LOW-ENERGY LIMIT OF THE γN-SCATTERING AMPLITUDE AND CROSSING SYMMETRY

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The low energy limit for the γN scattering amplitude is derived with the aid of single-nucleon invariant amplitudes. Subsequent terms in ν for $Q^2 = 0$ and the expression for the limiting value of the first derivative in Q^2 as $Q^2 \to 0$ can be obtained by taking into account the conditions of crossing symmetry.

1. Low, Gell-Mann, and Goldberger showed[1] that the condition of relativistic and gauge invariance makes it possible to express the limiting value of the amplitudes for the scattering of low energy γ quanta on spin-$\frac{1}{2}$ particles and the limiting value of the derivative of the amplitude with respect to the frequency as $\nu \to 0$ in terms of the charge and magnetic moment of the particle. This result was later generalized[2] to the case of elastic scattering of γ quanta by particles with other spins and also to the case of bremsstrahlung.[3] The result for elastic scattering also holds when only CP invariance is assumed. Consideration of the single-nucleon terms in the dispersion relations for γN scattering[4-6] also leads to the limit theorem. (A similar result holds for bremsstrahlung.[7])

In the present note, we derive the limit theorem for γN scattering on the basis of the single-nucleon terms. The requirement of crossing symmetry for the invariant functions $T_i(\nu, Q^2)$ (= 1, . . . , 6) makes it possible to obtain additional terms for the limiting values of the functions $R_i(\nu, 0)$, which characterize the γN scattering matrix in the center-of-mass system, and also the limiting values of the derivatives of the amplitudes with respect to Q^2 as $\nu \to 0$. (For the definition of the quantities T_i and R_i see, e.g.,[6].)

2. The invariant functions $T_i(\nu, Q^2)$ are related to the scalar functions $R_i(\nu, Q^2)$ ($i = 1, \ldots, 6$) in the following way:

$$T_1 - T_3 = \frac{8MW^2}{(W^2-M^2)^2}\left[\nu - \frac{W-M}{W+M}\frac{Q^2}{M}\right](R_3+R_4) - \frac{4W}{(M+W)}\left[1-\frac{4Q^2W^2}{(W^2-M^2)^2}\right](R_1+R_2),$$

$$T_2 - T_4 = \frac{8MW^2}{(W^2-M^2)^2}\left[1+\frac{2W}{M}\frac{Q^2}{(W+M^2)}\right](R_3+R_4) + \frac{4W}{(W+M)^2}\left[1-\frac{4Q^2W^2}{(W^2-M^2)^2}\right](R_1+R_2),$$

$$T_1 + T_3 = \frac{8MW^2}{(W^2-M^2)^2}\left[\nu - \frac{W-M}{W+M}\frac{Q^2}{M}\right](R_3-R_4) + \frac{16W^3Q^2}{(W+M)(W^2-M^2)^2}(R_1-R_2),$$

$$T_2 + T_4 = \frac{8MW^2}{(W^2-M^2)^2}\left[1+\frac{2W}{M}\frac{Q^2}{(W+M)^2}\right](R_3-R_4) - \frac{16W^3Q^2}{(W+M)^2(W^2-M^2)^2}(R_1-R_2),$$

$$\frac{M\nu+Q^2}{W^2}T_5 = \frac{8W^2Q^2}{(W^2-M^2)^2}(R_5-R_6) - (R_3-R_4),$$

$$\frac{M\nu+Q^2}{W}T_6 = \left(2-\frac{8W^2Q^2}{(W^2-M^2)^2}\right)(R_5+R_6) + (R_3+R_4), \quad (1)$$

where W is the total c.m.s. energy and ν and Q^2 are two invariants characterizing the kinematics of the process; $W^2 - M^2 = 2M\nu + 2Q^2$.

The pole terms for $T_i(\nu, Q^2)$ have the form[6]

$$T_1^0 = \frac{2e^2}{M}\frac{Q^2}{Q^4/M^2-\nu^2}, \qquad T_2^0 = \frac{e^2}{M}\frac{\nu}{Q^4/M^2-\nu^2}, \qquad T_3^0 = 0,$$

$$T_4^0 = -\frac{e^2(1+\lambda)^2}{M}\frac{\nu}{Q^4/M^2-\nu^2},$$

$$T_5^0 = MT_6^0 = \frac{e^2(1+\lambda)}{M}\frac{Q^2}{Q^4/M^2-\nu^2}, \quad (2)$$

where we have used the system of units in which $\hbar = c = 1$ and the magnetic moment is $\mu = e(1+\lambda)/2M$.

