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THE COLLECTIVE GYROMAGNETIC RATIO FOR ODD ATOMIC NUCLEI

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A general expression for the gyromagnetic ratio of even-odd deformed nuclei is obtained by taking pair-correlation effects into account. It is demonstrated that the gyromagnetic ratio of odd proton nuclei is greater, and that of odd neutron nuclei smaller, than the ratio of neighboring even-even nuclei. The results are compared with the experiments.

THE gyromagnetic ratios of several nuclei in the rare-earth region have already been measured.[1] The gyromagnetic ratios for excited rotational states of even-even nuclei are found to be of the order 0.2 — 0.4. For odd-proton nuclei, g_R^p is greater than the gyromagnetic ratio for the neighboring even-even nuclei, while for odd-neutron nuclei g_R^n is less.

Although the ratio Δ_p/Δ_n[2] ($\Delta_{n,p}$ is a quantity characterizing the pair correlation) can be calculated from the gyromagnetic ratio of an even-even nucleus, it is a difficult quantity to measure. It is therefore of interest to obtain a general expression for g_R of an odd nucleus and to relate it with the gyromagnetic ratio for the even nucleus.

The gyromagnetic ratio is defined by

$$g_R = \langle \mu \rangle / \langle \mathbf{I} \rangle, \tag{1}$$

where $\langle \mu \rangle$ and $\langle \mathbf{I} \rangle$ are the mean values of the magnetic moment and the angular momentum of the nucleus. One of the authors, together with Drozdov and Zaretskii,[3] has calculated the density matrix of an odd rotating system. With the aid of this matrix it is easy to calculate the mean values of the angular momentum and magnetic moment operators

$$\mu = \sum (g_l \mathbf{l} + g_s \mathbf{s}).$$

The summation is over all the nucleons of the nucleus.

If the angular velocity Ω of the rotating system is parallel to the x axis, which in turn is perpendicular to the symmetry axis z of the nucleus, then $\langle \mathbf{I} \rangle = J\Omega$, where J is the moment of inertia of the nucleus and

$$\langle \mu \rangle = [J_p + (g_s^p - 1) W^p + g_s^n W^n]\,\Omega. \tag{2}$$

The value of W is given by

$$W^{n,p} = \sum_{\lambda\lambda'} \frac{(E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} - \Delta^2_{n,p}) j^x_{\lambda\lambda'} s^x_{\lambda\lambda'} - i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda'}) s^x_{\lambda\lambda'} f_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}$$

$$+ \sum_\lambda \frac{2(E^2_{\lambda_0} + \varepsilon_\lambda \varepsilon_{\lambda_0} + \Delta^2_{n,p}) j^x_{\lambda\lambda_0} s^x_{\lambda_0\lambda} + i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda_0})(s^x_{\lambda\lambda_0} f_{\lambda_0\lambda} - s^x_{\lambda_0\lambda} f_{\lambda\lambda_0})}{E_{\lambda_0}(\varepsilon^2_\lambda - \varepsilon^2_{\lambda_0})}, \qquad E_\lambda = \sqrt{\varepsilon^2_\lambda + \Delta^2_{n,p}}. \tag{3}$$

Here ϵ_λ is the single-particle energy reckoned from the Fermi surface ϵ_0, and λ_0 is the state of the odd particle. The subscripts p and n denote summation in (3) over the proton and neutron states. The neutron and proton systems are assumed in this paper to be non-interacting.

