CONTRIBUTION TO THE THEORY OF ABSORPTION OF ULTRASONIC WAVES BY METALS IN A MAGNETIC FIELD

G. L. KOTKIN

Moscow State University

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The complex elastic modulus of a metal is computed under conditions when the electron mean free path is much greater than the acoustic wavelength. Some cases are considered when measurement of the sound absorption coefficient permits one to draw certain conclusions regarding the momenta and velocities of the electrons on the Fermi surface.

THE study of ultrasonic absorption in metals at low temperatures in the presence of a magnetic field makes it possible to clarify certain characteristics of the Fermi surface, as was first shown by Pippard.[1] A number of researches have been devoted to a theoretical consideration of this problem.[2-4] In the case in which the electron mean free path l is much greater than the acoustic wavelength λ, the transfer of energy from the lattice to the electrons is not connected with their collisions. A systematic theory of sound absorption under these conditions (without a magnetic field) has been constructed by Silin.[5] In the present work, the case of an infinite mean free path of the electrons is also considered, in contrast with,[2-4] where the length of the free path was essential.

In Sec. 1, general formulas will be obtained for the complex elastic modulus when $l \gg \lambda$. In Sec. 2, sound propagation is considered along the axis of symmetry in a magnetic field and also in the case when the acoustic wave vector **k** and the magnetic field **H** lie in the plane of symmetry of the Fermi surface. It is shown that in these cases the sound absorption coefficient vanishes in the approximation considered if the field exceeds the value for which the maximum displacement of the electron over the period of its motion in the magnetic field is equal to the acoustic wavelength. In Sec. 3, sound propagation is considered in the case of strong magnetic fields.

1. When the lattice is deformed, the energy of the electron is changed:

$$\varepsilon = \varepsilon_0 + \Lambda_{ik}\, \partial u_i/\partial x_k$$

(**u** is the lattice displacement vector). The electron distribution function satisfies the equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \left(e\mathbf{E} + \frac{e}{c}[\mathbf{vH}] - \frac{\partial}{\partial \mathbf{r}}\Lambda_{ik}\frac{\partial u_i}{\partial x_k}\right)\frac{\partial f}{\partial \mathbf{p}} = 0. \quad (1.1)^*$$

We assume that $\mathbf{u} \sim \exp i(\mathbf{k}\cdot\mathbf{r} - \omega t)$. Transforming to the variables introduced in the work of I. Lifshitz, Azbel' and Kaganov,[6] we get for the increment δf to the equilibrium distribution function

$$f_0 = 2(2\pi\hbar)^{-3}[\exp(\varepsilon_0 - \mu)/T + 1]^{-1}$$

the equation

$$\Omega\frac{\partial \delta f}{\partial \varphi} + i\mathbf{kv}_0\delta f = i\omega\delta f - e\mathbf{v}_0\mathbf{E}\frac{\partial f_0}{\partial \varepsilon} - i\mathbf{kv}_0 u_i k_m \Lambda_{im}\frac{\partial f_0}{\partial \varepsilon} + iu_i k_m \Omega\frac{\partial \Lambda_{im}}{\partial \varphi}\frac{\partial f_0}{\partial \varepsilon}. \quad (1.2)\dagger$$

Here φ/Ω is the time of motion of an electron with a given energy ϵ_0 and momentum projection in the direction of the magnetic field p_H, Ω is its circular frequency, and $\mathbf{v}_0 = \partial\epsilon_0/\partial\mathbf{p}$.

We restrict ourselves to closed Fermi surfaces. Taking into account the periodicity of $\delta f(\varphi)$, we find

$$\delta f = (iu_i k_m \Lambda_{im} + iek^{-2}\mathbf{kE})\,\partial f_0/\partial\varepsilon - \frac{\Omega^{-1}\int_{\varphi}^{\varphi+2\pi} d\varphi_1\,[eE_\alpha v_\alpha(\varphi_1)\,\partial f_0/\partial\varepsilon - i\omega\delta f(\varphi_1)]\exp\left(i\mathbf{k}\int_{\varphi}^{\varphi_1}\mathbf{v}(\varphi')\,d\varphi'/\Omega\right)}{\exp i\mathbf{k}\int_0^{2\pi}\mathbf{v}(\varphi)\,d\varphi/\Omega - 1}. \quad (1.3)$$

The indices 0 are omitted on v, while v_α denotes the components of the velocity perpendicular to **k**.

