CERTAIN PROBLEMS IN RELATIVISTIC GAS DYNAMICS OF CHARGED PARTICLES

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A number of general problems in relativistic gas dynamics of charged particles in external fields and self-consistent fields are considered. An analog is found for potential motion, and the theorem for conservation of magnetic flux along a fluid contour is generalized. The one-dimensional problem of relativistic breakup of a charged layer in external fields is solved.

1. INTRODUCTION

THE relativistic motion of a conducting gas is characterized by a number of features, which can lead to effects that are qualitatively different from those found for nonrelativistic motion. For example, Veksler[1] has shown that when an ionized gas mass collides with a concentration of magnetic force lines the ions transfer an appreciable part of their energy to the electrons; the electrons then become relativistic, even if the initial gas velocities are nonrelativistic. At relativistic gas velocities this limiting-current effect becomes still more important. It leads to a situation such that even for negligibly small collision frequencies ν_{eff} between the electrons and ions (i.e., high conductivity), in the reference system attached to a given gas element $\mathbf{E} + \mathbf{v} \times \mathbf{H}$ does not vanish and is determined by the inertia force which, in the ultra-relativistic limit, can be large because of the relativistic mass increase. The magnetic field flux through the fluid contour is therefore not conserved and the "freezing" of the magnetic lines of force is disturbed.

Thus, one of the peculiarities of relativistic motion of a conducting gas mass is that the magnetic lines of force may not be frozen. As a result, the description of this motion by equations which contain the two vectors $\mathbf{v}$ and $\mathbf{H}$, the hydrodynamic velocity and the magnetic field (relativistic magnetogasdynamics), is no longer possible. The remark pertains especially to transient processes in which the inertia force in the accompanying reference system may be large.

In order to investigate the relativistic motion of a conducting gas it is convenient to consider relativistic two-component gas dynamics in electromagnetic fields. By virtue of what has been indicated above, interest attaches in the relativistic case only to the situation in which the friction of one component against the other is negligibly small compared with the effect of the interaction with the self-consistent field. For this reason, we can consider the equations separately for each of the components in the electromagnetic fields, including in the latter external fields and the fields produced by all the gas components.

The relativistic gas dynamics of a neutral gas have been considered by Khalatnikov.[2] In Sec. 1 of the present work we extend the corresponding results of Khalatnikov to include self-consistent fields and external electromagnetic fields. In spite of the fact that the magnetic lines of force may not be frozen for relativistic motion of an electron-ion plasma, it is found that in those cases for which the ion current is small it is possible to extend the theorem of conservation of magnetic flux along a fluid contour.

In problems which are of interest in accelerator technology, for example the case of a charged electron gas, either the inertia term in the equations of motion is considerably greater than the term that contains the pressure derivatives, or else the external fields have an important effect on the motion. In both cases the characteristic dimensions and the time intervals, which determine the possibility of applying the hydrodynamic analysis, are, to a large extent, determined by the initial and boundary conditions, and by the variations in the external field.

In the present work we consider one-dimensional relativistic breakup of a charged gas layer in vacuum when the inertia term is the principal one. We also consider relativistic collisions between a charged layer and constant external fields. The electric fields produced by breakup of a quasineutral plasma layer in a vacuum are also analyzed.

2. SOME GENERAL PROBLEMS

By virtue of the above considerations, the equation of relativistic motion for a gas of charged particles in an electromagnetic field can be written in the form of an equation for each component s:[3]

$$\partial T^{(s)}_{\mu\nu}/\partial x_\nu = F_{\mu\nu} j^{(s)}_\nu, \qquad T^{(s)}_{\mu\nu} = w^{(s)} u^{(s)}_\mu u^{(s)}_\nu + p^{(s)} \delta_{\mu\nu},$$
$$n^{(s)}_{\text{lab}} = n^{(s)} / \sqrt{1 - v^{(s)2}}, \qquad (1)$$

where $u^{(s)}_\mu$ is the 4-velocity of the gas, $p^{(s)}$ is the pressure of the gas, $w^{(s)} = \epsilon^{(s)} + p^{(s)}$ is the heat function per unit proper volume, $\epsilon^{(s)}$ is the internal energy per unit proper volume, $j^{(s)}_\mu = e^{(s)} n^{(s)} u^{(s)}_\mu$ is the current produced by particles of a given type, $n^{(s)}$ is the proper density of particles of this type, $v^{(s)}$ is the three-dimensional velocity (the velocity of light is equal to unity). In addition, each component obeys a continuity equation

