates how many times y appears in the indices i and j).

3. We now consider limiting cases of Eq. (14). First we show that (14) yields the classical result of Sitenko and Stepanov.[1] We assume that f_0 is a Maxwellian distribution function and expand the difference

$$f_0(E_{k_z+q_z, n'}) - f_0(E_{k_z, n}) = -f_0(E_{k_z, n})(E_{k_z+q_z, n'} - E_{k_z, n}) \quad (17)$$

in terms of q_z and $n' - n$.

Since the operator $\hat{J}$ is unitary (11), we have

$$(N_0(2\pi)^2\mu\theta\alpha^2)^{-1}\sum_{n'n}\int dk_z f_0(k_z, n)(\mathbf{K}^*_{nn'})_i(\mathbf{K}_{n'n})_j = \delta_{ij}. \quad (18)$$

The components of the vectors $\mathbf{K}_{nn'}$ are replaced by their asymptotic expressions for large n, corresponding to the transition to the classical limit ($n \to \infty$, $\hbar \to 0$, $n\hbar$ is finite):

$$\lim\langle n | \hat{J} | n' \rangle = \lim \exp(-\alpha^2 q_x^2/4)(n'!\,n!)^{-1/2}$$
$$\times(\alpha q_x/\sqrt{2})^{n'-n} L_n^{n'-n}(\alpha^2 q_x^2/2)$$
$$= (-1)^{n'-n} J_{n'-n}(\alpha q_x\sqrt{n'+n+1}), \quad (19)$$

for $n' \geq n$. Here $L_n^{n'-n}(x)$ is the Laguerre polynomial and $J_{n'-n}(x)$ is the Bessel function of order $(n'-n)$.

Using the recurrence formulas for the Bessel functions we have

$$\langle n | [\hat{y}, \hat{J}]_+ | n' \rangle = \sqrt{2n}\frac{nJ_{n'-n}(\alpha q_x\sqrt{n'+n+1})}{q_x\sqrt{n+n'+1}},$$
$$\langle n | [\hat{p}_y, \hat{J}]_+ | n' \rangle = \frac{\hbar}{i\alpha}\sqrt{2n}J'_{n'-n}(\alpha q_x\sqrt{n'+n+1}),$$
$$(\hbar k_z + \hbar q_z/2)\langle n | \hat{J} | n' \rangle = \hbar k_z J_{n'-n}(\alpha q_x\sqrt{n'+n+1}); \quad (20)$$

in which we have taken account of the fact that $[\hat{y}, \hat{J}]_- = 0$ in the classical limit. Converting to the variables

$$(\hbar k_z)^2/2\mu\theta = y^2, \quad n\hbar\Omega/\theta = t^2, \quad s = \sqrt{3\theta/\mu},$$
$$\sqrt{3/2}(\omega - n\Omega)/(sq_z) = Z_n, \quad \lambda = \sqrt{2/3}\,sq_x/\Omega, \quad \omega_0^2 = 4\pi e^2 N_0/\mu,$$

we have from (14) in the classical limit

$$\varepsilon_{ij}(\omega, q) = \delta_{ij} - \frac{4\omega_0^2}{\sqrt{\pi}\omega^2}\sum_{n=-\infty}^{\infty}\int_0^\infty te^{-t^2}dt\int_{\curvearrowleft}\frac{e^{-y^2}dy}{Z_n - y}$$
$$\times(\mathbf{D}(n/\lambda;\ i\partial/\partial\lambda,\ y)J_n(\lambda t))_i$$
$$\times(\mathbf{D}(n/\lambda,\ -i\partial/\partial\lambda,\ y)J_n(\lambda t))_j;$$
$$\mathbf{D}(n/\lambda,\ i\partial/\partial\lambda,\ y) \equiv \{n/\lambda,\ i\partial/\partial\lambda,\ y\}, \quad (21)$$

which coincides with Eq. (11) of Sitenko and Stepanov.[1]

We now find the quasi-classical limit for ϵ_{ij} for a plasma with a Fermi distribution function. This case is of importance in the plasma model of a metal. For complete degeneracy we have

$$f_0(k_z+q_z, n') - f_0(k_z, n) = -\delta\,[\hbar^2 k_z^2/2\mu - \hbar\Omega(n_0-n)]\,[E_{k_z+q,n'} - E_{k_z,n}], \tag{22}$$

and, from Eq. (11),

$$(2\pi^2\mu N_0\alpha^2)^{-1}\sum_{n'n}\int dk_z\delta\left[\frac{\hbar^2 k_z^2}{2\mu} - \hbar\Omega(n_0-n')\right] \times (\mathbf{K}^*_{nn'})_i(\mathbf{K}_{n'n})_j = \delta_{ij}, \tag{23}$$

where $\hbar\Omega n_0 = E_0 - \hbar\Omega/2$, E_0 is the Fermi energy (more precisely, the chemical potential).

