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SPECTRUM OF GALACTIC AND SOLAR COSMIC RAYS

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A differential energy spectrum of galactic cosmic rays has been obtained in the form of a power law with an exponent $\gamma = 2.5$ by assuming that the source energy is equally divided between the magnetic field, turbulence, and cosmic rays. If a constant pressure is assumed to be maintained when the solar cosmic rays leave the region in which they were produced, a value $\gamma = 3.5$ is obtained for nonrelativistic cosmic rays and $\gamma = 5$ for ultra-relativistic rays.

To interpret the observed exponent of the cosmic-ray spectrum

$$N(E)\,dE = KE^{-\gamma}dE, \quad (1)$$

where E is the total energy of the particle, K is a constant, and γ is the spectrum exponent ($\gamma = 2.5$ for galactic cosmic rays), the scheme first suggested by Fermi[1] is usually employed. In this scheme γ is obtained under the assumptions that the energy of the particles increases exponentially with a time constant $1/\alpha$ during the acceleration process and that the absorption of the particles follows an experimental law with a lifetime T. Then

$$\gamma = 1 + 1/\alpha T. \quad (2)$$

Subsequently, the lifetime with respect to nuclear collisions, which led to a strong unobserved dependence of γ on the atomic number of the cosmic-ray nucleus, was replaced by the mean time T in which the particles leave the region of acceleration.[2] In order to obtain the spectrum (1) and (2) in this scheme, it has to be assumed, moreover, that the conditions of acceleration, i.e., the parameters, α and T, and, what is particularly important, the injection of the particles do not change over the time interval necessary for the particles to acquire the maximum observed energy. Under these assumptions and with a suitable choice of parameters α and T, one can obtain the value of γ required by experiment.

It should be noted that the rather severe assumptions on the character of the acceleration and injection processes and chiefly the strong dependence of γ on specific values of α and T make the foregoing scheme for the production of the cosmic-ray spectrum highly unconvincing.

As a matter of fact, the conditions of the acceleration of cosmic rays in powerful explosive processes such as the flare-up of supernovae, which are apparently the basic sources of cosmic radiation in our Galaxy,[3] are not stationary both during the early stages of the flare-up and during the subsequent expansion of the shells. The chromospheric flare process on the Sun is also known not to be a stationary one.[4] Therefore the assumption that the rate of acceleration is constant and that the particles are injected uniformly is doubtful and should somehow be generalized.[5] Furthermore, the similarity of the radio spectra of different discrete sources (radiogalaxies) is evidence, according to the present views, of the similarity of the cosmic-ray spectra in these sources. At the same time, the energies of the internal motion, the magnetic fields, and the size of various galaxies, which determine the parameters α and T, can differ basically. It is very unlikely that the parameters α and T for each of these objects take on purely by chance values leading precisely to $\gamma \approx 2.5$.

The foregoing remarks apply in no lesser degree to the spectrum of cosmic rays produced in solar flares. Despite considerable oscillations in the duration and strength of the flares, the spectrum of the produced cosmic rays always appears to be more or less stable.

It is therefore natural to assume that the constancy of the cosmic-ray spectrum from various objects is some general property of the dynamics of magnetized cosmic gaseous masses and the thermodynamics of the relativistic cosmic-ray gas. We present below some simple arguments indicating that the spectrum of galactic cosmic rays and cosmic rays of solar origin can be obtained, under certain assumptions, from the general thermodynamical requirements without regard to the specific mechanism of the acceleration of the cosmic rays.