$$\frac{\partial}{\partial x_\nu} n^{(s)} u^{(s)}_\nu = 0.$$

We determine the motion and the fields, by using in addition to Eq. (1) the Maxwell equations

$$\partial F_{\mu\nu} / \partial x_\nu = 4\pi \Big(j^{(0)}_\mu + \sum_s j^{(s)}_\mu \Big), \qquad (2)$$

where $j^{(0)}_\mu$ is the current that produces the external fields. Multiplying (1) scalarly by $u^{(s)}_\mu$ and taking account of the equations of continuity, the condition $u^{(s)}_\mu F_{\mu\nu} j^{(s)}_\nu = 0$, and $u^{(s)}_\mu u^{(s)}_\mu = -1$, we find by virtue of the thermodynamic law that entropy is conserved along the phase trajectories.

If at any instant of time (or on some hypersurface of the 4-dimensional manifold x_μ) the entropy is the same at all points, then the motion is isentropic. In what follows we confine ourselves to isentropic motion so that Eq. (1) can be written in simple form ($W^{(s)} = w^{(s)}/n^{(s)}$)

$$u^{(s)}_\mu \frac{\partial}{\partial x_\mu} W^{(s)} u_\nu = - \frac{\partial W^{(s)}}{\partial x_\nu} + e F_{\nu\mu} u^{(s)}_\mu. \qquad (3)$$

We find an analog for the condition for potential motion for a gas of charged particles in an electromagnetic field. As shown by Khalatnikov,[2] the relativistic analog of this condition for a neutral gas is $Wu_\mu = \partial\varphi/\partial x_\mu$. It is simple to generalize this result to the case in which electromagnetic fields are present. Introducing the 4-potential of the electrodynamic field A_μ, we see that Eq. (3) has a solution of the following form:

$$W^{(s)} u^{(s)}_\mu = - e A_\mu + \partial \varphi^{(s)} / \partial x_\mu, \qquad (4)$$

which is the required generalization. The fourth relation in (4) is an analog of the Bernoulli equation.

We show further that (4) is always satisfied for the one-dimensional nonstationary case. For one-dimensional motion all quantities depend on x_1 and x_4, and Eq. (3) can be written in the form

$$- u_4^2 \frac{\partial W^{(s)}}{\partial x_1} + u_4 \frac{\partial}{\partial x_4} W^{(s)} u_1 + W^{(s)} u_1 \frac{\partial u_1}{\partial x_1} = e u_4 \left(\frac{\partial A_4}{\partial x_1} - \frac{\partial A_1}{\partial x_4} \right) \qquad (5)$$

Since $u_4 \neq 0$, it follows from Eq. (5) that

$$\frac{\partial}{\partial x_1} (W^{(s)} u_4 + e A_4) = \frac{\partial}{\partial x_4} (W^{(s)} u_1 + e A_1), \qquad (6)$$

and this proves that the motion is potential.

We now consider the case of one-dimensional motion in the presence of a current perpendicular to the direction of motion. We assume that the charged fluid being considered is an electron fluid with charge partially or completely compensated by the ion component, whose contribution to the current density can be neglected. As before, we assume that all quantities depend only on x_1 and x_4 but $u_3 \neq 0$ and $A_3 \neq 0$. Then, the projection of Eq. (3) on the x_3 axis gives $x_0 = -ix_4$, $u^{(e)}_1 = v^{(e)} \gamma^{(e)}$, $\gamma^{(e)} = (1 - v^{(e)2} - v_3^{(e)2})^{-1/2}$

$$\frac{\partial}{\partial x_0} W^{(e)} v_3^{(e)} \gamma^{(e)} + v \frac{\partial}{\partial x_1} W^{(e)} v_3^{(e)} \gamma^{(e)} = e (E_3 + v^{(e)} H_2). \qquad (7)$$

It follows from Eq. (7) that the inertia force, which may be large at relativistic velocities, disturbs the freezing even if the electrical conductivity is infinite, $E_3 \neq - vH_2$. Instead of finding the relation between H_2 and E_3, which follows from Eq. (7), it is convenient to find the condition imposed on the potential. We write Eq. (7) in the form

$$\left(u_1 \frac{\partial}{\partial x_1} + u_4 \frac{\partial}{\partial x_4} \right) (W^{(e)} u_3^{(e)} + e A_3) = \frac{d}{ds} (W^{(e)} u_3^{(e)} + e A_3) = 0. \qquad (8)$$

Hence it follows that the quantity $p_3 = W^{(e)} u_3^{(e)} + eA_3$ is conserved along the trajectory. This result is valid not only for one-dimensional motion. To satisfy Eq. (8) it is sufficient that all quantities be independent of x_3 (cyclical coordinate).