Adiabatic "switching on" of the interaction at $t = -\infty$ corresponds to a passage about the poles determined by an infinitesimally small imaginary contribution to ω, or by an addition of the opposite

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.
†$\mathbf{kv} = \mathbf{k}\cdot\mathbf{v}$.

sign to $\mathbf{k}\cdot\mathbf{v}$. In the case in which $\mathbf{k} \perp \mathbf{H}$, it is shown that $\mathbf{k}\cdot\int_0^{2\pi}\mathbf{v}(\varphi)\,d\varphi \equiv 0$, and it is necessary to take the collisions into account. The condition for the applicability of Eq. (1.3) reduces to $kv\cos\theta \gg \nu$, where ν is the characteristic frequency of collisions for the electrons, and θ is the angle between $\mathbf{k}$ and $\mathbf{H}$.

By making use of Maxwell's equations, we obtain the following equations for the determination of $\mathbf{E}$:

$$\int \delta f\, d\mathbf{p} = -i\mathbf{k}\mathbf{u}N, \qquad (1.4)$$

$$e\int v_\alpha \delta f\, d\mathbf{p} = -i\omega e N u_\alpha - \frac{ic^2k^2E_\alpha}{4\pi\omega}. \qquad (1.5)$$

Here N is the number of electrons per unit volume.

We introduce the notation

$$-\int d\mathbf{p} A\partial f_0/\partial\varepsilon = \langle A\rangle, \qquad (1.6)$$

$$(2\pi)^{-1}\int_0^{2\pi} A(\varphi)\,d\varphi = \widetilde{A}, \qquad (1.7)$$

$$\left\langle \frac{A\Omega^{-1}\int_\varphi^{\varphi+2\pi} d\varphi_1 B(\varphi_1)\exp(i\mathbf{k}\int_\varphi^{\varphi_1}\mathbf{v}(\varphi')\,d\varphi'/\Omega)}{\exp 2\pi i\,\mathbf{k}\widetilde{\mathbf{v}}/\Omega - 1}\right\rangle = (A, B). \quad (1.8)$$

Substituting (1.3) in (1.4) and (1.5), we get

$$-iu_ik_m\langle\Lambda_{im}\rangle - ie\mathbf{k}\mathbf{E}k^{-2}\langle 1\rangle - eE_\alpha(1, v_\alpha) + \omega u_ik_m(1, \Lambda_{im}) + \omega e\mathbf{k}\mathbf{E}k^{-2}(1,1) = -i\mathbf{k}\mathbf{u}N, \qquad (1.9)$$

$$-e^2E_\alpha(v_\alpha, v_\beta) + e\omega u_ik_m(v_\beta, \Lambda_{im}) = i\omega eNu_\beta - ic^2k^2E_\beta/4\pi\omega. \qquad (1.10)$$

To determine E we use an expansion in powers of v_s/v (v_s is the speed of sound):

$$eE_\alpha = -i\omega e^2u_iB^{-1}_{\alpha\beta}(N\delta_{i\beta} - i(v_\beta, L_i)), \qquad (1.11)$$

$$\frac{e\mathbf{k}\mathbf{E}}{k^2} = \frac{u_ik_m}{\langle 1\rangle}(N\delta_{im} - \langle\Lambda_{im}\rangle) - i\omega u_i\frac{(1, L_i)}{\langle 1\rangle} + ieE_\alpha\frac{(1, v_\alpha)}{\langle 1\rangle}; \qquad (1.12)$$

$$B_{\alpha\beta} = -e^2(v_\alpha, v_\beta) - i(c^2k^2/4\pi\omega)\,\delta_{\alpha\beta}, \qquad (1.13)$$

$$L_i = k_m(\Lambda_{im} - \langle\Lambda_{im}\rangle\langle 1\rangle^{-1} + N\delta_{im}\langle 1\rangle^{-1}). \qquad (1.14)$$