The value of f is determined from the integral equation (see [4]). If we write $f = f^0 + f'$, where f^0 is the solution of the integral equation for the even-even nucleus and f' is a quasi-classical small change in the solution to account for the presence of the odd particle, then expression (3) can be rewritten

$$W^{n,p} = W_e^{n,p}(\varkappa_0) - \sum_{\lambda\lambda'} \frac{i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda'}) s^x_{\lambda\lambda'} f'_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}$$

$$+ \sum_\lambda \frac{2(E^2_{\lambda_0} + \varepsilon_\lambda \varepsilon_{\lambda_0} + \Delta^2_{n,p}) j^x_{\lambda\lambda_0} s^x_{\lambda_0\lambda} + 2i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda_0}) s^x_{\lambda\lambda_0} f^0_{\lambda_0\lambda}}{E_{\lambda_0}(\varepsilon^2_\lambda - \varepsilon^2_{\lambda_0})}; \tag{4}$$

$$W_e^{n,p}(\varkappa_0) = \sum_{\lambda\lambda'} \frac{(E_\lambda E_{\lambda'} - \varepsilon_\lambda \varepsilon_{\lambda'} - \Delta^2_{n,p}) j^x_{\lambda\lambda'} s^x_{\lambda'\lambda} - i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda'}) s^x_{\lambda\lambda'} f^0_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}, \tag{5}$$

while κ_0 is determined from the values of Δ and β of the odd nucleus; in order of magnitude we have $\kappa \sim \epsilon_0\beta/\Delta A^{1/3}$.

It is convenient to write the general expressions for the gyromagnetic ratios g_R^n and g_R^p of the odd-neutron and odd-proton nuclei in the form

$$g_R^n = g_R J_e/J_0 + g_s^n [W_e^n(\varkappa_0) - W_e^n(\varkappa_e)]/J_0 + \delta g_R^n, \tag{6}$$

$$g_R^p = 1 - (1 - g_R) J_e/J_0 + (g_s^p - 1)[W_e^p(\varkappa_0) - W_e^n(\varkappa_e)]/J_0 + \delta g_R^p;$$

$$g_R = [J_e^p + g_s^n W_e^n + (g_s^p - 1) W_e^p]/J_e, \tag{7}$$

g_R is the gyromagnetic ratio of the even nucleus. An analogous expression for g_R with account of f^0 was obtained by Nilsson and Prior.[5] It is readily seen from (4) and (5) that

$$\delta g_R^{n,p} = \frac{g_s^{n,p} - g_l^{n,p}}{J_0}\Big[\sum_\lambda \frac{2(E_{\lambda_0}^2 + \varepsilon_\lambda \varepsilon_{\lambda_0} + \Delta_{n,p}^2)\, j_{\lambda\lambda_0}^x s_{\lambda_0\lambda}^x + 2i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda_0}) s_{\lambda\lambda_0}^x f_{\lambda_0\lambda}^0}{E_{\lambda_0}(\varepsilon_\lambda^2 - \varepsilon_{\lambda_0}^2)} - \sum_{\lambda\lambda'} \frac{2i\Delta_{n,p}(\varepsilon_\lambda - \varepsilon_{\lambda'}) s_{\lambda\lambda'}^x f'_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})}. \qquad (8)$$

On going from the even nucleus to the odd one, the deformation remains practically unchanged, and the change in the pairing energy Δ is determined by the following expression[2] $[\Delta = (\Delta_o + \Delta_e)/2]$

$$(\Delta_e - \Delta_o)/\Delta = 1/2\,\rho_0\Delta \sim A^{-1/3}.$$

Since the function $W^{n,p}$ is smooth, its variation as N or Z changes by unity is

$$W(\varkappa_e) - W(\varkappa_o) \sim W(\varkappa_e)\,\delta\varkappa/\varkappa \sim W(\varkappa_e)\,A^{-1/3}.$$

Let us show further that $W \sim JA^{-2/3}$. For this purpose we consider the first term in (5) (without the small term containing f^0):

$$W_1 = \sum_{\lambda\lambda'}\Big[1 - g\Big(\frac{\varepsilon_\lambda - \varepsilon_{\lambda'}}{2\Delta}\Big)\Big] j_{\lambda\lambda'}^x s_{\lambda'\lambda}^x\, \delta(\varepsilon_\lambda).$$

