DEPOLARIZATION OF μ^+ MESONS AND POLARIZATION OF Σ^+ PARTICLES IN A MAGNETIZED PARAMAGNETIC GAS

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A formula is derived for the degree of depolarization of polarized μ^+ mesons in a paramagnetic gas situated in a magnetic field. Exchange of electrons between the gas atoms and a μ^+e^- atom is taken into account. The degree of depolarization depends on the fraction of μ^+-mesons capturing an electron, on the probability of electron exchange, and on the magnetic field strength. The degree of polarization in a paramagnetic gas of initially unpolarized Σ^+ particles is also computed.

1. INTRODUCTION

IN connection with the discovery of nonconservation of parity, the problem has arisen of the degree of depolarization of μ^+ mesons in a medium. In $\pi^+ \to \mu^+ + \nu$ decay the μ^+ mesons* are produced completely polarized. Such mesons are brought to rest in a time which is short compared to their lifetime. After slowing down† they capture electrons giving rise to muonium atoms‡ (μ^+e^-), and are partially depolarized as a result of the hyperfine interaction, i.e., the interaction between the magnetic moments of the meson and the electron. After the formation of muonium the meson polarization will be equal to ½, since in the $\mu^+_\uparrow e^-_\uparrow$ system the meson is not depolarized, while in the $\mu^+_\uparrow e^-_\downarrow$ system it is completely depolarized.

When unpolarized positively charged fermions, for example Σ^+ particles, are introduced into a magnetized paramagnetic gas the formation of atoms and the hyperfine splitting will lead to polarization of the particles, and the degree of polarization will be of the order of magnitude of the electron polarization and will be much greater than the polarization of Σ^+ corresponding to the magnetic field. Such a mechanism for the polarization of protons has been proposed by Zavoĭskii.[3]

Further depolarization depends on the electron exchange between an atom of μ^+e^- and the atoms of the surrounding medium and occurs only in a paramagnetic gas. At first sight it would seem that the greater the number of exchanges the more complete will be the depolarization. In actual fact the depolarization will be small if the probability of exchange is large. The non-occurrence of depolarization in the case of a large number of exchanges can be explained in the following manner: suppose that in a time corresponding to the hyperfine splitting, $\tau = \hbar/\epsilon$ (ϵ is half the hyperfine splitting energy), there occur n exchanges and $n > 1$. Then the depolarization during the time $\Delta t = \tau/n$ is equal to $(\Delta t/\tau)^2 = n^{-2}$, since transitions between states of the discrete spectrum occur in accordance with $1 - \cos(t/\tau)$. The depolarization during the time τ is equal to $1/n$. It can be seen that it tends to zero for a large number of exchanges. In other words, if the electrons are exchanged rapidly their magnetic field is averaged out and does not lead to a reversal of the meson spin. Such a possibility can be realized in a metal where the density of free electrons is great. However, if the probability of exchange is neither too great nor too small, then, since the meson lifetime is much greater than the time corresponding to the hyperfine splitting, there exists a broad range within which the mesons must be completely depolarized.

By placing a muonium atom in a magnetic field we decrease the average depolarization during a decay time. The effect of the magnetic field in the absence of exchange has been taken into account by Ferrell and Chaos.[4] In the work of Sens et al.[5] exchange has been taken into account. However, this has been taken into account only roughly: it was assumed that exchange occurs after exactly defined time intervals. Section 2 of the present paper gives exact formulas for the degree of depolarization taking exchange into account. Solutions of the equations are discussed in Sec. 3. A formula is given for the degree of polarization of initially

*The word "mesons" will henceforth refer to μ^+ mesons.