The lattice vibrations are described by the equation[5]

$$\rho\ddot{u}_i = \lambda^{(0)}_{imjl}\frac{\partial u_j}{\partial x_m\partial x_l} - eNE_i - i\frac{e}{c}N\omega[\mathbf{u}\mathbf{H}]_i + \frac{\partial}{\partial x_m}\int\Lambda_{im}f\,dp, \qquad (1.15)$$

where ρ is the density of matter, and $\lambda^{(0)}_{imjl}$ is the tensor of the elastic moduli of the lattice. By using the complex elastic modulus $\lambda_{imjl}(\omega, \mathbf{k})$, introduced by Silin,[5] we transform Eq. (1.5) to

$$\rho\omega^2u_i = k_lk_m\lambda_{imjl}u_j;$$

$$\lambda_{imjl}k_mk_l = \lambda^{(0)}_{imjl}k_mk_l + \delta\lambda_{imjl}k_mk_l, \qquad (1.16)$$

$$\delta\lambda_{imjl}k_mk_lu_j = eNE_i - iec^{-1}N\omega H_mu_je_{ijm} - ik_m\int\Lambda_{im}\delta f\,dp. \qquad (1.17)$$

Substituting (1.3), (1.11), and (1.12) in (1.17), we obtain

$$\delta\lambda_{imjl}k_mk_l = -k_mk_l\langle\Lambda_{im}\Lambda_{jl}\rangle - k_mk_l\langle 1\rangle^{-1}(\langle\Lambda_{im}\rangle - N\delta_{im})(\langle\Lambda_{jl}\rangle - N\delta_{jl}) + iec^{-1}N\omega H_me_{ijm} - i\omega(L_i, L_j) - i\omega e^2(N - i(L_i, v_\alpha))\,B^{-1}_{\alpha\beta}(N\delta_{j\beta} - i(v_\beta, L_j)). \qquad (1.18)$$

The first two terms in (1.18) have the same order of magnitude as $\lambda^{(0)}_{imjl}k_mk_l$, and determine the renormalization of the elasticity modulus, brought about by the interaction of the lattice with the electrons. This renormalization is shown not to depend on the magnetic field, and is identical with that obtained by Silin.[5] The remaining terms in (1.18), generally speaking, are much smaller than $\lambda^{(0)}_{imjl}k_mk_l$. They lead to dispersion and sound absorption, determined by the addition of $\delta\omega(\mathbf{k}) = \delta\omega' + i\delta\omega''$ to the normal frequency $\omega_0(\mathbf{k})$ found from (1.16) without account of the small terms in (1.18).

From (1.16), we find

$$\delta\omega' = (\delta\lambda_{imjl} + \delta\lambda^*_{jlim})\,k_mk_lu^*_iu_j/\omega_0\rho\,|u|^2, \qquad (1.19)$$

$$\delta\omega'' = -i(\delta\lambda_{imjl} - \delta\lambda^*_{jlim})\,k_mk_lu^*_iu_j/\omega_0\rho\,|u|^2. \qquad (1.20)$$

The effect of small additions to the elasticity modulus tensor on the polarization of normal waves is considered in the Appendix.

We now bring (A, B) to a form which is more suitable for investigation. We transform (1.8) by expanding the following function (which is periodic in φ) in a Fourier series:

$$B(\varphi)\exp\left\{i\mathbf{k}\int_0^\varphi \mathbf{v}(\varphi')\,d\varphi'/\Omega - i\mathbf{k}\widetilde{\mathbf{v}}\varphi/\Omega\right\}$$

$$(A, B) = -\frac{4\pi i}{(2\pi\hbar)^3}\int m^*\,dp_H\Omega^{-1}\sum_{n=-\infty}^{\infty}\frac{A^*_nB_n}{\mathbf{k}\widetilde{\mathbf{v}}/\Omega + n - i\delta}; \qquad (1.21)$$

$$B_n = (2\pi)^{-1}\int_0^{2\pi} d\varphi B(\varphi)\exp\left\{i\mathbf{k}\int_0^\varphi \mathbf{v}(\varphi')\,d\varphi'/\Omega - i\mathbf{k}\widetilde{\mathbf{v}}/\Omega - in\varphi\right\}. \qquad (1.22)$$

