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PRODUCTION OF HIGH-ENERGY π MESON BEAMS

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The cross section for the production of fast π^+ mesons in collisions between high-energy protons and nuclei in which a small momentum is transferred to the nucleus is calculated. Under these conditions, the main role in the interaction is played by the exchange of virtual photons. The cross section under consideration was therefore determined by means of available experimental data on the photoproduction cross section of π^+ mesons on protons.

FAST π meson beams are at present produced in proton accelerators as the result of an interaction of primary protons with nuclei. If these protons have an energy E_L of the order of 30 — 40 Bev then, as will be shown below, a considerable role is played by the Coulomb field of the nucleus in the production of π^+ mesons with energy $\epsilon_L \gtrsim 0.4 - 0.5\,E_L$. The reaction involved is

$$p + N \rightarrow N + n + \pi^+, \quad (1)$$

in which a small momentum is transferred to the nucleus N. If the condition

$$q^2 \ll \mu^2 A^{-2/3}, \quad (2)$$

is satisfied, where $q^2 = (P - P')^2$ are the 4-momenta of the nucleus before and after collision, A is the atomic weight of the nucleus, and μ the π meson mass, then, as has been shown earlier,[1] the main contribution to process (1) is due to the diagrams involving the exchange of virtual photons (see Fig. 1). Under these conditions, the differential cross section of process (1) $d\sigma$ may be determined by the Weizsäcker-Williams method, using the available experimental data on the photoproduction cross section of π mesons on the nucleus. According to the relation between these cross sections [see [1], Eq. (2.17)], we have

$$d\sigma = \frac{Z^2\alpha}{\pi}\frac{dw^2}{w^2 - m^2}\frac{dq^2}{(q^2)^2}\left[q^2 - \left(\frac{w^2 - m^2}{2E_L}\right)^2\right]d\sigma_p \qquad (\alpha = 1/137) \quad (3)$$

where Z is the nuclear charge, E_L is the energy of the incident proton in the laboratory system (l.s.), m is the proton mass, w is the energy of the neutron and the π^+ meson produced in the reaction (or of the proton and the π^0 meson) in the c.m.s., and $d\sigma_p$ is the differential cross section for the photoproduction

$$\gamma + p \rightarrow n + \pi^+, \quad (4)$$

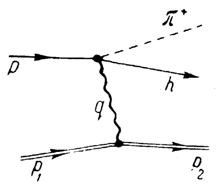

FIG. 1

which we shall write in the following way:

$$d\sigma_p = 2\pi\sigma_p(\omega, \theta_c)\sin\theta_c d\theta_c, \quad (5)$$

where θ_c is the angle between the momenta of the π meson and of the γ ray in the c.m.s., and ω is the frequency in the W coordinate system (the system in which the proton is at rest before the collision).

Taking into account that $w^2 = m^2 + 2m\omega$, we integrate Eq. (3) over q^2 in the limits between

$$q^2_{min} = \left(\frac{w^2 - m^2}{2E_L}\right)^2 = \left(\frac{m\omega}{E_L}\right)^2 \text{ and } q^2_{max} = \frac{\mu^2}{A^{2/3}}.$$

Denoting the result obtained by $\sigma_\pi(E_L, \omega, \theta_c)\,d\omega \sin\theta_c\,d\theta_c$, we have

$$\sigma_\pi(E_L, \omega, \theta_c)\,d\omega\sin\theta_c\,d\theta_c = 2Z^2\alpha\left[2\ln\frac{\mu E_L}{mA^{1/3}\omega} - 1 + \left(\frac{mA^{1/3}\omega}{\mu E_L}\right)^2\right]\frac{d\omega}{\omega}\sigma_p(\omega, \theta_c)\sin\theta_c\,d\theta_c. \quad (6)$$

It should be noted that, in view of condition (2) and since $w^2 - m^2 < 2E_L\sqrt{q^2}$, the permissible values of ω are determined by the condition

$$\omega < \mu E_L / mA^{1/3}. \quad (7)$$

In order to obtain the energy and angular distributions of π mesons in the l.s., we shall pass in Eq. (6) from θ_c and ω to the variables ϵ_L and θ_L (denoting the energy and the angle of emission of the produced π meson in the l.s. measured from the direction of the primary-proton momentum).