If the inertia term $W^{(e)} u_3^{(e)}$ can be neglected the conservation relation (8) reduces to the conservation of flux through the fluid contour ($A_3 \sim \Phi$). If p_3 is the same for all points at the initial time, then it remains so at all subsequent times. It can be shown that this one-dimensional nonstationary motion is always potential motion. The first equation in (3) can be written in the form

$$(1 + u_1^{(e)2}) \frac{\partial W^{(e)}}{\partial x_1} + u_4^{(e)} \frac{\partial}{\partial x_4} W^{(e)} u_1^{(e)} + W^{(e)} u_1^{(e)} \frac{\partial u_1^{(e)}}{\partial x_1}$$
$$= e u_4^{(e)} (\partial A_4 / \partial x_1 - \partial A_1 / \partial x_4) + e u_3^{(e)} \partial A_3 / \partial x_1. \qquad (9)$$

We use the identity $u_1^{(e)2} + u_3^{(e)2} + u_4^{(e)2} = -1$ and the relation

$$e \partial A_3 / \partial x_1 = - u_3^{(e)} \partial W^{(e)} / \partial x_1 - W^{(e)} \partial u_3^{(e)} / \partial x_1,$$

which follows from the condition

$$W^{(e)} u_3^{(e)} + e A_3 = \text{const}.$$

Equation (9) is then reduced to the form

$$\frac{\partial}{\partial x_1}(W^{(e)}u_4^{(e)} + eA_4) = \frac{\partial}{\partial x_4}(W^{(e)}u_1^{(e)} + eA_1), \tag{10}$$

which shows that the motion is potential.

Finally, we consider 3-dimensional motion and obtain a generalization of the Thomson theorem for the conservation of circulation of velocity. We use the notation $p_\mu^{(s)} = W^{(s)}u_\mu^{(s)} + eA_\mu$ and for the 4-curl of this quantity $p_{\mu\nu}^{(s)} = \partial p_\mu^{(s)}/\partial x_\nu - \partial p_\nu^{(s)}/\partial x_\mu$. We now find the substantive derivative of $p_{\mu\nu}^{(s)}$ with respect to proper time

$$\frac{d}{ds}\, p_{\mu\nu}^{(s)} = p_{\mu\alpha}^{(s)} \frac{\partial u_\alpha^{(s)}}{\partial x_\nu} - p_{\nu\alpha}^{(s)} \frac{\partial u_\alpha^{(s)}}{\partial x_\mu}. \tag{11}$$

In deriving the last relation we have used Eq. (3), $u_\alpha^{(s)}\partial u_\alpha^{(s)}/\partial x_\nu = 0$, and the Maxwell equations

$$\partial F_{\nu\mu}/\partial x_\alpha + \partial F_{\alpha\nu}/\partial x_\mu + \partial F_{\mu\alpha}/\partial x_\nu = 0.$$

It is apparent that in the nonrelativistic limit $W \approx m$, $u_i \approx v_i$, $u^4 \approx i$, Eq. (11) leads in the absence of electromagnetic fields to a relation for the conservation of circulation of velocity along the fluid contour in differential form.

3. SOME ONE-DIMENSIONAL PROBLEMS

a) Relativistic breakup of a layer of charged particles in vacuum. The relativistic breakup of a layer of neutral gas in vacuum has been considered by Landau[4] and Khalatnikov[2] in connection with the hydrodynamic theory for the multiple production of particles. In the decay of a cluster of charged particles of the same sign the collective effect of the total electric field produced by the particles may have a decisive effect on its decay.

We consider the limiting case of a monatomic gas at nonrelativistic temperatures. By virtue of the proof given above, the one-dimensional motions are always potential,

$$Wu_1 = -eA_1 + \partial\varphi/dx_1, \qquad Wu_4 = -eA_4 + \partial\varphi/dx_4.$$

Making use of gauge invariance, we can write $\varphi = 0$ so that the field and the motion of the gas are described by the potentials A_1 and $A_0 \equiv -iA_4$ only. The electric field is expressed simply in terms of W and $u \equiv u_1$ ($u_4 = i\sqrt{1+u^2}$):

$$eE = \frac{\partial}{\partial x_0} Wu + \frac{\partial}{\partial x_1} W\sqrt{1+u^2}. \tag{12}$$

From the electric field equations we obtain the relation

$$\frac{\partial E}{\partial x_0} + \frac{u}{\sqrt{1+u^2}}\frac{\partial E}{\partial x_1} = 0. \tag{13}$$

Rather than seek u as a function of x_1 and x_0, we seek x_1 as a function of x_0 and u: $x_1 = \psi(x_0, u)$.