We assume that the Fermi surface has a center of symmetry, so that $B(-\mathbf{p}) = \zeta_B B(\mathbf{p})$, where $\zeta_B = 1$ when $B = 1$, Λ_{ik} and $\zeta_B = -1$ when $B = v_i$. By making the substitutions $\mathbf{p} \to -\mathbf{p}$ and $n \to -n$ in (1.22), we can put (1.22) in the form

$$(A, B) = \frac{2\pi^2}{(2\pi h)^3} m^*\Omega^{-1}$$

$$\times \sum_{|n|<(\mathbf{k}\tilde{\mathbf{v}}/\Omega)_{max}} \left[(A_n^* B_n + \zeta_{AB} A_n B_n^*) \left| \frac{\partial}{\partial p_H} \frac{\mathbf{k}\tilde{\mathbf{v}}}{\Omega} \right|^{-1} \right]_{p_{Hn}}$$

$$- \frac{2\pi i}{(2\pi i h)^3} \mathrm{P} \int m^* dp_H \Omega^{-1} \sum_{n=-\infty}^{\infty} \frac{A_n^* B_n - \zeta_{AB} A_n B_n^*}{\mathbf{k}\tilde{\mathbf{v}}/\Omega + n}, \quad (1.23)$$

where p_{Hn} is determined by the condition

$$\mathbf{k}\tilde{\mathbf{v}}(p_{Hn}) + n\Omega(p_{Hn}) = 0. \quad (1.24)$$

We note that for $\zeta_{AB} = 1$, the quantity (A, B) is real, while for $\zeta_{AB} = -1$, it is imaginary.

With increase in the magnetic field, individual components fall out of the sum in (1.23). This corresponds to jumps in the coefficient of absorption or in its derivative with respect to the field H at $(\mathbf{k}\cdot\tilde{\mathbf{v}}/\Omega)_{max} = n$, as Gurevich has shown.[2] If the Fermi surface differs sharply from elliptic, then $(\partial/\partial p_H)(\mathbf{k}\cdot\tilde{\mathbf{v}}/\Omega)$ can vanish for certain $p_H^{(0)}$. When $\mathbf{k}\cdot\mathbf{v}(p_H^{(0)}) = n\Omega(p_H^{(0)})$, discontinuities also appear,[2] and then analysis of the collision integral close to the discontinuity is essential.

The last term in (1.18) arises from the transverse field E_α. Its role depends, as is well known, on the region of frequencies under consideration. In the region where λ is small in comparison with the skin depth δ in the anomalous skin effect, $B_{\alpha\beta} = -e^2(v_\alpha, v_\beta)$ and the last two components in (1.18) are of the same order. If now $\lambda \ll \delta$, then $B_{\alpha\beta} = -i(c^2k^2/4\pi\omega)\delta_{\alpha\beta}$ and the contribution of the transverse field, generally speaking, is much less than the contribution of the deformation potential.

2. Let z be the axis of symmetry of the Fermi surface of s-th order, and at the same time the axis of symmetry of the crystal, so that sound that is purely longitudinal or purely transverse is possible when $\mathbf{k} \parallel \mathbf{z}$. We consider the case in which the sound is propagated along z, and the magnetic field is directed parallel to this axis. Taking it into account that v_z and L_z do not change upon rotation through an angle $2\pi/s$ about the z axis, we find

$$(L_z)_n = (2\pi)^{-1} \sum_{l=0}^{s-1} e^{-2\pi i n l/s}$$

$$\times \int_0^{2\pi/s} d\varphi L_z(\varphi) \exp\left(ik \int_0^{\varphi} \frac{v_z d\varphi}{\Omega} - \frac{ik\tilde{v}_z\varphi}{\Omega} - in\varphi \right)$$

$$= (2\pi)^{-1} \int_0^{2\pi/s} d\varphi L_z(\varphi) \exp\left(ik \int_0^{\varphi} \frac{v_z d\varphi}{\Omega} - \frac{ik\tilde{v}_z\varphi}{\Omega} - in\varphi \right)$$

$$\times \begin{cases} s & \text{for } n = qs \\ 0 & \text{for } n \neq qs \end{cases} \quad (2.1)$$

(q is an integer).