From the conservation laws for the process (4), it follows that

$$\cos\theta = [(m+\omega)\varepsilon - m\omega - \tfrac{1}{2}\mu^2]/g\omega, \quad (8)$$

where ϵ and g are the energy and momentum of the produced π meson in the W system, and θ is the angle between the momenta of the π meson and of the γ ray in the same system. On the other hand

$$\varepsilon_L = m^{-1}(\varepsilon E_L - g p_L \cos\theta). \quad (9)$$

From Eqs. (8) and (9), we have

$$\varepsilon = \frac{\omega(1-\varepsilon_L/p_L) + \mu^2/2m}{1-(\omega/m)(E_L/p_L - 1)}. \quad (10)$$

Taking Eq. (7) into account, we can neglect the second term in the denominator of the expression for ϵ. In fact,

$$\frac{\omega}{m}\left(\frac{E_L}{p_L} - 1\right) \approx \frac{\omega}{m}\frac{m^2}{2E_L^2} < \frac{m}{2E_L^2}\frac{\mu E_L}{mA^{1/3}} = \frac{\mu}{2E_L A^{1/3}} \ll 1. \quad (11)$$

In addition, in the numerator of Eq. (10), we can neglect the term $\mu^2/2m$, since, at any rate, $\epsilon > \mu$. Consequently, with sufficiently good accuracy, we have

$$\varepsilon = \omega(1 - \varepsilon_L/E_L). \quad (12)$$

We shall now consider those π mesons which are emitted in the l.s. at small angles to the direction of primary protons, and which have a very high energy ϵ_L constituting a large fraction of the energy E_L. As we shall soon see, these π mesons will be relativistic in the W system, i.e., for them $\epsilon \approx g \gtrsim \mu$. Taking this into account, we obtain from (9) and (12)

$$\omega = m\varepsilon_L/(E_L - \varepsilon_L)[1 - (p_L/E_L)\cos\theta]. \quad (13)$$

Hence, for a given ϵ_L, we have for the minimum frequency ω_{min} (corresponding to $\theta = \pi$)

$$\omega_{min} = m\varepsilon_L/2(E_L - \varepsilon_L). \quad (14)$$

The minimum value of ϵ corresponding to ω_{min} is, according to Eq. (12), found to equal $m\epsilon_L/2E_L$. Therefore, the assumption made earlier that $\epsilon \gtrsim \mu$ is fulfilled for those π mesons whose energy ϵ_L in the l.s. satisfies the condition

$$\varepsilon_L \gtrsim 2\mu E_L/m = \varepsilon_1. \quad (15)$$

On the other hand, for the maximum frequency ω_{max} we obtain, from Eq. (13), the value of ω $[\omega_{max} = 2\epsilon_L E_L^2/m(E_L - \epsilon_L)]$ which, for the large ϵ_L of interest, is always much greater than ω_{max} determined by Eq. (7). Therefore, the integration with respect to ω in Eq. (6) should be carried out over the limits from ω_{min} to $\omega_{max} = \mu E_L/mA^{1/3}$. Moreover, the condition $\omega_{min} < \omega_{max}$ determines the upper limit for ϵ_L which can be attained if the condition (7) is satisfied:

$$(\varepsilon_L)_{max} = \frac{E_L}{1 + m/2\omega_{max}} = \frac{E_L}{1 + m^2A^{1/3}/2\mu E_L} = \varepsilon_2. \quad (16)$$

Thus, the production of fast π mesons with energy $\epsilon_L \gtrsim 2\mu E_L/m$ in the Coulomb field of the nucleus is only possible for a sufficiently large energy E_L of the incident protons

$$E_L \gtrsim mA^{1/3}/(1 + 2\mu/m). \quad (17)$$

Let us denote the velocity of the c.m.s. with respect to the W system by V and the velocity of the produced π mesons in the c.m.s. by v:

$$V = \frac{\omega}{m+\omega}, \quad v = \sqrt{(\omega - \mu^2/2m)^2 - \mu^2}/(\omega + \mu^2/2m). \quad (18)$$