Taking $d\psi/du \neq 0$ and $\chi \equiv (e/m)E(x_0, u)$, we transform Eqs. (12) and (13) to expressions in the variables x_0 and u:

$$\frac{\partial\chi}{\partial x_0} + \chi\frac{\partial\chi}{\partial u} = \frac{T_0}{m} G \frac{\partial\chi}{\partial u},$$

$$\frac{\partial\psi}{\partial x_0} + \chi\frac{\partial\psi}{\partial u} = \frac{u}{\sqrt{1+u^2}} + \frac{T_0}{m} G \frac{\partial\psi}{\partial u}; \tag{14}$$

$$G = 5\left(\frac{n}{n_0}\right)^{2/3}\left[\frac{1}{2} + \frac{1}{3}\frac{u}{n}\left(\frac{\partial n}{\partial x_0} - \frac{\partial n}{\partial u}\frac{\partial\psi/\partial x_0}{\partial\psi/\partial u}\right) + \frac{1}{3}\frac{\sqrt{1+u^2}}{n}\left(\frac{\partial\psi}{\partial u}\right)^{-1}\frac{\partial n}{\partial u}\right]. \tag{15}$$

Here we use $W = m + \frac{5}{2}T_0(n/n_0)^{2/3}$, where T_0 and n_0 are the initial temperature and density of the gas, m is the rest energy and the remaining terms are all of first order in T_0/m.

The density n must be inserted in (14) in the form

$$n = \frac{1}{r_0\sqrt{1+u^2}}\frac{\partial\chi/\partial u}{\partial\psi/\partial u}, \qquad r_0 = 4\pi\frac{e^2}{m}. \tag{16}$$

Since $T_0 \ll m$, the solution of (14) and (15) can be found in the form of an expansion in the parameter T_0/m; we keep only the first two terms:

$$\chi = \chi_0 + \chi_1 T_0/m, \qquad \psi = \psi_0 + \psi_1 T_0/m + \ldots \tag{17}$$

The equations for the successive approximation are obtained easily from (15). The system for the zeroth approximation

$$\frac{\partial\chi_0}{\partial x_0} + \chi_0\frac{\partial\chi_0}{\partial u} = 0, \qquad \frac{\partial\psi_0}{\partial x_0} + \chi_0\frac{\partial\psi_0}{\partial u} = \frac{u}{\sqrt{1+u^2}}$$

is found to be linear (the linearity of the equations is trivial for the higher approximation). The system (18) can be solved successively since the boundry or initial conditions for χ_0 can be given independently of physical considerations and apart from the constant e/m, χ_0 coincides with the magnitude of the electric field. From the value of χ_0 we then find the solution for the second equation in (18).

We present the results of analysis of the breakup of a charged layer of thickness $2l$. We assume that at the initial time $x_0 = 0$ the gas density is zero everywhere except for the layer $-l < x_1 < l$, where it is constant and equal to the value n_0. We assume that at the initial time the electric field inside the layer is a linear function of x_1. With these assumptions we find

$$\psi = u/n_0 r_0 x_0 + (x_0/u)(\sqrt{u^2+1} - 1), \qquad u = v\gamma = v/\sqrt{1-v^2}, \tag{19}$$

where v is the hydrodynamic velocity.

Taking $\psi = x_1$, we obtain an implicit expression for the velocity v as a function of x_1 and x_0. The solution in (19) applies when $|\psi| \le l + (ln_0r_0)^{-1}\times(\sqrt{1 + l^2n_0^2r_0^2x_0^2} - 1)$. The equality sign corresponds to motion of the layer boundary. The boundary of the layer moves with uniform acceleration when $x_0 \ll 1/l n_0 r_0$ and inertially when $x_0 \gg 1/l n_0 r_0$.

It follows from Eq. (19) that in the nonrelativistic limit ($\gamma = 1 + \epsilon$, $\epsilon \ll 1$)

$$\varepsilon = n_0^2 r_0^2 x_0^2 x_1^2 / 2 (1 + n_0 r_0 x_0^2 / 2)^2. \quad (20)$$

The energy of the particles increases as the square of the distance from the center of the layer. At a fixed point it first increases with time and then decreases. This last situation is explained as follows: at a given point, for large values of x_0 slow particles start to arrive from the center (these may not have high energies in the weak self-consistent fields at the center) whereas the fast particles leave this point. The characteristic time interval for the growth of energy at a given point is $\sqrt{2/n_0 r_0}$. If $x_0 \ll 1/l n_0 r_0$ and $2 n_0 r_0 l^2 \ll 1$ the maximum energy of the particles at the layer of the boundary $\epsilon_{max} = \frac{1}{2} n_0^2 r_0^2 x_0^2 l_0^2 \ll 1$ is nonrelativistic. When $n_0 r_0 l^2 \gg 1$ and $l \gg x_0 \gg 1/l n_0 r_0$ the energy becomes relativistic.

