ANALYSIS OF THE DISTRIBUTIONS OF THE TRANSVERSE MOMENTA OF PIONS AND STRANGE PARTICLES

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The experimental distributions of the transverse momenta of π mesons and strange particles are analyzed on the basis of various theoretical models. According to the quasi-unidimensional hydrodynamic theory, π mesons and strange particles should be emitted at the same temperature $kT \approx m_\pi c^2$, whereas according to the field theory the radius of the region of π-meson production should be approximately twice the size of the region of hyperon and K-meson emission.

MANY recently published papers are devoted to an interpretation of the distribution of the transverse momenta of shower particles from the point of view of various theoretical representations.[1-3] From an analysis of these papers it follows that apparently the distribution of $p_\perp$ is not very sensitive to the type of interaction responsible for multiple generation. Nonetheless, information on the $p_\perp$ spectra can prove to be quite useful, for through the use of certain theoretical concepts one can obtain, with good accuracy, the values of certain physical parameters of interest to the theory under consideration from the distribution of the transverse momenta. We shall attempt to analyze the spectra of the transverse momenta of pions and strange particles from the point of view of two interaction models, which in a certain sense are contrary.

1. Let us assume that the process of multiple generation of particles is described by a quasi-unidimensional variant[1] of the hydrodynamic theory. In this case, owing to the absence of hydrodynamic liquid flow in the transverse direction, the $p_\perp$ distribution will have the form

$$\frac{dN_\perp}{N_\perp} = \frac{p_\perp dp_\perp}{(mc)^2}\frac{mc^2}{kT}\sqrt{\left(\frac{p_\perp}{mc}\right)^2+1}\cdot \times \frac{\sum\limits_{n=0}^{\infty}(\mp 1)^{n-1}K_1\left(n\frac{mc^2}{kT}\sqrt{(p_\perp/mc)^2+1}\right)}{\sum\limits_{n=0}^{\infty}(\mp 1)^{n-1}K_2(nmc^2/kT)/n} \qquad (1)$$

and will be characterized by a mean-square value

$$\langle p_\perp^2\rangle = 2\frac{kT}{mc^2}\frac{\sum\limits_{n=0}^{\infty}(\mp 1)^{n-1}K_2(nmc^2/kT)/n^2}{\sum\limits_{n=1}^{\infty}(\mp 1)^{n-1}K_2(nmc^2/kT)/n} \qquad (2)$$

where T is the dispersal temperature of the system, m is the mass of the investigated particle, $K_1(x)$, $K_2(x)$, and $K_3(x)$ are Bessel functions of imaginary argument, and the $\pm$ signs correspond to Bose and Fermi particles.

Comparing the spectrum[1] at different values of the dispersal temperature with the experimental distribution of $p_\perp$ for pions[2,4,5] and for a mixture of Λ^0, θ^0, and $\Sigma^\pm$ particles[6] (Figs. 1a and b) we readily see that the hydrodynamic theory describes satisfactorily both spectra at a temperature kT on the order of the pion rest mass.

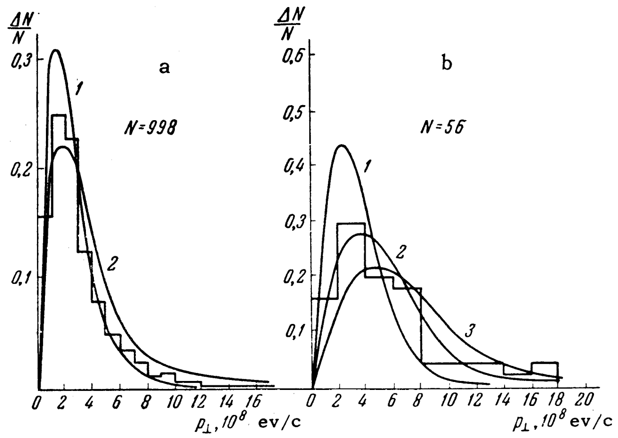

FIG. 1. a – $p_\perp$ distribution of pions registered in penetrating cosmic-ray showers with the aid of magnetic cloud chambers.[2,4,5] Curves 1 and 2 correspond to $kT/m_\pi c^2 = 1$ and 1.5. b – $p_\perp$ distribution of mixture of strange particles registered in penetrating cosmic-ray showers with the aid of a magnetic cloud chamber.[6] Curves 1, 2, and 3 correspond to $kT/m_\pi c^2 = 0.5$, 1, and 1.5. N – number of particles included in the histogram.