Then

$$v\cos\theta_c = \left[g\cos\theta - \frac{\omega}{m+\omega}\varepsilon\right] \Big/ \left[\varepsilon - \frac{\omega}{m+\omega}g\cos\theta\right]. \quad (19)$$

With the same accuracy with which we obtained Eq. (13), we have

$$\cos\theta_c \approx \left[\cos\theta - \frac{\omega}{m+\omega}\right] \Big/ \left[1 - \frac{\omega}{m+\omega}\cos\theta\right]$$
$$= 1 - \frac{\varepsilon_L}{E_L}\left(2 + \frac{m}{\omega}\right). \quad (20)$$

Furthermore,

$$E_L\varepsilon_L - \sqrt{(E_L^2 - m^2)(\varepsilon_L^2 - \mu^2)}\cos\theta_L = m\varepsilon. \quad (21)$$

Hence, taking Eqs. (12) and (15) into account, we obtain

$$\cos\theta_L = \frac{\varepsilon_L E_L}{\sqrt{(E_L^2 - m^2)(\varepsilon_L^2 - \mu^2)}}\left(1 - \frac{m\varepsilon}{\varepsilon_L E_L}\right)$$
$$\approx \left(1 + \frac{m^2}{2E_L^2} + \frac{\mu^2}{2\varepsilon_L^2}\right)\left[1 - \frac{m\omega}{\varepsilon_L E_L}\left(1 - \frac{\varepsilon_L}{E_L}\right)\right]$$
$$\approx 1 + \frac{m^2}{2E_L^2} - \frac{m\omega}{\varepsilon_L E_L}\left(1 - \frac{\varepsilon_L}{E_L}\right). \quad (22)$$

Consequently, for small θ_L ($\theta_L \ll 1$), which is the only range important for the process under consideration, we have

$$\theta_L^2 = \frac{2m\omega}{\varepsilon_L E_L}\left(1 - \frac{\varepsilon_L}{E_L}\right) - \frac{m^2}{E_L^2}. \quad (23)$$

The maximum angle $\theta_L^{max}(E_L, \epsilon_L)$ of emission of a pion with energy ϵ_L permitted by Eq. (7) is therefore

$$[\theta_L^{max}(E_L, \varepsilon_L)]^2 = \frac{2\mu}{A^{1/3}\varepsilon_L}\left(1 - \frac{\varepsilon_L}{E_L}\right) - \frac{m^2}{E_L^2}. \quad (24)$$

Equations (20) and (23) enable us to pass to variables ϵ_L and θ_L in Eq. (6), as a result of which we find

Table I. Cross section $\sigma_\pi(E_L, \epsilon_L) \times 10^{-30}$ cm²/Bev

E_L, Bev \ ε_L/E_L	0.30	0.35	0.40	0.50
20	131	59	15	0.02
30	194	108	36	1.9
40	226	129	50	5.3

$$\sigma_\pi(E_L, \varepsilon_L, \theta_L)\, d\varepsilon_L\, d\Omega_L = \frac{4Z^2\alpha}{\pi}\left[2\ln\frac{\omega_{max}}{\omega} - 1 + \left(\frac{\omega}{\omega_{max}}\right)^2\right]\sigma_p(\omega, \theta_c)\frac{\theta_L^2 + m^2/\varepsilon_L E_L}{E_L(\theta_L^2 + m^2/E_L^2)^2}\, d\varepsilon_L\, d\Omega_L. \quad (25)$$

In the above expression, ω and θ_c are determined for given ϵ_L and θ_L by Eqs. (20) and (23), and $d\Omega_L = 2\pi\theta_L d\theta_L$.