†The depolarization of μ^+ mesons in the course of slowing down has been investigated by Ferrell, Lee, and Pal.[1]

‡The results of Hughes et al.[2] show that at least in some gases 100% production of muonium occurs.

unpolarized Σ^+ hyperons or of other positively charged fermions in a magnetized paramagnetic gas with exchange taken into account.*

It is not clear whether similar formulas can be used to explain the depolarization of μ^+ mesons in a solid, for example in an emulsion. From the formulas obtained it follows that the polarization must decrease with time exponentially. Swanson's experiments[6] show that in the majority of substances this does not occur. However, the formulas for polarization averaged over the meson decay time agreed well with the experimental data.[7]

2. DERIVATION OF EQUATIONS

We consider the mechanism of depolarization. We take the μ^+e^- atom to be situated in a magnetic field parallel to the z axis. Such a system obeys the following Schrödinger equation:

$$\mathcal{H}\psi = E\psi,$$

$$\mathcal{H} = \mathcal{H}_0 + \tfrac{16}{3}\pi g_1 g_2 \left(\tfrac{1}{4} + \mathbf{s}_1\mathbf{s}_2\right)\delta(\mathbf{r}) - g s_z H - G S_z H. \quad (1)$$

Here $\mathbf{r}_i$, m_i, $\mathbf{s}_i$, g_i are respectively the position vector, the mass, the spin and the gyromagnetic ratio in units of $e\hbar/2m_i c$ for the electron (subscript 1) and for the μ meson (subscript 2):

$$g = \tfrac{1}{2}(g_1 - g_2);\quad G = \tfrac{1}{2}(g_1 + g_2);\quad \mathbf{s} = \mathbf{s}_1 - \mathbf{s}_2;$$
$$\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2;\quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2.$$

If the energy is measured in units of $\epsilon = 8\pi g_1 g_2 |\psi(0)|^2/3 = 1.84 \times 10^{-5}$ ev, and the magnetic field is measured in units of $H_0 = \epsilon/g = 1580$ gauss, $x = H/H_0$, then the Hamiltonian assumes the following form

$$\mathcal{H} = \mathcal{H}_0 + \tfrac{1}{2}(1 + 4\mathbf{s}_1\mathbf{s}_2) - x s_z - G g^{-1} x S_z. \quad (2)$$

The eigenfunctions of this Hamiltonian $\mathcal{H}$ and its energy levels have the following form

$$\Psi_{11} = \varphi(r)\alpha(1)\alpha(2),\quad E_{11} = E_0 - G g^{-1} x;$$
$$\Psi_{1-1} = \varphi(r)\beta(1)\beta(2),\quad E_{1-1} = E_0 + G g^{-1} x;$$
$$\Psi_+ = \varphi(r)\,[(f+g)\alpha(1)\,\beta(2) + (f-g)\beta(1)\,\alpha(2)]/\sqrt{2},$$
$$E_+ = E_0 + \Delta;$$
$$\Psi_- = \varphi(r)\,[(f-g)\alpha(1)\,\beta(2) - (f+g)\beta(1)\,\alpha(2)]/\sqrt{2},$$
$$E_- = E_0 - \Delta, \quad (3)$$

where $\alpha(\beta)$ means that the particle spin is directed up (down),

$$f = \sqrt{{}^1/_2(1 + 1/\Delta)},\quad g = \sqrt{{}^1/_2(1 - 1/\Delta)},$$
$$\Delta = \sqrt{1 + x^2}. \quad (4)$$

*We note that although the formation of the Σ^+e^- atom increases the effect of polarization of Σ^+, nevertheless, because of the short lifetime of Σ^+ this effect is small and, apparently, cannot be observed experimentally.

The spin state of the system μ^+e^- is described by the density matrix $\rho(t)$ which has for its system of basis vectors

$$\chi_1 = \alpha_\mu\alpha_e,\quad \chi_2 = \alpha_\mu\beta_e,\quad \chi_3 = \beta_\mu\alpha_e,\quad \chi_4 = \beta_\mu\beta_e$$

and of all the off-diagonal elements only ρ_{23} and ρ_{32} differ from zero, while all the other off diagonal ρ_{ik} are equal to zero, since as the system develops freely in time states with different components of the spin along the z axis do not mix. The collisions in which the component of the spin of the system along the z axis are altered are incoherent ones and, therefore, the wave functions which after the collision correspond to different spin components are not related by definite phase relations.

The density matrix is found in the usual manner. If the system has the probability w^γ of being in the state $\Psi^\gamma = \sum_i c_i^\gamma \Psi_i$ where all the states Ψ^γ are incoherent with respect to each other, i.e., are not related by definite phase relations, then we have

$$\rho_{ik} = \sum_\gamma w^\gamma c_i^\gamma c_k^{\gamma *}.$$

For the average value of any physical quantity O specified by a matrix with respect to the same set of basis vectors we have $\langle O \rangle = \mathrm{Tr}(\rho O)$.