In place of the transverse components v_α, it is better to make use of the quantities $v_\pm = v_x \pm iv_y$, which satisfy the condition

$$v_\pm(\varphi + 2\pi/s) = \exp(\pm 2\pi i/s)\, v_\pm(\varphi); \quad (2.2)$$

$$(v_\pm)_n = (2\pi)^{-1} \int_0^{2\pi/s} d\varphi v_\pm(\varphi) \exp\left(ik \int_0^{\varphi} \frac{v_z d\varphi}{\Omega} - \frac{ik\tilde{v}_z\varphi}{\Omega} - in\varphi \right)$$

$$\times \begin{cases} s & \text{for } n = qs \pm 1 \\ 0 & \text{for } n \neq qs \pm 1 \end{cases}. \quad (2.3)$$

Consequently, $(v_\alpha)_n = 0$ for $n \neq qs \pm 1$.

Similar equalities are valid for L_i. Thus the components in (1.23) vanish for $n \neq qs$, $qs \pm 1$, and the corresponding discontinuities in the elasticity modulus or its derivative with respect to the field are absent. For example, we have for longitudinal sound in the case $\lambda \gg \delta$

$$k^2\lambda_{zzzz} = -i\omega(L_z, L_z) = -\frac{4\pi^2\omega}{(2\pi\hbar)^3} \sum_{n=0,\, s,\ldots < (k\tilde{v}_z/\Omega)_{max}} m^*\Omega^{-1}$$

$$\times \left[|(L_z)_n|^2 \left| \frac{\partial}{\partial p_H} \frac{k\tilde{v}_z}{\Omega} \right|^{-1} \right]_{p_{Hn}} \quad (2.4)$$

and the discontinuities appear s times less often.

If the magnetic field exceeds the value for which $(k\tilde{v}_z/\Omega)_{max} = 1$, then only the component for $n = 0$ remains in the sum in (1.23). If s is even, then it vanishes for all the (A, B) which are necessary, according to (1.18), for consideration of transverse sound. It is shown that

$$(v_\alpha, v_\beta) = -(v_\beta, v_\alpha), \quad (L_\alpha, L_\beta) = -(L_\beta, L_\alpha),$$

$$(L_i, v_\alpha) = (v_\alpha, L_i). \quad (2.5)$$

In both limiting cases, $\lambda \gg \delta$ and $\lambda \ll \delta$, we obtain $k^2\delta\lambda''_{\alpha z\beta z} = -k^2\delta\lambda''_{\beta z\alpha z}$, so that $\delta\omega''$ vanishes for transverse sound. Thus, in the case of the transverse sound propagation along a symmetry axis of even order of the Fermi surface in a longitudinal magnetic field, the absorption disappears in the approximation considered if the field exceeds

$$H_1 = ck(m^*\tilde{v}_z)_{max}/e.$$

For the free electron model, it was shown by Kjeldaas[7] that the absorption coefficient of transverse sound vanishes for $\mathbf{k} \parallel \mathbf{H}$ if $H > ckmv/e$. We note that the sound absorption coefficient vanishes smoothly or with a discontinuity, depending on whether $(m^*\tilde{v}_z)_{max}$ is achieved for $p_z = p_{z\,max}$, or for $p_z < p_{z\,max}$. If the $\omega(\mathbf{k})$ coincide for the two transverse polarizations, then rotation of the plane of polarization of the sound should be observed for $H > H_1$. The angle of rotation per unit distance is

$$\varkappa = k^3 \delta\lambda_{xzyz} / 2\rho\omega^2. \tag{2.6}$$

According to estimate, $\kappa \sim Nmvk^2/\rho\omega$, which has the same order of magnitude as the angle found by Vlasov[8] for the free electron model.

Let yz be the plane of symmetry of the Fermi surface (and at the same time the plane of symmetry of the crystal). We further consider the case in which **k** and **H** lie in the yz plane. For the substitution $\varphi \to -\varphi$, the values of v_x and L_x change sign, while v_y, v_z, L_y, L_z do not change sign, so that we get

$$(v_x)_n = -(v_x)_{-n}, \qquad (v_y)_n = (v_y)_{-n} \text{ etc.} \tag{2.7}$$

In view of the assumed symmetry of the crystal a purely transverse wave polarized along the x axis, is possible without the magnetic field. The frequencies of the other two waves with the given **k** will, generally speaking, be different, so that the turning on of the magnetic field produces only a small distortion of the wave under consideration.