In order to obtain the energy distribution of the π mesons produced, it is more convenient to change in Eq. (6) directly from the variable θ_c to ϵ_L using Eq. (20), and to integrate the expression obtained with respect to ω between the limits given by Eqs. (7) and (14). We then obtain

$$\sigma_\pi(E_L, \varepsilon_L)\, d\varepsilon_L = d\varepsilon_L \int_{\omega_{min}}^{\omega_{max}} \sigma_\pi(\omega, \theta_c)\sin\theta_c \frac{d\theta_c}{d\varepsilon_L}\, d\omega$$

$$= 2Z^2\frac{d\varepsilon_L}{E_L}\alpha\int_{\omega_{min}}^{\omega_{max}}\left[2\ln\frac{\omega_{max}}{\omega} - 1 + \left(\frac{\omega}{\omega_{max}}\right)^2\right]\sigma_p(\omega, \theta_c)\left(2 + \frac{m}{\omega}\right)\frac{d\omega}{\omega}. \quad (26)$$

It is convenient to represent the available data on the cross section for photoproduction of π mesons on nucleons in the form

$$\sigma_p(\omega, \theta_c) = A(\omega) + B(\omega)\cos\theta_c + C(\omega)\cos^2\theta_c, \quad (27)$$

where $A(\omega)$, $B(\omega)$, and $C(\omega)$ are values known from experiments (see[2], where data on the photoproduction of π^+ mesons for γ ray energies less than 1 Bev are collected). Taking (20) into account, this enables us to carry out a numerical integration in Eq. (26) and to obtain the energy spectrum of π mesons with energy ϵ_L in the range from $2\mu E_L/m$ to $E_L/(1 + m^2A^{1/3}/2\mu E_L)$ [see Eqs. (15) and (16)] and emitted in a narrow cone with aperture $\theta_L^{max}(E_L, \epsilon_L)$ [see Eqs. (24)].

The results for the cross section $\sigma_\pi(E_L, \epsilon_L)$ calculated in this manner for nuclei with $Z = 82$ and $A = 207$ (Pb) and primary-proton energies $E_L = 20$, 30, and 40 Bev for different values of ϵ_L are shown in Table I.

In connection with these calculations, it should be mentioned that the frequency ω corresponding to the $\frac{3}{2}$, $\frac{3}{2}$ resonance is of the order of 2μ. Therefore, if $\omega_{min} \lesssim 2\mu$, the resonance range falls within the integration limits in (26). According to Eq. (14), this will take place if

$$\varepsilon_L/E_L \lessapprox (1 + m/4\mu)^{-1} \approx 0.4. \quad (28)$$

The main contribution to the integral for $\sigma_\pi(E_L, \epsilon_L)$ will then be due to frequencies ω close to ω_{min}. Therefore, if, in addition to (28), the following condition is satisfied

$$\omega_{max}/\omega_{min} \gg 1, \quad (29)$$

then, for the important values of ω, the first factor in the integrand of Eq. (26) also will be large (the logarithmic term being large). If conditions (28) and (29) are satisfied, we can assume that the mechanism due to the action of the Coulomb field of the nucleus will contribute greatly, and may constitute the main contribution, to the production of fast π mesons (i.e., may be greater than the contribution of purely nuclear interactions).

For π mesons with energy ϵ_L satisfying condition (28), the inequality (29) becomes

$$E_L/2mA^{1/3} \gg 1. \quad (30)$$

It is essential to satisfy (29) also because of the following reasons: passing from Eq. (3) to Eq. (6), we have used the exact equality, which determines the upper limit of integration with respect to q^2, instead of the inequality (2) which gives the condition of applicability of Eq. (3). The inaccuracy due to this procedure will be negligible if the argument of the logarithm in the brackets of Eq. (6) ω_{max}/ω will be much greater than unity. Therefore, the corresponding error will also be small for $\sigma_\pi(E_L, \epsilon_L)$ if Eq. (29) is satisfied.

For a primary proton energy E_L equal, for example, to 30 Bev, and an energy of the produced π mesons $\epsilon_L = 0.4\, E_L = 12$ Bev, condition (28) is satisfied and $\omega_{max}/\omega_{min} \approx 2.5$. We therefore see that, under these conditions, the mechanism discussed contributes greatly to the production of π mesons of the corresponding energies emitted at angles not greater than about 2° [see Eq. (24)]. Clearly, this mechanism will remain important in the range of π meson energies $\epsilon_L \gtrsim 0.4\, E_L$, which will form an increasingly large energy fraction with increasing E_L.