1. GALACTIC COSMIC RAYS

An important result of observations and their interpretation is the conclusion that both in individual galactic nebulae producing strong radio waves and in our Galaxy as a whole, and also, apparently, in other galaxies, the energy is distributed equally between the kinetic energy of the gaseous masses, magnetic energy, and energy of relativistic particles.[5] The equality of the magnetic and kinetic energies of turbulent motion of a magnetized conducting medium is apparently a general consequence of the behavior of magnetohydrodynamic systems.* As regards the establishment of the energy equilibrium between cosmic rays and the magnetic field, thus far there have been only qualitative discussions.[8,9]

One can make the following arguments in favor of the establishment of such an equilibrium. The production of cosmic rays and their acceleration is apparently a fundamental property of a turbulent magnetized plasma on a cosmic scale. If such a plasma occupies a limited volume, then, as the cosmic-ray energy increases, a limit will be reached at which it will no longer be possible to retain the cosmic rays in this volume. In this case, the equilibrium in the volume will be preserved at the expense of the escape of part of the cosmic rays as soon as the energy begins to exceed some critical value depending on the magnetic field intensity. It is natural to assume (if only from dimensional considerations) that this state of equilibrium is reached when there is a uniform distribution of energy between the above-mentioned three components. Thus we henceforth assume that the magnetized gaseous cloud with intense internal motion attains relatively rapidly a state of equilibrium in which

$$W_{\text{c.r.}} = W_{\text{turb}} = W_{\text{magn}} = W/3, \tag{3}$$

*The present state of the problem of magnetohydrodynamic turbulence and the existing ideas are discussed in a review article by the author.[6] Although the question of the character of stationary turbulence has not yet been clarified theoretically, all the available data indicate that, at least in cosmic conditions, the magnetic energy tends to be equal to the total kinetic energy of the turbulence, and not to the energy of the smallest eddies, as is sometimes assumed (see e.g. reference 7).

where W is the total energy of the cloud and consists of the turbulent energy W_{turb}, the magnetic field energy W_{magn}, and the cosmic-ray energy $W_{\text{c.r.}}$.

According to present ideas (see, e.g., reference 3), the clouds producing cosmic rays are, first of all, nebulae and supernova shells. It is also not impossible that a similar role can be played by the rapidly rotating region close to the galactic nucleus.

We shall assume, further, that the injection of new fast particles has ceased and the decrease in energy of the cloud occurs mainly through the escape of relativistic particles. The latter may be due to the diffusion of relativistic particles towards the boundaries of the cloud or to the ejection of "bunches" of cosmic rays as a result of local disturbances of the magnetic field and the boundaries of the cloud. Then the equation of the energy balance based on (3) has the form

$$dW \equiv d(3n\overline{E}_k) = \overline{E}_k dn, \tag{4}$$

where n is the number of relativistic particles in the nebula and $\overline{E}_k$ is the average kinetic energy. Here $-\overline{E}_k\,dn$ is the drop in energy due to the escape of cosmic-ray particles. Hereafter, only the ultrarelativistic energy region in which $E_k \approx E$ will be considered for galactic cosmic rays.

Equation (4) gives the following relation between the number of cosmic-ray particles remaining in the cloud and their average energy:*

$$n = \text{const}\cdot\overline{E}^{-1.5}. \tag{5}$$

The differential spectrum of the particles emitted from the nebula then has the form

$$N(E)\,dE = -dn = \text{const}\cdot E^{-2.5}dE. \tag{6}$$

Here we have replaced the average energy of the particles by their true energy E. Such a substitution, of course, is valid if the particle spectrum inside the cloud is close to being monoenergetic, which would correspond to the quite natural assumption that all the accelerated particles are injected during a small interval of time,[11] and, consequently, the acceleration of all the relativistic particles is approximately the same. This assumption, however, is not a necessary one. For any particle spectrum in the cloud, the spectrum outside it (6) is preserved if the mechanism of acceleration of the cosmic rays is such that the increase in energy is proportional to the energy

*In the general case, if in the state of equilibrium the cosmic-ray energy amounts to a fraction δ of the remaining forms of energy, then, as shown by the author,[10] $n = \text{const}.\ \overline{E}^{-(1+\delta)}$.

of the particle (see Appendix). Such a law for the increase of energy is very common. In particular, it is valid for all regular mechanisms of acceleration (mechanisms of the first order) and also for the Fermi statistical acceleration in the relativistic region and the statistical betatron mechanism (see, e.g., reference 3).

