

EFFECT OF ROTATION ON PAIR CORRELATION IN NUCLEI

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Submitted to JETP editor February 4, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) 41, 445-450 (August, 1961)

Corrections are determined to the quantity Δ characterizing the pair correlation, in the second order of perturbation theory in the rotation. Corrections are estimated to the rotation spectrum, arising from the dependence of Δ and consequently of the moment of inertia on the nuclear spin.

It is well known that a magnetic field reduces the magnitude of the energy gap 2Δ in a superconductor. Since a magnetic field is equivalent to a rotation, the rotation of a system of "paired" particles will also lead to this same effect. The reduction of Δ results in an increase in the moment of inertia. Since the change of Δ depends on the rate of rotation or the spin I of the system, this will lead to corrections to the moment of inertia which are proportional to $I(I+1)$ and consequently to corrections to the energy proportional to $I^2(I+1)^2$.

The calculation of the change in Δ is conveniently done by the method of Gor'kov and Migdal,^[1] using the equation for the Green's function. In the present paper we shall find the diagonal corrections to the solutions of the Gor'kov equation in the second order of perturbation theory and find the dependence of Δ on I . Using the dependence $\Delta(I)$ thus found, we can estimate the corrections to the energy of the rotating system proportional to $I^2(I+1)^2$.

We shall write these equations for the Green's functions

$$G(x_1, x_2) = -i \langle T [\psi(x_1) \psi^+(x_2)] \rangle,$$

$$F = -i \langle T [\psi^+(x_1) \psi^+(x_2)] \rangle \exp(-i2\mu t)$$

in the usual form

$$\begin{aligned} (i\partial/\partial t - H) G - i\Delta F &= \delta(\mathbf{r}_1 - \mathbf{r}_2), \\ (i\partial/\partial t + H^* - 2\mu) F + i\Delta^* G &= 0, \end{aligned} \quad (1)$$

where $\psi(x)$ and $\psi^+(x)$ are the operators for annihilation and creation of particles, μ is the chemical potential, and Δ is a quantity characterizing the pair correlation. The quantity Δ is found from the equation

$$\Delta = \gamma \int_C F^*(\mathbf{r}, \mathbf{r}, \omega) \frac{d\omega}{2\pi}. \quad (2)$$

Here $F(\mathbf{r}, \mathbf{r}, \omega)$ is the time Fourier component of the function F , γ is the interaction constant of the particles, while the contour C consists of the real

axis and the infinite semicircle in the upper half plane.

The total Hamiltonian of the system of particles has the form

$$H = H^0 + H' = H^0 - M^x \Omega,$$

where H is the Hamiltonian of the nucleus in the rotating system, M^x is the angular momentum along the x axis, coinciding with the axis of rotation, and Ω is the angular velocity.

Treating the term $M^x \Omega$ as a perturbation, it is easy to calculate the diagonal corrections (which are the only ones we are interested in for what follows) in the second order of perturbation theory in the functions G and F . For the case where the functions G and F are expanded in eigenfunctions of the Hamiltonian H^0 , these corrections have the form

$$\begin{aligned} G_{\lambda\lambda}^0 &= \sum_{\lambda_1} \{ H'_{\lambda\lambda_1} H'_{\lambda_1\lambda} [G_\lambda G_{\lambda_1} G_\lambda - G_\lambda F_{\lambda_1} F_\lambda - F_\lambda D_{\lambda_1} F_\lambda - F_\lambda F_{\lambda_1} G_\lambda] \\ &+ i H'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} [G_\lambda G_{\lambda_1} F_\lambda - G_\lambda F_{\lambda_1} G_\lambda - F_\lambda F_{\lambda_1} F_\lambda - F_\lambda D_{\lambda_1} G_\lambda] \\ &+ i \Delta'_{\lambda\lambda_1} H'_{\lambda_1\lambda} [-F_\lambda G_{\lambda_1} G_\lambda + F_\lambda F_{\lambda_1} F_\lambda + G_\lambda D_{\lambda_1} F_\lambda + G_\lambda F_{\lambda_1} G_\lambda] \\ &+ \Delta'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} [F_\lambda G_{\lambda_1} F_\lambda - F_\lambda F_{\lambda_1} G_\lambda - G_\lambda F_{\lambda_1} F_\lambda - G_\lambda D_{\lambda_1} G_\lambda] \\ &+ i \Delta'_{\lambda\lambda} 2F_\lambda G_\lambda - 2\mu'' F_\lambda F_\lambda, \end{aligned} \quad (3)$$