The first row of Table II shows the values of the cross section $\sigma'_\pi(E_L)$ which were obtained by a numerical integration of $\sigma_\pi(E_L, \epsilon_L)$ within the limits given by Eqs. (15) and (16):

$$\sigma'_\pi(E_L) = \int_{\varepsilon_1}^{\varepsilon_2} \sigma_\pi(E_L, \varepsilon_L)\, d\varepsilon_L. \quad (31)$$

Table II. Cross sections $\sigma'_\pi(E_L)$ and $\sigma_\pi(E_L)$, 10^{-27} cm^2

E_L, Bev	20	30	40	50	60	75
σ'_π	0.146	0.395	0.656			
σ_π	1.09	2.54	3.88	5.05	6.08	7.38

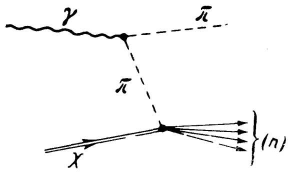

FIG. 2

The second row of the table shows the data which indicate the dependence of the cross section $\sigma_\pi(E_L)$ on E_L corresponding to the production in the reaction (1) of π mesons with arbitrary momenta satisfying condition (2). This cross section corresponds to π mesons which, in the l.s., can also have small energy ($\epsilon_L < 2\mu E_L/m$). The magnitude of this cross section is obtained by introducing the total cross section $\sigma_p(\omega)$ for the reaction (4) into Eq. (6), instead of the differential cross section $2\pi\sigma_p(\omega, \theta_c) \sin\theta_c d\theta_c$, and then carrying out a numerical integration with respect to ω between the limits $\omega_1 = \mu(1 + \mu/2m)$ [the energy threshold of reaction (4)] and ω_{max} [determined by Eq. (7)]

$$\sigma_\pi(E_L) = \frac{Z^2\alpha}{\pi} \int\limits_{\omega_1}^{\omega_{max}} \left[2\ln\frac{\omega_{max}}{\omega} - 1 + \left(\frac{\omega}{\omega_{max}}\right)^2\right] \sigma_p(\omega)\frac{d\omega}{\omega}. \quad (32)$$

The cross sections $\sigma_\pi(E_L, \epsilon_L)$ and $\sigma'_\pi(E_L)$ can be calculated using the available experimental data only for a primary proton energy $E_L < 40$ Bev. According to Eq. (7), ω_{max} which, for A = 207, is greater than 1 Bev, corresponds to the energy E_L greater than 40 Bev. There are so far no corresponding data for the photoproduction cross section $\sigma_p(\omega, \theta_c)$ for a γ ray energy greater than 1 Bev. The cross section $\sigma_\pi(E_L)$ may, with good accuracy, also be calculated for $E_L > 40$ Bev. This is because the main contribution to the integral in the right-hand side of (32) is due to relatively low frequencies corresponding to the $^3\!/_2$, $^3\!/_2$ resonance.

In a recently published article,[3] Drell came to the conclusion that accelerated electrons rather than protons should result in a greater yield of ultra-high energy π mesons even at γ ray (produced in the bremsstrahlung of electrons) and proton energies $E_L = 25$ Bev. However, in estimating the cross section for the production of fast π mesons by collision of protons with nucleons, Drell based himself on the results of the statistical model. The applicability of this model for processes in which all the energy of the colliding particles is transmitted to a single particle, is clearly very doubtful. On the other hand, the photoproduction of π mesons on nuclei was calculated by Drell using the polar approximation, i.e., taking into account the diagrams of the type shown in Fig. 2 [on which X and (n) correspond to the nucleus in the initial and final states]. In application to processes involving nuclei, the validity of such an approximation is especially doubtful (see the remarks of Gell-Mann[4] referring to the article of Drell), since it is possible that the nearest singularity in the value of the momentum transferred to the nucleus in the amplitude of such processes is not due to the mass μ of the π mesons but to a smaller quantity $1/R = \mu/A^{1/3}$ (where R is the nuclear radius).

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