We write down the equation for the density matrix, taking into account the fact that the change of the density matrix per unit time is equal to the change of the density matrix as a result of its free development in time without collisions, $(d\tilde{\rho}/dt)$ plus its change as a result of collisions:

$$d\rho/dt = d\tilde{\rho}/dt + w(\rho' - \rho). \quad (5)$$

The first term may be obtained if we know the energy levels of the system in the magnetic field. By considering collisions of the μ^+e^- atom with the atoms of the surrounding medium under the condition that the medium contains equal numbers of atoms with any arbitrary spin component parallel to the direction of the magnetic field ($\mu H/kT \ll 1$) we shall obtain ρ' (ρ' is the density matrix after the collision). Since Coulomb forces play the principal role in collisions, we need not take into account the spin of the μ meson and the interaction of the magnetic moment of the electron with the gas atom. Moreover, the equations involve the amplitudes a_+ and a_- for the exchange of an electron with a gas atom in states of total angular momentum respectively given by $j + \frac{1}{2}$ and $j - \frac{1}{2}$ (j is the spin of the gas atom). It turns out that the equations contain only the quantity w which is the probability that during the time τ the process $\mu^+e^-_\uparrow + A \rightarrow \mu^+e^-_\downarrow + A'$ will occur. It can be shown that w is proportional to $|a_+ - a_-|^2$, and this is natural, since it is specifically this quantity that is associated with the reversal of spin.

As a result of this we obtain the following system of equations (we have taken $\tau = \hbar/\epsilon$ for the unit of time):

$$\begin{gathered} d\rho_{11}/dt = -w(\rho_{11} - \rho_{22}), \qquad d\rho_{44}/dt = -w(\rho_{44} - \rho_{33}), \\ d\rho_{22}/dt = -w(\rho_{22} - \rho_{11}) + i(\rho_{23} - \rho_{32}), \\ d\rho_{33}/dt = -w(\rho_{33} - \rho_{44}) - i(\rho_{23} - \rho_{32}), \\ d\rho_{23}/dt = -2(w + i|x|)\rho_{23} + i(\rho_{22} - \rho_{33}), \\ d\rho_{32}/dt = -2(w - i|x|)\rho_{32} + i(\rho_{33} - \rho_{22}). \end{gathered} \tag{6}$$

In the formation of the μ^+e^- atom the meson is totally polarized, so that the initial conditions will be the following:

$$\rho_{11}(0) = 1/2, \qquad \rho_{22}(0) = 1/2, \qquad \text{all other } \rho_{ik}(0) = 0. \tag{7}$$

In the case when the particles capturing the electrons are unpolarized, but the gas is magnetized, the initial conditions will be

$$\begin{gathered} \rho_{11}(0) = n_\uparrow/2(n_\uparrow + n_\downarrow), \qquad \rho_{22}(0) = n_\downarrow/2(n_\uparrow + n_\downarrow), \\ \rho_{33}(0) = n_\uparrow/2(n_\uparrow + n_\downarrow), \qquad \rho_{44}(0) = n_\downarrow/2(n_\uparrow + n_\downarrow), \\ \text{all other } \rho_{ik} = 0. \end{gathered} \tag{8}$$

Here $n_\uparrow$ and $n_\downarrow$ are the numbers of electrons with the spins respectively up and down.

The polarization of the μ^+ mesons at time t is equal to

$$P(t) = \rho_{11}(t) + \rho_{22}(t) - \rho_{33}(t) - \rho_{44}(t). \tag{9}$$

On averaging over the μ^+ meson lifetime we obtain

$$\overline{P} = \lambda \int_0^\infty P(t) e^{-\lambda t} dt, \tag{10}$$

where λ is the probability of decay in the time τ, $\lambda \approx 10^{-5}$ for a μ meson.