Let the frequency be so large that $\lambda \ll \delta$. The sound absorption coefficient is equal to

$$k\delta\omega''/\omega = 2\,(L_x, L_x)\,\rho^{-1} \tag{2.8}$$

and for $H > ck\,(m^*\tilde{v}_H)_{max} \cos\theta/e$ vanishes. (More accurately, the distortions of the transverse wave lead to an absorption coefficient of the order of $\delta\omega''^2 k/\omega^2$, where $\delta\omega''$ refers to the other polarization.) Thus, in the cases considered, one can determine the value of $(m^*\tilde{v}_H)_{max}$ by measuring the field for which the sound absorption decreases sharply. The completely determined dependence of the absorption limit on the angle θ makes it possible to control the applicability of the theory.

3. In the case of strong fields ($\Omega \gg kv$) we expand (1.8) in powers of kv/Ω:

$$\begin{aligned}(A, B) &= (1 + \zeta_{AB})\, 2\pi^2 (2\pi\hbar)^{-3} \int m^* dp_H \delta(\mathbf{k}\tilde{\mathbf{v}})\, \tilde{A}\tilde{B} \\ &- (1 - \zeta_{AB})\, 2\pi\, (2\pi\hbar)^{-3} i \mathrm{P} \int m^* dp_H \frac{\tilde{A}\tilde{B}}{\mathbf{k}\tilde{\mathbf{v}}} \\ &+ (1 - \zeta_{AB})(2\pi\hbar)^{-3} i \int m^* dp_H\, \Omega^{-1} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi_1 \\ &\times A(\varphi)\, B(\varphi_1) \left[\delta(\mathbf{k}\tilde{\mathbf{v}}) \int_{\varphi}^{\varphi_1} \mathbf{kv}(\varphi')\, d\varphi' - \sigma(\varphi - \varphi')/2 \right] \\ &+ O((kv/\Omega)^2),\end{aligned} \tag{3.1}$$

where

$$\sigma(x) = \begin{cases} -1 & \text{for } x < 0 \\ 1 & \text{for } x > 0 \end{cases}.$$

For (v_α, v_β) the first terms of the expansion vanish. An estimate shows that the first term in Eq. (1.13) can be neglected for $H \gg (4\pi m N v v_s)^{1/2}$ $\sim 10^5$ oe. The contribution of the transverse field in (1.18) is shown to be equal to

$$\begin{aligned}&4\pi\omega^2 e^2 N^2 c^{-2} k^{-2} (\delta_{ij} - \operatorname{tg}\theta\, \delta_{y'i} k_j k^{-1} \\ &- \operatorname{tg}\theta\, \delta_{y'j} k_i k^{-1} + \operatorname{tg}^2\theta\, k_i k_j k^{-2}).\end{aligned} \tag{3.2*}$$

Here y′ is the direction in the plane of **k** and **H** perpendicular to **k**. This leads to dispersion in the sound velocity, which is large for $\omega < 10^8$/sec and falls off with increase in frequency. In the range of fields $\omega vmc/ev \ll H \lesssim 10^5$ oe, both components are important in (1.13). In this region, both large dispersion and sound absorption are observed.

For $\omega > 10^8$/sec, the contribution of the transverse field becomes small, and the sound absorption is determined by the deformation potential. The absorption coefficient is of the same order as for H = 0:

$$\begin{aligned}\gamma &= 2k\,(L_i, L_j)\, u_i^* u_j / \rho\omega\, |H|^2 \\ &= \int m^* dp_H\, \delta(v_H)\, \tilde{L}_i L_j \frac{\operatorname{Re} u_i^* u_j}{|u|^2 \pi\hbar^3 \rho\omega \cos\theta}.\end{aligned} \tag{3.3}$$

For a given direction of **H** we have $\gamma \sim \cos^{-1}\theta$. The case of degeneration of the frequencies $\omega(\mathbf{k})$ corresponds to that considered in the Appendix. We only note that for small θ the normal vibrations will be elliptically polarized, and for θ close to $\pi/2$, linearly polarized.

In conclusion, I express my deep gratitude to V. P. Silin for suggesting the topic, and for constant assistance in the research.