For $Q^2 = 0$, it follows from (1) that

$$(T_1 - T_3)_0 = \frac{2W^2}{M\nu}(R_3+R_4)_0 - \frac{4W}{W+M}(R_1+R_2)_0,$$

$$(T_2 - T_4)_0 = \frac{2W^2}{M\nu^2}(R_3+R_4)_0 + \frac{4W}{(W+M)^2}(R_1+R_2)_0,$$

$$(T_1 + T_3)_0 = \frac{2W^2}{M\nu}(R_3-R_4)_0, \quad (T_2+T_4)_0 = \frac{2W^2}{M\nu^2}(R_3-R_4)_0,$$

$$(T_5)_0 = -\frac{W^2}{M\nu}(R_3-R_4)_0,$$

$$(T_6)_0 = \frac{W}{M\nu}[2(R_5+R_6)+R_3+R_4].$$

Differentiating the relations in (1) with respect to Q^2, we obtain, in the limit $Q^2 = 0$,

$$(T_1 - T_3)'_0 = \frac{2W^2}{M\nu}(R_3 + R_4)'_0 - \frac{4W}{W+M}(R_1 + R_2)'_0$$

$$- \frac{2(2W^3 + M^2W + M^3)}{M^2\nu^2(W+M)}(R_3 + R_4)_0$$

$$+ \frac{4}{W(W+M)}\left[\frac{W^4}{M^2\nu^2} - \frac{M}{W+M}\right](R_1 + R_2)_0,$$

$$(T_2 - T_4)'_0 = \frac{2W^2}{M\nu^2}(R_3 + R_4)'_0 + \frac{4W}{(W+M)^2}(R_1 + R_2)'_0$$

$$+ \frac{4}{M^2\nu^2}\left[\frac{W^3}{(W+M)^2} - M - \frac{M^2}{\nu}\right](R_3 + R_4)_0$$

$$- \frac{4}{(W+M)^2}\left[\frac{W^3}{M^2\nu^2} - \frac{1}{W} + \frac{2}{M+W}\right](R_1 + R_2)_0,$$

$$(T_1 + T_3)'_0 = \frac{2W^2}{M\nu}(R_3 - R_4)'_0$$

$$- \frac{2(2W^3 + WM^2 + M^3)}{M^2\nu^2(W+M)}(R_3 - R_4)_0$$

$$+ \frac{4W^3}{(W+M)M^2\nu^2}(R_1 - R_2)_0,$$

$$(T_2 + T_4)'_0 = \frac{2W^2}{M\nu^2}(R_3 - R_4)'_0$$

$$+ \frac{4(R_3 - R_4)_0}{M^2\nu^2}\left[\frac{W^3}{(W+M)^2} - M - \frac{M^2}{\nu}\right]$$

$$- \frac{4W^3}{(W+M)^2}\frac{(R_1 - R_2)_0}{M^2\nu^2},$$

$$(T_5)'_0 = \frac{W^2}{M\nu}\left[\frac{2W^2}{M^2\nu^2}(R_5 - R_6)_0 - (R_3 - R_4)'_0 + \frac{M}{W^2\nu}(R_3 - R_4)_0\right],$$

$$(T_6)'_0 = \frac{W}{M\nu}\left[-\frac{2W^2}{M^2\nu^2}(R_5 + R_6)_0 + (2R_5 + 2R_6 + R_3 + R_4)' - \frac{M+\nu}{W^2\nu}(2R_5 + 2R_6 + R_3 + R_4)\right]. \quad (4)$$

It is seen from (2) that the terms $T_1 - T_3$ and $T_2 + T_4$ do not contain poles for $Q^2 = 0$. It then follows from (1) that $(R_1 + R_2)_0$ and $(R_3 \pm R_4)_0/\nu$ are finite when $\nu \to 0$.

Since the functions $(T_2 \mp T_4)_0$ have a singularity of the form

$$\left[-\frac{e^2}{M} \mp \frac{e^2}{M}(1+\lambda)^2\right]\frac{1}{\nu},$$

it then follows from (1) that

$$\frac{(R_3 \pm R_4)_0}{\nu} = -\frac{e^2}{2M^2}\left[1 \pm (1+\lambda)^2\right] \quad (5)$$

as $\nu \to 0$, which is in accordance with the limit theorem.

Since T_5 and T_6 do not contain poles when $Q^2 = 0$, the quantity $(R_5 + R_6)/\nu$ should remain constant as $\nu \to 0$. Similarly, from the condition that $(T_1 \pm T_3)'_0$ contains a pole of the second order

$$(T_1 \pm T_3)'_{0p} = -2e^2/M\nu^2,$$

and that $\nu(T_2 - T_4)'_0$ does not contain a pole, it follows that

$$(R_1 \pm R_2)_0 = -e^2/M, \quad (6)$$

and $(R_3 \pm R_4)'_0 \to$ const and $\nu(R_1 \pm R_2)'_0 \to$ const as $\nu \to 0$.

Since

$$(T_5)'_{0p} = M(T_6)'_{0p} = -e^2(1+\lambda)/2M^2,$$

we conclude that

$$(R_5 \pm R_6)_0 = \pm e^2(1+\lambda)\nu/2M^2, \quad (7)$$

and $(2R_5 + 2R_6 + R_3 + R_4)'_0 \to$ const as $\nu \to 0$.