Hence, regardless of the character of the acceleration and the energy spectrum of the particles inside the nebula, the cosmic-ray spectrum in the surrounding space will have the form (6), in good agreement with what is observed for galactic cosmic rays.

We note that within the framework of the scheme, the cosmic-ray spectrum inside the sources is in no way connected with the observed galactic cosmic-ray spectrum, and is, in principle, quite arbitrary. At the same time, in the interpretation of radio waves of galactic nebula as synchrotron radiation of relativistic electrons, the relativistic electron spectrum in the nebula-shells of supernovae usually proves to be close to the spectrum of galactic cosmic rays. Thus if we assume the same mechanism for the production of cosmic rays and relativistic electrons or the secondary nature of the latter (production of relativistic electrons in collisions between cosmic rays and atoms of the nebula gas), we can conclude that the cosmic-ray spectrum in the sources is similar to the cosmic ray spectrum observed at the earth.

It should be mentioned, however, that the spectra of radio-wave electrons in galactic nebulae sometimes differ essentially from the galactic cosmic-ray spectrum. Thus, for example, Crab nebula, which according to present notions is one of the most typical sources of cosmic rays, has a relativistic-electron spectrum characterized by an exponent $\gamma = 1.7$, while it follows from general considerations that the relativistic-electron spectrum, either from cosmic-ray origin or in the case of their secondary nature, can be only a soft cosmic-ray spectrum. Moreover, the very question of the nature of the relativistic electrons of galactic nebulae is still far from clear. Hence there is not yet any serious basis to require that the mechanism by which the spectrum is produced must lead to the same spectrum for relativistic particles both inside and outside the source.

2. COSMIC RADIATION FROM SOLAR FLARES

As distinct from galactic gaseous nebulae, parts of the solar chromosphere responsible for cosmic rays should be regarded as energetically isolated. In fact, the chromospheric flares usually leading to the appearance of solar cosmic rays represent a local phenomenon on the sun's surface; as a rule, they occur close to sunspots and are apparently mainly caused by the magnetic fields of the spots.[12] It is reasonable to assume that, as a result of some accelerating process in the solar chromosphere, a cavity filled with fast particles and maintained by the pressure of the external magnetic field is formed. Under sufficiently large pressure from these particles and a slow dissipation of their energy, the retaining magnetic field is ruptured and the contents of the cavity are ejected into the space around the sun. Then, if we consider the external pressure resulting from the strong magnetic field of the spots to be practically constant during the process of ejection of the particles from the cavity, we can use the thermodynamical relation

$$dH = \overline{E}_k dn, \tag{7}$$

where H is the heat function of the system, n is the number of particles, and $\overline{E}_k$ is the average energy of the particles in the cavity. The presence of the heat function in (7) at once reflects the fact that the system is not an isolated one; as more particles leave, the average energy of the particles in the cavity increases at the expense of the work done by the external pressure.

The cosmic-ray gas can be regarded as an ideal gas with a constant specific-heat ratio κ. Then

$$H = \varkappa \mathscr{E} = \varkappa n \overline{E}_k, \tag{8}$$

where $\mathscr{E}$ is the total kinetic energy of the particles in the cavity. From Eqs. (7) and (8) it follows that the relation between the number of particles in the cavity and the average energy per particle is

$$n = \mathrm{const} \cdot \overline{E}_k^{-\varkappa/(\varkappa-1)} . \tag{9}$$

The spectrum of the particles leaving the cavity therefore has the form

$$N(E_k)\, dE_k = -dn = \mathrm{const} \cdot E_k^{-\gamma}, \tag{10}$$

where

$$\gamma = (2\varkappa - 1)/(\varkappa - 1). \tag{11}$$

As in the derivation of the expression (6), the substitution of the true energy in (10) for the average energy is valid under very general assumptions (see Appendix).