As shown by Migdal[2] with a rectangular potential well model, it is necessary to retain in this sum only the values of λ' for which $m' = m \pm 1$ and $\nu = \nu'$, where m is the eigenvalue of the projection of the angular momentum on the symmetry axis and ν is the totality of the remaining quantum numbers. With quasi-classical accuracy the matrix element s^x for such transitions is equal to

$$s_{\lambda\lambda'}^x = \pm j_{\lambda\lambda'}^x/2l,$$

where the plus sign corresponds to $j = l + 1/2$ and the minus sign to $j = l - 1/2$ (l is the quantum number of the orbital angular momentum), and therefore W_1 will contain two sums over $j = l + 1/2$ and $j = l - 1/2$, with opposite signs; these sums cancel in the quasi-classical approximation. W_1 is equal to the next term in the expansion in $A^{-1/3}$, the order of which is $JA^{-2/3}$. Analogous arguments can be cited for the second term in W, since $f^0 \sim j^x$; as a result we get $W \sim JA^{-2/3}$. Naturally, the same estimate for W is obtained also in the second limiting model, an oscillator nuclear potential with spin-orbit coupling.

For heavy nuclei $g_s - g_l$ is numerically of the order of $A^{1/3}$, so that we obtain for the second term in (6) and (7) the estimate $(g_s - g_l)\,[W(\kappa_o) - W(\kappa_e)]/J \sim A^{-2/3}$.

Let us examine the expression for δg_R. Since $f' = f_0/2\rho_0\Delta \sim f_0 A^{-1/3}$, we obtain for the same reason as before

$$(g_s - g_l)\sum_{\lambda\lambda'} \frac{i\Delta(\varepsilon_\lambda - \varepsilon_{\lambda'}) s_{\lambda\lambda'}^x f'_{\lambda'\lambda}}{2E_\lambda E_{\lambda'}(E_\lambda + E_{\lambda'})} \sim JA^{-2/3},$$

and since the first term in (8) contains a single sum, its order is $A^{-1/3}$. Terms containing f^0 in the expression for δg_R do not exceed 20 percent of the principal term and can therefore be estimated by using the expression derived by Migdal[2] for f^0 in an oscillator nuclear potential.

Discarding terms of order $A^{-2/3}$ in (6) and (7) we obtain an approximate expression for the gyromagnetic ratio of odd nuclei when $\epsilon_{\lambda_0} = 0$:

$$g_R^n = g_R J_e/J_o + \delta g_R^n, \qquad (9)$$

$$g_R^p = 1 - (1 - g_R) J_e/J_o + \delta g_R^p; \qquad (10)$$

$$\delta g_R^{n,p} = \frac{(g_s - g_l)}{J_o\Delta}\sum_\lambda s_{\lambda\lambda_0}^x j_{\lambda_0\lambda}^x \Big(\frac{1}{\nu_\lambda^2} - \frac{g_1 + g_2}{g_1\nu_1^2 + g_2\nu_2^2}\Big);$$

$$\nu_\lambda = \frac{(\varepsilon_\lambda - \varepsilon_{\lambda 0})}{2\Delta}, \qquad \nu_1 = \frac{\omega_0 3}{2\Delta}, \qquad \nu_2 = \frac{\omega_0}{\Delta},$$

$$g_x = \frac{\operatorname{arg\,sh} x}{x\sqrt{1 + x^2}}, \qquad (11)^*$$

where $g_1 = g(\nu_1)$ and $g_2 = g(\nu_2)$.

In (9) and (10) δg_R is always smaller than the first term, by a factor $A^{-1/3}$. Consequently we can neglect in the approximate analysis its contribution to $g_R^{n,p}$. Recognizing that in the rare-earth region we have in the mean $J_e^n/J_o^n = 0.65$, $J_e^p/J_o^p = 0.9$ and $g_R = 0.3$, we get $g_R^n \approx 0.2$ and $g_R^p \approx 0.35$, i.e., the relation $g_R^p > g_R > g_R^n$ is satisfied in the mean.