APPENDIX

We shall regard

$$\begin{aligned}\rho\delta\gamma_{ij} &= iec^{-1} N\omega H_m e_{ijm} - i\omega (L_i, L_j) \\ &- i\omega^2 (N\delta_{i\alpha} - i(L_i, v_\alpha))\, B^{-1}_{\alpha\beta} (N\delta_{j\beta} - i(v_\beta, L_j))\end{aligned}$$

as a small correction to

$$\begin{aligned}\rho\gamma_{ij} &= \lambda^{(0)}_{imjl} k_m k_l - k_m k_l \langle \Lambda_{im} \Lambda_{jl} \rangle \\ &+ k_m k_l \langle 1 \rangle^{-1} (\langle \Lambda_{im} \rangle - N\delta_{im})(\langle \Lambda_{jl} \rangle - N\delta_{jl}).\end{aligned}$$

Let all three normal frequencies, corresponding to a given **k** (without account of this correction) be different, and let the normal vibrations be polarized along the ξ, η, ζ axes. Account of $\delta\gamma_{ij}$ in (1.15) leads to a small distortion of the normal vibrations. Instead of a wave polarized along ξ, u = (u, 0, 0), an elliptically polarized wave is obtained:

$$u = (u,\; u\delta\gamma_{\xi\eta} (\gamma_{\xi\xi} - \gamma_{\eta\eta})^{-1},\; u\delta\gamma_{\xi\zeta} (\gamma_{\xi\xi} - \gamma_{\zeta\zeta})^{-1}).$$

*tg = tan.

The ratio of the axes of the ellipse $[\delta\gamma''^2_{\xi\eta}(\gamma_{\xi\xi}-\gamma_{\eta\eta})^{-2} + \delta\gamma''^2_{\xi\zeta}(\gamma_{\xi\xi}-\gamma_{\zeta\zeta})^2]^{1/2} \ll 1$, and the direction of the major axis is $[1, \delta\gamma'_{\xi\eta}(\gamma_{\xi\xi}-\gamma_{\eta\eta})^{-1}, \delta\gamma'_{\xi\zeta}(\gamma_{\xi\xi}-\gamma_{\zeta\zeta})^{-1}]$.

Let two normal frequencies $\omega(\mathbf{k})$ corresponding to polarization along ξ, η be the same. The set of equations

$$(\delta\gamma_{\xi\xi}-\delta\omega^2)\,u_\xi + \delta\gamma_{\xi\eta}u_\eta = 0,$$
$$\delta\gamma_{\eta\xi}u_\xi + (\delta\gamma_{\eta\eta}-\delta\omega^2)\,u_\eta = 0$$

serves for the determination of $\delta\omega$ and the polarization of the normal vibrations in this case.

Let us consider in some detail the important case in which $\delta\gamma'_{ij} = 0$. This exists, for example, both for $\lambda \gg \delta$ and for $\lambda \ll \delta$. Depending on the sign of $a^2 = (\delta\gamma''_{\xi\xi} + \delta\gamma''_{\eta\eta})^2 - 4\delta\gamma''_{\xi\eta}\delta\gamma''_{\eta\xi}$, we have two possible cases. For $a^2 > 0$, the normal waves are plane polarized, the direction of polarization lying in the $\xi\eta$ plane and the angles β and $\beta + \pi/2$ coinciding with the ξ axis, where $\tan\beta = (\delta\gamma''_{\xi\xi} - \delta\gamma''_{\eta\eta} + a)/2\delta\gamma''_{\xi\eta}$; their absorption coefficients are $\gamma = k\delta\omega''/\omega = (\delta\gamma''_{\xi\xi} + \delta\gamma''_{\eta\eta} \pm a)/4\omega^2$, and the frequencies are identical.

For $a^2 < 0$, the normal vibrations are elliptically polarized and their absorption coefficients are the same, equal to

$$\gamma = (\delta\gamma''_{\xi\xi} + \delta\gamma''_{\eta\eta})\,k/4\omega^2,$$

while the frequencies are different:

$$\omega = \omega_0 \pm ia/4\omega_0.$$

In particular, if $\delta\gamma''_{ij} = -\delta\gamma''_{ji}$ the absorption is absent and the normal waves are circularly polarized. This leads to a rotation of the direction of the polarization of the plane polarized waves. The angle of rotation per unit distance is equal to $\kappa = k\delta\gamma''_{\xi\eta}/2\rho^2\omega^2$ (see [5]).

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