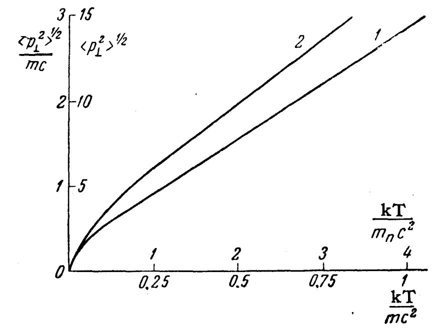

FIG. 2. Curve 1 – dependence of $\langle p_\perp^2\rangle^{1/2}/mc$ on kT/mc^2. Curve 2 – dependence of $\langle p_\perp^2\rangle^{1/2}$ on $kT/m_\pi c^2$ for the mixture of Λ^0, θ^0, and $\Sigma^\pm$ observed in.[6] The transverse momenta are in units of 10^8 ev/c.

We can determine the dispersal temperature numerically by using the mean-square momenta $p_\perp$ of pions and strange particles with the aid of the curves shown in Fig. 2. Curve 1 corresponds to the dependence of $\langle p_\perp^2\rangle^{1/2}$ on kT for bosons of arbitrary mass, since both variables are plotted on this curve in mass units. At temperatures kT several times smaller than the nucleon mass, the curve can also be applied to baryons. Curve 2 represents the same dependence of $\langle p_\perp^2\rangle^{1/2}$ on kT for the mixture of strange particles observed in [6] (46% Λ^0, 32% θ^0, and 23% $\Sigma^\pm$). In Fig. 2 the value of $\langle p_\perp^2\rangle^{1/2}$ is in units of 10^8 ev/c, and kT is given in pion masses.

The mean-squared momentum $p_\perp$ corresponding to the histogram 1a has a value

$$\langle p_\perp^2\rangle^{1/2} = \left(3.86 {+0.48 \atop -0.05}\right)\cdot 10^8 \text{ ev/c},$$

Assuming that all the particles included in the distribution are pions, we obtain from curve 1 of Fig. 2 a critical temperature

$$kT = \left(0.96 {+0.14 \atop -0.01}\right) m_\pi c^2.$$

An analogous estimate can be made also for the dispersal temperature of strange particles, using curve 2 of Fig. 2 and the experimental value of $\langle p_\perp^2\rangle^{1/2}$ corresponding to the histogram 1b:

$$\langle p_\perp^2\rangle^{1/2} = \left(6.56 {+2.01 \atop -0.39}\right) 10^8 \text{ ev/c},$$

which leads to

$$kT = \left(1.1 {+0.6 \atop -0.1}\right) m_\pi c^2.$$

We see thus that the $p_\perp$ distribution gives a sufficiently accurate value of the dispersal temperature of pions and strange particles, provided that the hydrodynamic theory fits the process of multiple generation of particles at high energies.

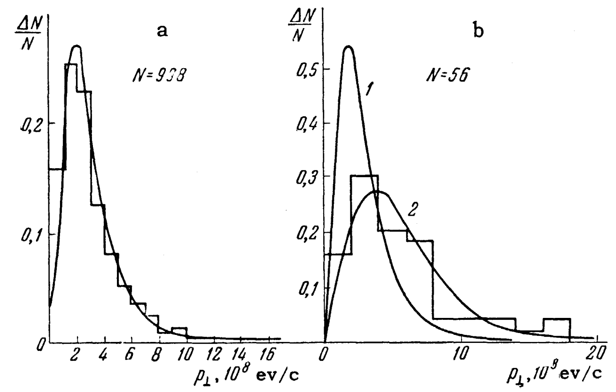

FIG. 3. a – $p_\perp$ distribution of pions. The curve corresponds to an exponential distribution of source density with $\langle r^2\rangle^{1/2} = 0.8 \times 10^{-13}$ cm. b – $p_\perp$ distribution of a mixture of strange particles. Curves 1 and 2 correspond to an exponential distribution of the source density with $\langle r^2\rangle^{1/2} = 0.8 \times 10^{-13}$ cm and $\langle r^2\rangle^{1/2} = 0.4 \times 10^{-13}$ cm respectively.