In a more exact calculation of the gyromagnetic ratio of odd nuclei we must take δg_R into account. Although this term is smaller than the first term, it fluctuates more strongly, owing to the matrix element $s_{\lambda\lambda_0}^x$, and in some cases it may make an appreciable contribution to (9) and (10).

The gyromagnetic ratios $g_R^{n,p}$ were calculated from formulas (9), (10), and (11), and since the spin-orbit coupling is essential for δg_R, this quantity was calculated in the Nilsson model.[6] The values of Δ_n and Δ_p were taken from the paper by Nilsson and Prior.[5] The values of the gyromagnetic ratio g_R of an even-even nucleus having one less particle than the original odd nucleus were taken from the same paper,[5] while

*sh = sinh.

Table I

Nucleus	State of odd particle $Nn_z\ \Lambda\Omega$	J_e/J_o	g_R	δg_R	g_{R1}	g_R^p	
						Theory	Experiment
$_{63}Eu_{90}^{153}$	4 1 3 5/2	0.64	0.34	—0.14	0.58	0.44	0.45±0.01
$_{65}Tb_{94}^{159}$	4 1 1 3/2	0.82	0.32	—0.07	0.44	0.37	0.24±0.09
$_{67}Ho_{98}^{165}$	5 2 3 7/2	0.83	0.30	0.11	0.42	0.53	0.30±0.07
$_{71}Lu_{104}^{175}$	4 0 4 7/2	0.99	0.31	—0.03	0.32	0.29	0.30±0.05
$_{73}Ta_{108}^{181}$	4 0 4 7/2	0.97	0.28	—0.08	0.30	0.22	0.33±0.01
$_{75}Re_{110}^{185}$	4 0 2 5/2	0.95	0.28	—0.02	0.31	0.29	0.41±0.04
$_{75}Re_{112}^{187}$	4 0 2 5/2	0.93	0.35	—0.03	0.40	0.37	0.41±0.04

Table II

Nucleus	State of odd particle $Nn_z\ \Lambda\Omega$	J_e/J_o	g_R	δg_R	g_{R1}	g_R^n	
						Theory	Experiment
$_{64}Gd_{91}^{155}$	5 2 1 3/2	0.59	0.37	0.05	0.22	0.27	0.34±0.07
$_{64}Gd_{93}^{157}$	5 2 1 3/2	0.74	0.33	0.04	0.24	0.28	0.22±0.06
$_{66}Dy_{95}^{161}$	6 4 2 5/2	0.43	0.32	—0.05	0.14	0.09	0.25±0.10
$_{68}Er_{99}^{167}$	6 3 3 7/2	0.65	0.30	—0.08	0.19	0.11	0.12±0.05
$_{70}Yb_{103}^{173}$	5 1 2 5/2	0.87	0.31	0.01	0.27	0.28	0.20±0.09
$_{72}Hf_{105}^{177}$	5 1 4 7/2	0.85	0.25	0.04	0.21	0.25	0.21±0.01
$_{72}Hf_{107}^{179}$	6 2 4 9/2	0.71	0.25	—0.08	0.18	0.10	0.20±0.03

the moments of inertia were obtained by experiment. It was found that δg_R amounts in the mean to 20 — 25 percent of the first term in (9) or (10). The values obtained for the gyromagnetic ratios of odd nuclei are listed in Tables I and II for protons and neutrons respectively. In these tables $g_{R_1}^{n,p}$ corresponds to the first term in the expressions (9) or (10).

The theory is in qualitative agreement with experiment. No pretense of quantitative agreement can be made at present since the known experimental values of $g_R^{n,p}$ are subject to considerable errors. In addition, this quantity is not measured in the experiments directly, but is obtained indirectly from the probabilities of the magnetic transitions and the magnetic moments. On the other hand, it must be noted that the wave functions of Nilsson potential do not yield correct values of the magnetic moments of the odd nuclei. We are therefore not even assured of the correctness of the values of δg_R calculated in the Nilsson model. This circumstance may change the theoretical value of g_R by 10 — 20 percent.

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