$$\begin{aligned} F_{\lambda\lambda}^0 &= \sum_{\lambda_1} \{ H'_{\lambda\lambda_1} H'_{\lambda_1\lambda} (D_\lambda D_{\lambda_1} F_\lambda + D_\lambda F_{\lambda_1} G_\lambda + F_\lambda G_{\lambda_1} G_\lambda - F_\lambda F_{\lambda_1} F_\lambda) \\ &+ i H'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} (D_\lambda F_{\lambda_1} F_\lambda + D_\lambda D_{\lambda_1} G_\lambda + F_\lambda G_{\lambda_1} F_\lambda - F_\lambda F_{\lambda_1} G_\lambda) \\ &+ i \Delta'_{\lambda\lambda_1} H'_{\lambda_1\lambda} (F_\lambda D_{\lambda_1} F_\lambda + F_\lambda F_{\lambda_1} G_\lambda + D_\lambda G_{\lambda_1} G_\lambda - D_\lambda F_{\lambda_1} F_\lambda) \\ &- \Delta'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} (F_\lambda F_{\lambda_1} F_\lambda + F_\lambda D_{\lambda_1} G_\lambda + D_\lambda G_{\lambda_1} F_\lambda - D_\lambda F_{\lambda_1} G_\lambda) \\ &+ 2\mu'' D_\lambda F_\lambda + i \Delta'' (F_\lambda F_\lambda - D_\lambda G_\lambda). \end{aligned} \quad (4)$$

Here G_λ , F_λ , and D_λ are given by the formulas

$$\begin{aligned} G_\lambda &= \frac{(E_\lambda + \epsilon_\lambda)}{2E_\lambda(\omega - E_\lambda + i\delta)} + \frac{(E_\lambda - \epsilon_\lambda)}{2E_\lambda(\omega + E_\lambda - i\delta)}, \\ F_\lambda &= -\frac{i\Delta}{2E_\lambda} \left[\frac{1}{\omega - E_\lambda + i\delta} - \frac{1}{\omega + E_\lambda - i\delta} \right], \\ D_\lambda &= \frac{(E_\lambda - \epsilon_\lambda)}{2E_\lambda(\omega - E_\lambda + i\delta)} + \frac{(E_\lambda + \epsilon_\lambda)}{2E_\lambda(\omega + E_\lambda - i\delta)}. \end{aligned} \quad (5)$$

The equation for determining Δ'' has the form

$$\Delta''(\mathbf{r}) = \gamma \sum_{\lambda} F_{\lambda\lambda}''(0) \varphi_{\lambda}(\mathbf{r}) \varphi_{\lambda}^*(\mathbf{r}). \quad (6)$$

In the present paper we shall be interested in the quantity Δ'' averaged over the nuclear volume V :

$$\bar{\Delta}'' = \frac{1}{V} \int \Delta''(\mathbf{r}) dV.$$

From formula (6) we then obtain an equation for $\bar{\Delta}''$:

$$\bar{\Delta}'' = \gamma \sum_{\lambda} F_{\lambda}''(0). \quad (6')$$

Using Eqs. (6) and (2) for the unperturbed Δ , we easily find (assuming that $\Delta''(\mathbf{r})$ depends smoothly on \mathbf{r}) that

$$\sum_{\lambda} F_{\lambda}''(0) + \bar{\Delta}'' \sum_{\lambda} \frac{1}{2E_{\lambda}} = 0. \quad (7)$$