3. DISCUSSION OF THE SOLUTIONS

On solving Eqs. (6) subject to the initial conditions (7) we obtain

$$\begin{gathered} \overline{P} = a\lambda \left[\frac{1}{\lambda + \alpha} - \frac{\alpha(\lambda + \alpha)}{\alpha\lambda^2 + (4w - \alpha)\alpha\lambda + 4w}\right] \\ - \frac{\alpha\lambda^2}{\alpha\lambda^2 + (4w - \alpha)\alpha\lambda + 4w}, \\ a = (2w - \alpha)/[2w - \alpha^2(2w - \alpha)], \end{gathered} \tag{11}$$

where α is the real root of the equation

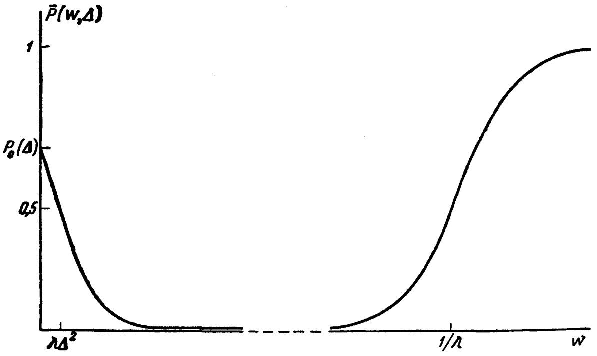
$$\mu^3 - 4w\mu^2 + 4(w^2 + \Delta^2)\mu - 4w = 0, \tag{12}$$

Δ is defined by (4).

It follows from equation (12) that

$$w/(w^2 + \Delta^2) \leqslant \alpha \leqslant 4w/(w^2 + 4\Delta^2)$$

and that there are no other real roots. By neglecting terms of order λ^2 and λ we obtain



Graph of the function $\overline{P}(w, \Delta)$ for $\lambda \ll 1$, $\Delta \ll 1/\lambda$; $P_0(\Delta) = 1 - 1/2\Delta^2$.

$$P(t) = ae^{-\alpha t}, \qquad \overline{P} = \frac{2w - \alpha}{2w - \alpha^2(2w - \alpha)} \frac{\lambda}{\lambda + \alpha}. \tag{13}$$

We see that $\overline{P}$ is not small only when $\alpha \lesssim \lambda$, i.e., for $w \ll \Delta$ and $w \gg \Delta$ (cf. diagram). In the case $w \ll \Delta$ we have

$$\overline{P} = \left(1 - \frac{1}{2\Delta^2}\right) \frac{1}{1 + w/\lambda\Delta^2}. \tag{14}$$

If $w \ll \lambda\Delta^2$, then the usual formula is obtained (cf. reference 4)

$$\overline{P} = (1 - 1/2\Delta^2). \tag{15}$$

For $w \gg \Delta$

$$\overline{P} = (1 + 1/\lambda w)^{-1}. \tag{16}$$

If $w \gg 1/\lambda$, then $\overline{P} = 1$. When $\alpha \gg \lambda$ we have $\overline{P} \approx 0$.

If we take into account the fact that not all the μ mesons capture electrons we obtain

$$P = (1 - f) + f\overline{P}, \tag{17}$$

where f is the fraction of mesons capturing electrons.

On solving (6) subject to the initial conditions (8) we obtain

$$\begin{gathered} \overline{P} = pb\left[\frac{\lambda}{\lambda + \alpha} - \frac{\lambda\alpha(\alpha + \lambda)}{\alpha\lambda^2 + (4w - \alpha)\alpha\lambda + 4w}\right], \\ b = \frac{\alpha}{2w + \alpha^2(\alpha - 2w)}, \qquad p = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow}. \end{gathered} \tag{18}$$

In the case $w \ll \Delta \ll \lambda$

$$\overline{P} = p\left[\frac{2}{\lambda^2} + \frac{w(4 - 3/\Delta^2)}{2\Delta^2\lambda}\right]. \tag{19}$$

In the case $w \ll \lambda \ll \Delta$

$$\overline{P} = \frac{p}{2\Delta^2}\left[1 + \left(1 - \frac{1}{2\Delta^2} - \frac{1}{\lambda^2}\right)\frac{w\lambda}{\Delta^2}\right]. \tag{20}$$

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