Let us consider briefly the choice of the specific-heat ratio κ for the cosmic-ray gas. As shown by Ginzburg and the author,[3] for a strictly regular acceleration between approaching parallel walls, $\kappa = 3$ in the nonrelativistic case and $\kappa = 2$ in the relativistic case. For regular betatron acceleration, the corresponding values are $\kappa = 2$ and $\kappa = 3/2$. These values were obtained under the assumption that the energy is channeled into one (in the case of walls) or two (for betatron acceleration) degrees of freedom of the particle. Actually, if the walls are not strictly parallel, and also owing to scattering on inhomogeneities of the magnetic field, the end result is that all degrees of freedom of the particle are equally privileged. Then, as should be the case for an ideal gas, $\kappa = 5/3$ in the nonrelativistic case and $\kappa = 4/3$ in the relativistic case. For these values of κ, the spectrum of solar cosmic rays, owing to (10) and (11), will be exponential with $\gamma = 3.5$ in the region of nonrelativistic energies and $\gamma = 5$ in the region of ultrarelativistic energies.

The experimental data indicate[13] that γ is already close to five for solar cosmic rays in the region of comparatively low energies ($E_k \gtrsim 100$ Mev for protons). This means that the spectrum turns out to be softer than would follow from the arguments developed above. One should keep in mind, however, that the solar cosmic-ray spectrum can become softer as the cosmic rays travel from the sun to the earth, owing to the large velocity of diffusion of the faster particles. Moreover, any losses in the cavity considered above (radiation, viscous, and Joule dissipation) and the fact that the external pressure does not remain constant as the cavity contracts should also lead to a softening of the spectrum.

APPENDIX

Let the differential energy spectrum of relativistic particles in the source have the form

$$\nu(E, t)\, dE = n(t)\, f(E, t)\, dE, \qquad \int_0^\infty f(E, t)\, dE = 1. \quad \text{(A.1)}$$

Then, after all the relativistic particles have left the source, their spectrum outside the nebula will be

$$N(E)\, dE = -dE \int_0^\infty f(E, t) \frac{dn}{dt} dt. \quad \text{(A.2)}$$

Using the relation between the average energy of a particle in a nebula $\bar{E}(t) = \int_0^\infty f(E, t)\, E\, dE$ and their number $n(t)$ [see (5) and (9)], which we write in the general form

$$n(t) = \text{const} \cdot [\bar{E}(t)]^{-(\gamma-1)}, \quad \text{(A.3)}$$

we obtain, by (A.2),

$$N(E)\, dE = \text{const} \cdot dE \int_{\bar{E}(0)}^\infty f(E, t)\, [\bar{E}(t)]^{-\gamma}\, d\bar{E}(t). \quad \text{(A.4)}$$

For particles with a monochromatic spectrum in the nebula $f(E, t) = \delta[E - \bar{E}(t)]$, we at once obtain

$$N(E)\, dE = \text{const} \cdot E^{-\gamma} dE, \qquad E > E(0). \quad \text{(A.5)}$$

If $f(E, t)$ is an arbitrary function which, as a result of the acceleration of the particle, varies with time according to the law

$$E(t) = E(0)\, \varphi(t), \quad \text{(A.6)}$$

then

$$f(E, t) = \frac{1}{\varphi(t)} f\left(\frac{E}{\varphi(t)}\right), \qquad \bar{E}(t) = \bar{E}(0)\, \varphi(t)$$

and we obtain from (A.4)

$$N(E)\, dE = \text{const} \cdot dE \int_1^\infty f\left(\frac{E}{\varphi}\right) \frac{d\varphi}{\varphi^{\gamma+1}}$$

$$= \text{const} \cdot E^{-\gamma} dE \int_0^E f(E_0)\, E_0^{\gamma-1}\, dE_0. \quad \text{(A.7)}$$

If in the high-energy region the initial spectrum of the particles in the source drops sufficiently rapidly with an increase in energy (faster than $E^{-\gamma}$) then, for $E \gg \bar{E}(0)$, we once again obtain the spectrum (A.5).

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