Using formulas (4) and (5) this equation takes the form

$$\begin{aligned} & \sum_{\lambda\lambda_1} \left\{ H'_{\lambda\lambda_1} H'_{\lambda_1\lambda} \left[\frac{\Delta(\epsilon_{\lambda_1} - \epsilon_{\lambda})^2 (\epsilon_{\lambda_1} \epsilon_{\lambda} - \Delta^2)}{4(E_{\lambda} E_{\lambda_1})^3 (E_{\lambda} + E_{\lambda_1})} + \frac{\Delta(\Delta^2 + \epsilon_{\lambda} \epsilon_{\lambda_1} - E_{\lambda} E_{\lambda_1})}{4(E_{\lambda} E_{\lambda_1})^2 (E_{\lambda} + E_{\lambda_1})} \right]_1 \right. \\ & + (H'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} - \Delta'_{\lambda\lambda_1} H'_{\lambda_1\lambda}) \left[\frac{(\epsilon_{\lambda} - \epsilon_{\lambda_1})}{4E_{\lambda} E_{\lambda_1} (E_{\lambda} + E_{\lambda_1})} \right. \\ & + \left. \frac{\Delta^2 (\epsilon_{\lambda_1} - \epsilon_{\lambda}) (2\Delta^2 + (\epsilon_{\lambda_1} - \epsilon_{\lambda})^2 + 2\epsilon_{\lambda} \epsilon_{\lambda_1} + E_{\lambda} E_{\lambda_1})}{4(E_{\lambda} E_{\lambda_1})^3 (E_{\lambda} + E_{\lambda_1})} \right]_2 \\ & + \Delta'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} \left[\frac{\Delta(\epsilon_{\lambda} - \epsilon_{\lambda_1})^2 (\epsilon_{\lambda} \epsilon_{\lambda_1} - \Delta^2)}{4(E_{\lambda} E_{\lambda_1})^3 (E_{\lambda} + E_{\lambda_1})} \right. \\ & + \left. \frac{\Delta(E_{\lambda} E_{\lambda_1} + \epsilon_{\lambda} \epsilon_{\lambda_1} + \Delta^2)}{4(E_{\lambda} E_{\lambda_1})^2 (E_{\lambda} + E_{\lambda_1})} \right]_3 \left. \right\} \\ & + \mu'' \Delta \sum_{\lambda} \frac{\epsilon_{\lambda}}{2E_{\lambda}^3} - \bar{\Delta}'' \Delta^2 \sum_{\lambda} \frac{1}{2E_{\lambda}^3} = 0. \quad (8) \end{aligned}$$

The corrections associated with the change in the chemical potential μ'' can be calculated from the equation

$$\sum_{\lambda} G_{\lambda\lambda}'' = 0. \quad (9)$$

Evaluating μ'' from this equation, we can show that the corrections to $\bar{\Delta}''$ resulting from a change in the chemical potential will be $\sim (\Delta/\epsilon_0)^2 \sim A^{-4/3}$ and can be neglected.

The quantities in square brackets which appear in (8) have sharp maxima, as a function of ϵ_{λ} for fixed $\epsilon_{\lambda} - \epsilon_{\lambda_1} = d$, of width $\sim \Delta$ at the point $\epsilon_{\lambda} + \epsilon_{\lambda_1}$. Thus these quantities can be calculated by the quasiclassical method developed by Migdal.^[1] One easily sees, by calculating the elementary integrals, that the sums in square brackets can be replaced by the expressions

$$[\dots]_1 \rightarrow -\frac{(1+2x^2)g-1}{2\Delta(1+x^2)} \delta(\epsilon_{\lambda}), \quad (10)$$

$$[\dots]_2 \rightarrow -\frac{x(x^2g-1)\delta(\epsilon_{\lambda})}{2\Delta(1+x^2)}, \quad (11)$$

$$[\dots]_3 \rightarrow -\frac{(x^2g-1)}{(1+x^2)} \delta(\epsilon_{\lambda}), \quad (12)$$

where

$$g(x) = \frac{\ln|x + \sqrt{1+x^2}|}{x\sqrt{1+x^2}}, \quad x = \frac{\epsilon_{\lambda} - \epsilon_{\lambda_1}}{2\Delta}.$$

It is easy to calculate the sum in the last term in (8) by changing to an integral over ϵ_{λ} :

$$\sum_{\lambda} \frac{1}{2E_{\lambda}^3} = \frac{\rho_0}{\Delta^2}, \quad (13)$$

where ρ_0 is the level density at the Fermi surface. As a result, Eq. (8) for $\bar{\Delta}''$ takes the form

$$\begin{aligned} \rho_0 \bar{\Delta}'' = & - \sum_{\lambda\lambda_1} \left\{ \frac{(1+2x^2)g-1}{2\Delta(1+x^2)} H'_{\lambda\lambda_1} H'_{\lambda_1\lambda} \right. \\ & + \left. \frac{x^2g-1}{2\Delta(1+x^2)} [(H'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda} - \Delta'_{\lambda\lambda_1} H'_{\lambda_1\lambda})x + \Delta'_{\lambda\lambda_1} \Delta'_{\lambda_1\lambda}] \right\} \delta(\epsilon_{\lambda}). \quad (14) \end{aligned}$$

We shall carry out the further calculations for the model of an axially symmetric deformed oscillator.

In this model the operator \hat{M}^X is different from zero only for transitions with $n'_X = n_X \pm 1$, $n'_Z = n_Z \pm 1$. To quasiclassical accuracy all possible matrix elements $\hat{M}^X_{\lambda\lambda_1}$ are equal, and the energies of the transitions are $\epsilon_{\lambda} - \epsilon_{\lambda_1} = \pm(\omega_Z \pm \omega_Y)$.