Using the asymptotic forms of (19) and (20) and assuming that $n = n_0\sin^2\vartheta$ we have from Eq. (14)

$$\varepsilon_{ij}(\omega, q_x, q_z) = \delta_{ij} - \frac{3\omega_0^2}{2\omega}\sum_{n=-\infty}^{\infty}\int_0^{\pi}\frac{\sin\vartheta\,d\vartheta}{v_0 q_z\cos\vartheta + n\Omega - \omega} \times \left(\mathbf{D}\left(\frac{n}{x}, \frac{i\partial}{\partial x}, \cos\vartheta\right)J_n(x\sin\vartheta)\right)_i \times \left(\mathbf{D}\left(\frac{n}{x}, -\frac{i\partial}{\partial x}, \cos\vartheta\right)J_n(x\sin\vartheta)\right)_j. \tag{24}$$

We note that two components of Eq. (15) for $\sigma_{ij}(\omega, \mathbf{q})$ have been obtained by Mattis and Dresselhaus [3] for a purely transverse field (σ_{xx} and σ_{zz}). This equation has also been obtained in the quasi-classical limit being considered here by Cohen, Harrison, and Harrison. [4] In both of these papers $\mathrm{Im}\,\omega = 1/\tau$ was assumed to be known and independent of magnetic field, and the frequency ω was identified with the frequency of the collisions between the electrons and the impurities or the lattice. There is reason to believe that these assumptions do not apply in the quantum region (strong magnetic fields) or in the short-wave region, where spatial dispersion is important.

We now consider the limiting quantum case corresponding to small quantum numbers n or strong magnetic fields. In this case $n = n' = 0$ if $n_0 < 1$; this means that all the particles are in the $n = 0$ level and that all $n > 0$ levels are empty. For these magnetic field strengths

$$\varepsilon_{ij}(\omega, q_x, q_z) = (1-\omega_0^2/\omega^2)\delta_{ij} - \exp(-\alpha^2 q_x^2/2)\,\omega_0^2\omega^{-2}(2\pi^2\alpha^2\mu N_0)^{-1} \times \lim_{\Delta\to 0}\int dk_z\frac{f_0(k_z+q_z) - f_0(k_z)}{\hbar^2(q_z^2+2k_z q_z)/2\mu - \hbar\omega - i\Delta} \times (\mathbf{D}(0, i\hbar q_z, 2\hbar k_z + \hbar q_z))_i(\mathbf{D}(0, -i\hbar q_z, 2\hbar k_z + \hbar q_z))_j. \tag{25}$$

It follows from this formula that the tensor ϵ_{ij} reduces to a scalar and there is no spatial dispersion for waves characterized by $q_z = 0$ and $q_x \neq 0$, i.e., waves that propagate at right angles to the magnetic field:

$$\varepsilon_{ij}(\omega) = (1-\omega_0^2/\omega^2)\delta_{ij}. \tag{26}$$

This result follows from the fact that a strong magnetic field inhibits particle motion perpendicular to the field. This feature does not apply for waves characterized by $q_z \neq 0$.

The quantity n_0 increases as the magnetic field is reduced, and as soon as $n_0 \geq 1$ motion across the magnetic field develops discontinuously. When $1 \leq n_0 < 2$ [for waves that propagate across the field ($q_x \neq 0$, $q_z = 0$)] the expression for ϵ_{ij} becomes

$$\varepsilon_{xx} = 1 - \frac{\omega_0^2}{\omega^2}\left\{1 - \frac{\Omega^2}{\omega^2-\Omega^2}\exp\left(-\frac{\alpha^2 q_x^2}{2}\right)\right\},$$