We see that formulas (5)—(7) obtained from consideration of the pole terms (2) contain the results of the limit theorem for $Q^2 = 0$.

3. It is of interest to note that with the aid of the conditions of crossing symmetry one can obtain additional information on the low energy limit. It follows from crossing symmetry that, for example, the quantity $T_1 - T_3$ should be an even function of ν. If in the first relation of (3) we make the substitution

$$(R_1 + R_2)_0 = -\frac{e^2}{M} + \alpha_1\nu + \ldots,$$

$$(R_3 + R_4)_0 = -\frac{e^2}{2M^2}\left[1 + (1+\lambda)^2\right]\nu + \alpha_3\nu^2 + \ldots \quad (8)$$

and take into account the fact that $W = (M^2 + 2M\nu)^{1/2} \approx M(1 + \nu/M)$ for small ν, then from the condition that there is no linear dependence on ν we obtain the relation

$$\alpha_3 M - \alpha_1 = (e^2/M)\left[\tfrac{1}{2} + (1+\lambda)^2\right]. \quad (9)$$

It follows from the requirement of crossing symmetry that the quantity $\nu(T_2 - T_4)$ should be an even function of ν.

The absence of a linear dependence of the terms on ν leads to the relation

$$\alpha_3 M = (e^2/M)\left[\tfrac{3}{2} + (1+\lambda)^2\right]. \quad (10)$$

From (8)—(10) we have

$$(R_1 + R_2)_0 = -\frac{e^2}{M}\left(1 - \frac{\nu}{M}\right) + O(\nu^2),$$

$$(R_3 + R_4)_0 = -\frac{e^2}{2M^2}\left(1 - \frac{3\nu}{M}\right)\nu - \frac{e^2}{2M^2}(1+\lambda)^2\left(1 - \frac{2\nu}{M}\right)\nu + O(\nu^3). \quad (11)$$

The functions $T_1 + T_3$, T_5, $(T_2 + T_4)$, and T_6 should be even functions of ν. Similar considerations lead to

$$(R_3 - R_4)_0 = -\frac{e^2}{2M^2}\left[1 - (1+\lambda)^2\right]\left(1 - \frac{2\nu}{M}\right)\nu + O(\nu^3), \quad (12)$$

and

$$[2(R_5+R_6)+R_3+R_4]_0$$

$$=-\frac{e^2}{2M^2}\lambda^2\left(1-\frac{\nu}{M}\right)\nu+O(\nu^3), \qquad (13)$$

$$(R_5+R_6)_0=\frac{e^2(1+\lambda)}{2M^2}\nu$$

$$+\frac{e^2}{4M^3}[\lambda^2-3-2(1+\lambda)^2]\nu^2+O(\nu^3). \qquad (13')$$

The function $(T_1-T_3)'_0$ is an even function of ν. Inserting in (4)

$$(R_3+R_4)'_0=\alpha'_3+\ldots, \qquad (R_1+R_2)'_0=\alpha'_1/\nu+\ldots,$$

we obtain from the condition that there is no term proportional to $1/\nu$

$$2M\alpha'_3-2\alpha'_1=(e^2/M)[1-2(1+\lambda)^2].$$

A similar condition for the even function $\nu(T_2-T_4)'_0$ leads to

$$2M\alpha'_3=-(e^2/M)[3+2(1+\lambda)^2].$$

Then $\alpha'_1=-2e^2/M^2$, and therefore

$$(R_1+R_2)'_0=-2\frac{e^2}{M^2}\frac{1}{\nu}+O(1),$$

$$(R_3+R_4)'_0=-\frac{e^2}{2M^2}[3+2(1+\lambda)^2]+O(\nu). \qquad (14)$$

The condition that the poles of the first order in the even functions $(T_1+T_3)'_0$ and $\nu(T_2+T_4)$ vanish leads to

$$(R_1-R_2)_0=-\frac{e^2}{M}\left(1-\frac{3\nu}{M}\right)+O(\nu^2),$$

$$(R_3-R_4)'_0=\frac{e^2}{2M^3}[-3+2(1+\lambda)^2]+O(\nu). \qquad (15)$$

Similar conditions for the functions $(T_5)'_0$ and $(T_6)'_0$ require that

$$(R_5-R_6)_0=-\frac{e^2(1+\lambda)}{2M^2}\nu+\frac{e^2}{4M^3}[-2+8(1+\lambda)$$

$$+(1+\lambda)^2]\nu^2+O(\nu^3),$$

$$(2R_5+2R_6+R_3+R_4)'_0=-\frac{e^2}{2M^3}(2\lambda^2-2\lambda-1). \qquad (16)$$

It should be kept in mind that the expression for the limiting energy is valid for amplitudes in the center-of-mass system. The result obtained can be useful for analysis of the scattering of γ quanta by nucleons with the aid of the dispersion relation technique.

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