2. In the Heisenberg theory,[7] however, $\langle p_\perp^2\rangle^{1/2}$ determines the transverse dimension of the region where particles are generated, in accordance with the uncertainty principle

$$\langle p_\perp^2\rangle^{1/2}\langle r_\perp^2\rangle^{1/2} \geqslant \hbar.$$

The values which we have used for the mean squares of the pion and strange-particle momenta lead to a certain structural representation of the region in which the radius of the volume from which pions are emitted is

$$\langle r_\perp^2\rangle^{1/2} \geqslant 0.52\cdot 10^{-13} \text{ cm},$$

and the radius for strange particles is

$$\langle r_\perp^2\rangle^{1/2} \geqslant 0.30\cdot 10^{-13} \text{ cm}.$$

Neglecting the interaction of the emitted particles (which makes the following analysis essentially contradict the hydrodynamic analysis), we can relate not only $\langle p_\perp^2\rangle^{1/2}$ but the entire distribution of the transverse momenta with the structural generation region* by assuming a) that there exists a certain coordinate system in which the momentum particle spectrum $|\varphi(p)|^2$ is independent of the angle of emission and b) that the connection between the momentum spectrum in this coordinate system and the probability density $|\rho(r)|^2$ of observing a particle at a distance r from the center of the region is given by the Fourier integral

$$\varphi(p) = \int \rho(r)\, e^{-i\mathbf{pr}} d\mathbf{r}. \qquad (3)$$

*A similar attempt is cited by Nagai and Ito.[8]

By specifying the form of the function $|\rho(r)|^2$ and integrating the resultant function $\varphi(p)$ with respect to the longitudinal momentum $p_{||}$ we can obtain the distribution over $p_\perp$ and compare it with the experimental histogram. It turns out that for many functions by suitable choice of the parameters (in particular, $\langle r_\perp^2\rangle^{1/2}$) we can reconcile the resultant $p_\perp$ spectra with those experimentally observed. Agreeing best with the distribution of the transverse pion momenta is a probability density that decreases exponentially with the distance

$$|\rho(r)|^2 = \exp[-(12/\langle r^2\rangle)^{1/2} r] \quad (4)$$

with a parameter

$$\langle r^2\rangle^{1/2} = 0.8\cdot 10^{-13} \text{ cm}.$$

It is interesting to note that the function (4) with the same value of $\langle r^2\rangle^{1/2}$ describes best of all the electromagnetic structure of protons investigated by scattering electrons with energies up to 550 Mev.[9]

If it is assumed that the function $|\rho(r)|^2$ has the same form for strange particles, the corresponding mean-squared radius has approximately half the value, about 0.4×10^{-13} cm.

The foregoing is illustrated in Figs. 3a and b, on which the $p_\perp$ distribution corresponding to an exponential density is compared with the histogram of the transverse momenta of pions and strange particles.

3. The foregoing analysis of the $p_\perp$ distributions leads to the following conclusions:

a) No preference can be given to any of the multiple-generation theories on the basis of the distribution of the transverse momenta of pions and strange particles.

b) From the point of view of the quasi-unidimensional hydrodynamic constant, the $p_\perp$ spectra of pions and strange particles show that the system breaks up at a temperature $kT \approx m_\pi c^2$.

c) If the field-theoretical description of the process of multiple generation is correct, it must be assumed that the region for the emission of strange particles has approximately half the radius as that for the generation of pions.

d) Finally, if it is assumed that the amplitudes of the momentum spectrum of the generated particles and of the spatial distribution of the source can be related by a Fourier integral, then the $p_\perp$ spectrum of the pions is in best agreement with an exponential distribution of the source density, with $\langle r^2\rangle^{1/2} = 0.8 \times 10^{-13}$ cm.

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