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APPLICATION OF THE POLE METHOD TO THE ANALYSIS OF EXPERIMENTAL DATA ON πp INTERACTIONS

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Various kinematic characteristics of the process $\pi + N \rightarrow \pi + \pi + N$ are examined. On this basis, a method is proposed for separating other events from events due to the pole diagram considered by Chew and Low.[1] The problem of improving the extrapolation procedure proposed by Chew and Low is discussed.

THE pole method proposed by Chew and Low[1] for studying the πp interactions has lately achieved great popularity. The inelastic interaction of a π meson with a nucleon, for example the reaction $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ ($\pi^- + \pi^+ + n$), is described in this method by the diagram of Fig. 1.

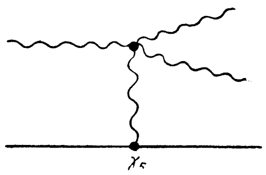


FIG. 1. The pole diagram for the process $\pi + N \rightarrow \pi + \pi + N$. The straight lines correspond to nucleons; the wavy lines correspond to π mesons.

The cross section $\sigma_{\pi p}$ for the πp interaction is in this case related to other quantities in the following manner[1]:

$$F(\omega^2, \Delta^2) = \frac{\partial^2 \sigma_{\pi p}}{\partial \Delta^2 \partial \omega^2} (\Delta^2 + \mu^2)^2$$

$$= \frac{f^2}{4\pi} \frac{\Delta^2}{q_0^2 \mu^2} \sqrt{\omega^2 (\omega^2 - 4\mu^2)}\, \sigma_{\pi\pi}(\omega), \tag{1}$$

where f^2 is the coupling constant, equal to 0.08; μ is the π-meson mass; m is the nucleon mass; q_0 is the momentum of the primary π meson in the laboratory coordinate system (l.s.); ω is the energy of the two secondary π mesons in their center of mass system (c.m.s.); $\sigma_{\pi\pi}$ is the cross section for the mutual scattering of two π mesons; Δ^2 is the square of the four-momentum of the intermediate π meson related to the recoil energy of the nucleon: $\Delta^2 = 2mT$ (T is the kinetic energy of the recoil nucleon in the l.s.).

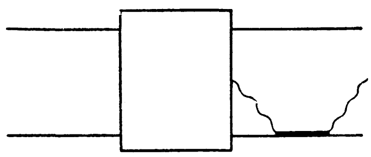
Thus, by measuring the quantities $\sigma_{\pi p}$, Δ^2, and ω one can obtain information about $\sigma_{\pi\pi}(\omega)$. For this it is sufficient, as has been shown in [1], to extrapolate the quantity $F(\omega^2, \Delta^2)$ from the region $\Delta^2 > 0$ to the point $\Delta^2 = -\mu^2$. At this point

$$\sigma_{\pi\pi} = -4\pi f^{-2} F(\omega^2, -\mu^2)\, q_0^2 [\omega^2 (\omega^2 - 4\mu^2)]^{-1/2}.$$

If all the experimentally recorded cases are indeed due to the diagram of Fig. 1 then, in accordance with (1), $F(\omega^2, \Delta^2)$ must depend linearly on Δ^2, and the extrapolation procedure in this case is simple and does not introduce a large error.

However, in addition to the possibility represented by the diagram of Fig. 1 the process may occur in other ways as well. For example, the following competing processes have been discussed in the literature: a) the so-called head-on collisions described by the statistical theory[2] and characterized by an isotropic distribution of the secondary particles in the c.m.s.; b) processes in which there is a πN interaction in the final state, the diagram for which is given in Fig. 2.

FIG. 2. Diagram of the process taking into account the πN interaction in the final state (the thick line represents the isobar 3/2, 3/2).



Moreover, terms can appear in the cross section which represent interference between matrix elements of different diagrams.* If the aforementioned processes make the same contribution to the cross section (or, more accurately, to the quantity $F(\omega^2, \Delta^2)$) as does the diagram of Fig. 1, then F will no longer depend linearly on Δ^2, but will be given in the general case by the polynomial

$$F(\omega^2, \Delta^2) = a_0 + a_1 (\Delta^2/\mu^2) + a_2 (\Delta^2/\mu^2)^2 + \ldots.$$

*It is impossible to assume that the contribution due to the diagram of Fig. 1 will be predominant for small Δ^2, since in accordance with (1) the quantity $F(\omega^2, \Delta^2)$ also tends to zero as $\Delta^2 \rightarrow 0$.

It should be emphasized that in this case all the terms of the polynomial (up to the fourth-degree term) must be of the same order (in absolute value) within the region $|\Delta^2/\mu^2| \lesssim 1$.

In this connection it appears to us to be reasonable to pick out in advance from the experimental material those cases which are indeed due to the process shown in Fig. 1.* We consider below one of the possible methods of making such a selection. The method is based on the qualitative difference in the kinematic characteristics of the various processes. At the present time it is impossible to consider all the processes. Therefore, in addition to the pole process we shall discuss only the most "competitive" processes a) and b).

We investigate the kinematic characteristics of the process shown in Fig. 1 for a π-meson energy $E_{lab} = 1$ Bev in the l.s., since the experimental data[3,4] refer to this energy. We first assume that $\sigma_{\pi\pi}(\omega)$ is constant in the range $0.3 \leq \omega \leq 0.7$ Bev. Then on integrating (1) with respect to Δ^2 over the range consistent with the conservation laws we can easily obtain the quantity $d\sigma_{\pi p}/d\omega$, i.e., the distribution of events with respect to the magnitude of ω. This distribution is shown in Fig. 3. It is bell-shaped with a maximum at $\omega = 0.55$ Bev and has a half width $\Delta\omega \sim 0.3$ Bev.