$$\varepsilon_{yy} = 1 - \frac{\omega_0^2}{\omega^2}\left\{1 - \frac{\Omega^2(1-\alpha^2 q_x^2/2)}{\omega^2-\Omega^2}\exp\left(-\frac{\alpha^2 q_x^2}{2}\right)\right\},$$

$$\varepsilon_{zz} = 1 - \frac{\omega_0^2}{\omega^2}\left\{1 + \tfrac{4}{3}n_0\frac{\Omega^2}{\omega^2-\Omega^2}\exp\left(-\frac{\alpha^2 q_x^2}{2}\right)\right\},$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = \frac{i\omega_0^2}{\omega^2-\Omega^2}\frac{\Omega}{\omega}\left(1-\frac{\alpha^2 q_x^2}{2}\right)\exp\left(-\frac{\alpha^2 q_x^2}{2}\right). \tag{27}$$

Furthermore, $\epsilon_{zj} = 0$ when $j \neq z$.

As the magnetic field is reduced, n_0 increases still further and when $n_0 = m$ (m is a whole number) the tensor $\epsilon_{ij}(\omega, \mathbf{q})$ changes discontinuously and a new level becomes populated.

4. The investigation of the dissipative properties of the plasma is based on the anti-Hermitian part of the tensor $\epsilon_{ij}(\omega, \mathbf{q})$ which we denote by $\epsilon''_{ij}(\omega, \mathbf{q})$ below. Using the general formulas (14) and (15) we find

$$\varepsilon''_{ij}(\omega, \mathbf{q}) = \frac{\omega_0^2}{2\pi N_0\mu\alpha^2\omega^2}\sum_{nn'}\int dk_z\{f_0(k_z+q_z, n') - f_0(k_z, n)\}(\mathbf{K}^*_{nn'})_i(\mathbf{K}_{n'n})_j\,\delta[E_{k_z+q_z,n'} - E_{k_z,n} - \hbar\omega]. \tag{28}$$

For waves propagating across the magnetic field ($q_z = 0$)

$$\varepsilon''_{ij}(\omega, q_x) = \frac{\omega_0^2}{\omega^2}(2\pi\alpha^2 N_0\mu)^{-1}\sum_{nn'}\int dk_z[f_0(k_z, n') - f_0(k_z, n)] \times (\mathbf{K}^*_{nn'})_i(\mathbf{K}_{n'n})_j\,\delta(\hbar\Omega(n'-n) - \hbar\omega). \tag{29}$$

Using the asymptotic expressions for the vectors $\mathbf{K}_{nn'}$ as $n\to\infty$ and assuming a Maxwellian distribution function we have from Eq. (29)

$$\varepsilon''_{ij}(\omega, q_x) = 4\sqrt{\pi}\frac{\omega_0^2}{\omega^2}\int_0^\infty dt\cdot te^{-t^2}\left(\mathbf{D}\left(\frac{\omega}{\lambda\Omega}, i\frac{\partial}{\partial\lambda}, 0\right)J_{\omega/\Omega}(\lambda t)\right)_i$$
$$\times\left(\mathbf{D}\left(\frac{\omega}{\lambda\Omega}, -\frac{i\partial}{\partial\lambda}, 0\right)J_{\omega/\Omega}(\lambda t)\right)_j$$
$$+\delta_{zz}\frac{2\pi\omega_0^2}{\omega\Omega}\int_0^\infty dt\cdot te^{-t^2}J^2_{\omega/\Omega}(\lambda t),$$
$$\varepsilon''_{zj} = 0 \quad \text{for } j \neq z, \tag{30}$$

while for a degenerate Fermi distribution we have

$$\varepsilon''_{ij}(\omega, q_x) = \frac{3\pi\omega_0^2}{2\omega\Omega}\int_0^{\pi/2}\left(\mathbf{D}\left(\frac{\omega}{x\Omega}, \frac{i\partial}{\partial x}, \cos\vartheta\right)J_n(x\sin\vartheta)\right)_i$$
$$\times\left(\mathbf{D}\left(\frac{\omega}{x\Omega}, -\frac{i\partial}{\partial x}, \cos\vartheta\right)J_n(x\sin\vartheta)\right)_j\sin\vartheta d\vartheta, \tag{31}$$

where $x = q_x v_0/\Omega$. We may note that ϵ''_{ij} in Eq. (31) approaches zero when the magnetic field is switched off; this follows from the asymptotic behavior of the Bessel functions.