In this model, as shown by Migdal,^[1]

$$\Delta' = -i \frac{\alpha}{2\Delta} \hat{M}^X \Omega, \quad (15)$$

where

$$\alpha = \frac{g_1 + g_2}{g_1 x_1^2 + g_2 x_2^2}, \quad g_1 = g(x_1), \quad g_2 = g(x_2), \quad x_1 = \frac{\omega_Z - \omega_X}{2\Delta},$$

$$x_2 = \frac{\omega_X + \omega_Y}{2\Delta} \approx \frac{\omega_0}{\Delta},$$

while the rigid moment of inertia J_0 is equal to

$$J_0 = \frac{x_1^2 + x_2^2}{x_1^2 x_2^2} \frac{1}{8\Delta^2} \sum_{\lambda\lambda_1} |\hat{M}^X_{\lambda\lambda_1}|^2 \delta(\epsilon_{\lambda}). \quad (16)$$

Then calculating the sum (14) by using (15) and (16), we get

$$\begin{aligned} \frac{\bar{\Delta}''}{\Delta} = & - \frac{J_0 I (I+1)}{2\rho_0 \Delta^2 J^2} \left\{ \frac{x_1^2 x_2^2}{(x_1^2 + x_2^2)} \left[\frac{(x_1^2 g_1 - 1)(1 + 2\alpha x_1^2 - \alpha^2 x_1^2)}{x_1^2 (1 + x_1^2)} + \frac{g_1}{x_1^2} \right. \right. \\ & \left. \left. + \frac{(x_2^2 g_2 - 1)(1 + 2\alpha x_2^2 - \alpha^2 x_2^2)}{x_2^2 (1 + x_2^2)} + \frac{g_2}{x_2^2} \right] \right\} \quad (17) \end{aligned}$$

or, abbreviated,

$$\frac{\bar{\Delta}''}{\Delta} = -\frac{J_0 I (I+1)}{2\rho_0 \Delta^2 J^2} L(x_1, x_2). \quad (17a)$$

Formula (17) supplies the answer to our problem. From (17) we see that in order of magnitude $\overline{\Delta}''/\Delta$ is equal to the ratio of the rotational energy $I(I+1)/2J$ to the total pairing energy $-\rho_0\Delta^2/4$. The numerical coefficient in the curly brackets is approximately equal to 0.4 for actual deformed nuclei. Formula (17) enables us to estimate the change in Δ at the point of transition from the superfluid state to the normal state. This change is of the order of 40%. In fact, at the transition point we have the relation

$$-\rho_0\Delta^2/4 + I_c(I_c + 1)/2J = I_c(I_c + 1)/2J_0. \quad (18)$$

In our qualitative estimates we neglect the slight difference in Δ for neutrons and protons, and we always write the total density ρ_0 . For a more precise computation it would be necessary to write the energy of the pair correlation $-\rho_0\Delta^2/4$ separately for protons and neutrons. However, this produces an insignificant change in the results.

The critical spin for the transition is equal to

$$I_c(I_c + 1) = \rho_0\Delta^2 J J_0 / 2(J_0 - J). \quad (19)$$

For angular momenta $I < I_c$ the superconducting state will be stable and $J < J_0$, while for $I > I_c$ the normal state of the system is stable, in which $\Delta = 0$ and $J = J_0$.

In first approximation we shall assume that Δ is not changed at the transition point. Then, substituting (19) in formula (17), we get an estimate for the change of Δ at the transition point:

$$\frac{\overline{\Delta}_c''}{\Delta} \approx -\frac{L}{(4J/J_0)(1 - J/J_0)}. \quad (20)$$

Since $J/J_0 \approx 1/2$, $\overline{\Delta}_c''/\Delta \approx 0.4$, i.e., it is only in first approximation that we can regard the value of Δ as being unchanged at the transition point.

Let us compute I_c on this assumption. To quasiclassical accuracy $\rho_0 = 3A/2\epsilon_0$. The Fermi energy of the nucleus is $\epsilon_0 \approx 36$ Mev, while $J/J_0 = 0.5$. The moments of inertia in the region of the rare earths are $J \approx 33$ Mev⁻¹, while in the region of heavy elements $J \approx 72$ Mev⁻¹. Estimating I_c according to formula (19), we find $I_c \approx 11$ for the rare earths, while $I_c \approx 17$ for the heavy elements.