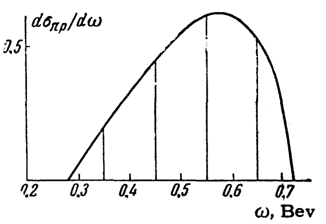


FIG. 3. The distribution $d\sigma_{\pi p}/d\omega$ calculated for the diagram of Fig. 1; ω is the energy of the two π mesons in their c.m.s.

The angular distribution of the nucleons in the c.m.s. of the nucleon and the primary π meson can be easily obtained by assuming $\omega = \overline{\omega} = 0.55$ Bev. It has the form

$$d\sigma \sim \frac{1.13 - \cos\theta}{(1.2 - \cos\theta)^2} d\cos\theta.$$

From this it can be seen that the angular distribution is fairly broad. Therefore, this characteristic cannot be utilized for discriminating between the pole process and, for example, a "head-on" collision.

The distribution of the π mesons with respect to the angle φ between them (in the c.m.s.) is shown in Fig. 4. Curve 1 corresponds to the process of Fig. 1;* curve 2 corresponds to the process for which a "πN isobar" of mass $\mathfrak{M} = 1.3$ m, isospin $T = 3/2$ and spin $J = 3/2$ is formed in the final stage. This process corresponds to the diagram of Fig. 2.† The dotted curve was obtained on the assumption of isotropic distribution of all the particles (π mesons and nucleon) in the c.m.s. taking the conservation of momentum into account.[6] It can be seen from the diagram that curves 2 and 3 are close to one another, but curve 1 is appreciably different.

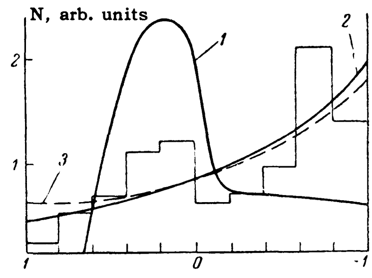


FIG. 4. Distribution with respect to the angle φ between two π mesons in the c.m.s. The curves correspond to: 1 – the pole diagram of Fig. 1, 2 – the process in which the isobar participates, 3 – isotropic angular distribution. The histogram represents the experimental data of[4]. All the curves are normalized in the same way (N is the number of events).

We note that an estimate of the interference between the matrix elements of the processes considered here can also be made using the curves of Fig. 3. Indeed, we can always choose the angle φ as one of the independent angular variables. Then the regions of overlap of the curves of Fig. 3 give some information about the order of magnitude of the interference terms. It can be seen from the graph that even if all the processes make equal contributions the interference between them is still not very great. It is impossible to estimate in advance the total contribution of the diagram of Fig. 1 to the process of πN interaction since it

*Of course, in this case, the statistical accuracy in the determination of $\sigma_{\pi p}$ [or more precisely, of the quantity $F(\omega^2, \Delta^2)$] will be reduced, but the accuracy of the extrapolation will be significantly increased.

*In obtaining this it was assumed that the angular distribution of the π mesons in their c.m.s. (denoted in the following by $\pi\pi$ system) is isotropic. In actual fact (for example, if there exists a resonance $\pi\pi$ interaction in the $T = 1$ and $J = 1$ state) this distribution can depend on the angle ϑ measured from the direction of motion **n** of the primary π meson in the $\pi\pi$ system. However, in our case this fact is immaterial, for owing to the broad angular distribution in the c.m.s. the distribution of the directions **n** in the $\pi\pi$ system is nearly isotropic.

†Analogous calculations of angular correlations have already been carried out earlier by Rus'kin and Usik.[5] They were utilized in one of the variants of the statistical theory of multiple production.

depends on the magnitude of $\sigma_{\pi\pi}$, which must itself be determined from experiment.

Experimentally the angular correlation of the π mesons has been measured only by Walker, Hushfur and Shepard[4] at an energy of $E_{lab} = 1$ Bev. Their data are represented by the histogram in Fig. 4. It can be seen that there exist two well resolved maxima: near $\cos\varphi = 0.2$ and $\cos\varphi = -0.8$. Comparison with the curves shows that the region near the first maximum is due to processes taking place in accordance with the diagram of Fig. 1, while the neighborhood of the second maximum is due to other processes. The area under the first maximum amounts to approximately one-third of the total area. Thus, the pole part does not give rise to a dominant contribution to the cross section, but amounts to approximately one-third of the cross section.

The whole discussion has been carried out on the assumption that $\sigma_{\pi\pi}(\omega)$ is a smooth function of the energy ω in the range $\omega \sim 0.3 - 0.7$ Bev. However, at the present time there are some indications of the resonance character of this function. If the resonance occurs in the neighborhood of $\omega_r \sim 0.55$ Bev,[3] then this merely improves the conditions for resolving the maxima in the angular distribution with respect to φ (Fig. 4). If the resonance occurs at $\omega_r \sim 0.7$ Bev, then at an energy of $E_{lab} = 1$ Bev the criteria for performing the separation become worse, but they will again be good at a higher energy. For this it is sufficient to have $\omega_r < \omega_{max}$, where ω_{max} is the maximum value of ω consistent with the conservation laws: $\omega_{max} = (2E_{lab}m + m^2 + \mu^2)^{1/2} - m$. We note that the separation of the maxima in the distribution over the angle φ is significantly improved if we do not utilize all the experimental material, but pick out from it the cases corresponding to small values of Δ^2.

From the above discussion it follows that the angular correlation of the mesons in the c.m.s. can serve as a good criterion for ensuring that the selected cases actually correspond to the diagram of Fig. 1.

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111