The results of the present work can be easily extended to the case of several particle species subjected to Coulomb interactions in a fixed magnetic field.

5. We discuss briefly the potential φ of a charge e moving with velocity $\mathbf{v}_0$ in a medium for which $\epsilon_{ij}(\omega, \mathbf{q})$ is given. The expression for φ is

$$\varphi(\mathbf{r}, t) = (2\pi)^{-3}$$
$$\times\int 4\pi e\, d\mathbf{q}\, d\omega e^{i\mathbf{qr}-i\omega t}(q_i q_j \varepsilon_{ij}(\omega, \mathbf{q}))^{-1}\delta(\omega - \mathbf{q}\mathbf{v}_0). \tag{32}$$

The denominator in (32) is the longitudinal dielectric constant $\epsilon(\omega, \mathbf{q})$:

$$q^2\varepsilon(\omega, \mathbf{q}) = q_i q_j \varepsilon_{ij}(\omega, \mathbf{q}), \tag{33}$$

which, by (14), is

$$\varepsilon(\omega, \mathbf{q}) = 1 - \frac{4\pi e^2}{2\pi^2\alpha^2}\lim_{\Delta\to 0}\sum_{nn'}|\langle n|\hat{J}|n'\rangle|^2$$
$$\times\int dk_z\frac{f_0(k_z+q_z, n') - f_0(k_z, n)}{E_{k_z+q_z, n'} - E_{k_z, n} - \hbar\omega - i\Delta}. \tag{34}$$

In the classical limit and with $\mathbf{v}_0 = 0$ we use (32) and (34) to find the isotropic exponential decay of potential with distance (Debye shielding with radius r_D). This case corresponds to the inequality $r_D \gg \lambda\!\!\!^-_F$ ($\lambda\!\!\!^-_F$ is the de Broglie wavelength of an electron at the Fermi surface). When $r_D \sim \lambda\!\!\!^-_F$ the potential oscillates and the shielding differs considerably from Debye shielding, depending on the magnetic field if such a field is present. The oscillations of the potential of a point charge in a plasma are associated with the diffraction of the electron de Broglie waves in the inhomogenous potential produced by the point charge; this effect does not appear in the classical limit.

In strong magnetic fields the Larmor radius r_L plays the role of the Debye radius. The inequality $r_L < \lambda\!\!\!^-_F$ holds for Fermi statistics with $n_0 \lesssim 1$ and we obtain the following expression for the potential φ (cylindrical coordinates):

$$\varphi(\rho, z) = \frac{e}{\pi}\int_0^\infty dq_\perp q_\perp J_0(\rho q_\perp)\int_{-\infty}^{+\infty} dq_z e^{iq_z z}$$
$$\times\left\{q^2 - \frac{1}{2\pi}(2\mu e^2/\hbar)^2\frac{\Omega}{\hbar q_z}\exp(-\alpha^2 q_\perp^2/2)\right.$$
$$\left.\times\left[\ln\frac{-\sqrt{2n_0} + q_z\alpha/2}{\sqrt{2n_0} + \alpha q_z/2} + \ln\frac{\sqrt{2n_0} - \alpha q_z/2}{-\sqrt{2n_0} - \alpha q_z/2}\right]\right\}^{-1},$$

where $q_\perp^2 = q_x^2 + q_y^2$ while $\epsilon(\omega, \mathbf{q})$ is computed in [5]. Analysis of this formula shows that when $n_0 \lesssim 1$ the anisotropic shielding is important only in a region of space characterized by dimension a. Electromagnetic wave propagation can be studied in the quantum case by means of the dispersion equation which follows from (13); the determinant of the system is set equal to zero.

[1] A. G. Sitenko and K. N. Stepanov, JETP **31**, 642 (1956), Soviet Phys. JETP **4**, 512 (1957).

[2] V. P. Silin, JETP **40**, 1768 (1961), Soviet Phys. JETP **13**, 1244 (1961).

[3] D. C. Mattis and G. Dresselhaus, Phys. Rev. **111**, 403 (1958).

[4] Cohen, Harrison and Harrison, Phys. Rev. **117**, 937 (1960).

[5] P. S. Zyryanov, JETP **40**, 1065 (1961), Soviet Phys. JETP **13**, 751 (1961).

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