Similar estimates of this effect were first made by Mottelson.^[3] These moments agree with the maximum moments observed in experiments on Coulomb excitation.^[4]

The observation of rotational states with $I > I_c$ by using electromagnetic transitions from states with $I < I_c$ will be difficult, since we must then excite a state in which $\Delta = 0$ and consequently there must occur a marked readjustment of the internal state of the nucleus. As we have pointed

out earlier,^[5] in a transition with a marked change in Δ , a retardation factor

$$K = \left[\prod_{\lambda} (u_{\lambda}^i u_{\lambda}^f + v_{\lambda}^i v_{\lambda}^f) \right]^2 = \exp \left\{ 2 \sum_{\lambda} \ln (u_{\lambda}^i u_{\lambda}^f + v_{\lambda}^i v_{\lambda}^f) \right\} \quad (21)$$

appears, where $u_{\lambda} = 1/2(1 + \epsilon_{\lambda}/E_{\lambda})$, $v_{\lambda} = 1/2(1 - \epsilon_{\lambda}/E_{\lambda})$, while the superscripts *i* and *f* denote the initial and final states, in which the nucleus has different values of Δ . Assuming, for example, that in the final state $\Delta^f = 0$, we obtain, by changing from summation to integration,

$$2 \sum_{\substack{\lambda > 0 \\ \epsilon_{\lambda} - \epsilon_0 > 0}} \ln \frac{1}{2} \left(1 + \frac{|\epsilon_{\lambda}|}{\sqrt{\Delta^2 + \epsilon_{\lambda}^2}} \right) = \int_0^{\infty} \rho_0 \ln \frac{1}{2} \left(1 + \frac{\epsilon}{\sqrt{\epsilon^2 + \Delta^2}} \right) d\epsilon = -\frac{\rho_0 \Delta (\pi - 2)}{2}, \quad (22)$$

$$K = \exp \{-\rho_0 \Delta (\pi - 2)\}. \quad (23)$$

Since, when we include the change in Δ for large values of I , $\rho_0 \Delta (\pi - 2) \sim 5$ for the heavy elements and ~ 4 for the rare earths, $K \sim 10^{-2}$.

We can also estimate the corrections to the rotational spectrum of the system, $\delta E = -BI^2(I+1)^2$, resulting from the change in Δ and the consequent change in the moment of inertia.

As was shown by Migdal,^[1] moments of inertia can be described well by the expression $J = Cx_1$, where $C = \text{const}$. Then the change in the rotational energy when Δ is changed will be

$$\delta E \approx -\frac{\hbar^2 I(I+1)}{2J^2} \delta J = \frac{\hbar^2 I(I+1)}{2J} \frac{\delta \Delta}{\Delta}. \quad (24)$$

Substituting for $\delta \Delta$ in (24) from formula (17), we get

$$\delta E \approx -\frac{J_0 L (x_1, x_2)}{4\rho_0 \Delta^2 J^3} I^2 (I+1)^2. \quad (25)$$

Thus we obtain as an estimate for the coefficient B

$$B \approx -J_0 L / 4 \rho_0 \Delta^2 J^3. \quad (26)$$

The usual nonadiabatic correction to the rotation spectrum, omitting the pair correlation, was calculated previously by us^[5] and has the form (for a deformation $\beta \gg A^{-2/3}$):

$$B \sim 1/\rho_0 J_0^2 (\omega_0 \beta)^2. \quad (27)$$

Since $\omega_0 \beta \sim \Delta \sim \epsilon_0 A^{-2/3}$, the two corrections have just the same order of magnitude with respect to A . Numerically estimated coefficients B from formula (26) also turn out to be close to the experimentally observed values.

For the rare earths

$$B_{\text{th}} \approx 25 \times 10^{-3} \text{ kev}, \quad B_{\text{exp}} \approx 20 \times 10^{-3} \text{ kev},$$

while for heavy elements

$$B_{\text{th}} \approx 6 \times 10^{-3} \text{ kev}, B_{\text{exp}} \approx (3-5) \times 10^{-3} \text{ kev}.$$

Thus rotation has a sizable effect on the pairing correlation, and this effect must be taken into account in computing the nonadiabatic corrections proportional to $I^2(I+1)^2$. In the case of the estimate of I_C we can neglect the change in Δ only in first approximation. However, for more precise computations this change must be taken into account.

The author expresses his gratitude to V. M. Galitskii and A. B. Migdal for interesting discussions.

¹A. Migdal, JETP **37**, 249 (1959), Soviet Phys. JETP **10**, 176 (1960).

²Yu. Grin', JETP **39**, 138 (1960), Soviet Phys. JETP **12**, 100 (1961).

³B. Mottelson and J. Valatin, Phys. Rev. Letters **5**, 511 (1960).

⁴Stephens, Diamond, and Perlman, Phys. Rev. Letters **3**, 435 (1959).

⁵Yu. Grin', JETP **41**, 222 (1961), Soviet Phys. **14**, 162 (1961).

Translated by M. Hamermesh