It is also easy to find the energy distribution in the ultrarelativistic case. The results are as follows: the energy increases linearly with increasing x_1. The distribution of particle density can be found from Eq. (16):

$$\frac{n}{n_0} = \left\{ \sqrt{1 + u^2} + n_0 r_0 x_0^2 \left[1 - \frac{(u^2 + 1)(\sqrt{1 + u^2} - 1)}{u^2 \sqrt{1 + u^2}} \right] \right\}^{-1} \quad (21)$$

In order to determine the region of applicability of the results obtained above, using Eqs. (14) and (15) we find the next approximation in T_0/m. The criterion is $T_0/m \ll n_0 r_0 l^2$. The method described here has also been used to solve the problem of breakup in vacuum with other density distributions, in particular, a layered structure.

The other limiting case, in which temperature effects are basically responsible for the expansion of the layer while the interaction with the self-consistent field is treated by perturbation methods (high temperatures), is of interest only in considerations of multiple production of particles and will be considered in a separate paper.

b) Breakup of a neutral plasma layer in vacuum. The ions and electrons will be treated as two charged fluids which, at the initial time, have the same fixed density in a layer of thickness $2l$. The initial temperatures are assumed to be different. By virtue of the above, the motion of electron and ion fluids is potential motion:

$$W^{(e)} u_\mu^{(e)} = -eA_\mu + \partial \varphi^{(e)} / \partial x_\mu, \quad W^{(i)} u_\mu^{(i)} = eA_\mu + \partial \varphi^{(i)} / \partial x_\mu,$$

where the index e refers to electrons while the index i refers to ions. Carrying out a gauge transformation of the electromagnetic potentials $A_\mu = A'_\mu + \partial\chi/\partial x_\mu$, we can write $\chi = \varphi^{(e)}$ and $\varphi = \varphi^{(i)} + \varphi^{(e)}$. Then the motion of the electron fluid is described by the electromagnetic field potentials, while the potential φ characterizes the total motion of the two fluids:

$$W^{(e)} u_\mu^{(e)} = -eA_\mu, \qquad W^{(i)} u_\mu^{(i)} + W^{(e)} u_\mu^{(e)} = \partial\varphi/\partial x_\mu; \quad (22)$$

$$eE = \frac{\partial}{\partial x_0} W^{(e)} u^{(e)} + \frac{\partial}{\partial x_1} W^{(e)} \sqrt{1 + u^{(e)2}}, \quad u^{(e)} \equiv u_1^{(e)}. \quad (23)$$

The electric fields produced in breakup tend to restore the disturbed neutrality of the plasma. We consider breakup while the disturbance of neutrality is still small. Here, as an approximation, we can assume the electron and ion velocities and densities to be the same. It then follows from Eq. (22) that

$$(\overline{W}^{(i)} + \overline{W}^{(e)}) u_\mu = \partial\varphi/\partial x_\mu, \qquad \mu = 1,4,$$

where $\overline{W}^{(i)}$ and $\overline{W}^{(e)}$ are the values of the heat functions at the same densities while u_μ is the common velocity. Substitution in the continuity equations (for the electrons and ions) leads to the standard solutions in which, however, we have in place of the heat functions $\overline{W}^{(i)} + \overline{W}^{(e)}$. This leads to a change in the rate of decay of the layer because we must substitute here in place of the velocity of sound

$$\frac{n}{\overline{W}^{(i)} + \overline{W}^{(e)}} \frac{d}{dn} (\overline{W}^{(i)} + \overline{W}^{(e)}) = c_{acous}^2.$$

At nonrelativistic temperatures

$$\overline{W}^{(i)} + \overline{W}^{(e)} = m^{(i)} + m^{(e)} + \tfrac{5}{2} (T^{(i)} + T^{(e)}) (n/n_0)^{2/3}.$$

When $T^{(e)} \gg T^{(i)}$ we have $c_{acous}^2 = 5T^{(e)}/3m^{(i)}$. In other words, the initial stage of breakup is described by a rarefaction wave that moves over the layer with a velocity determined by the temperature of the electrons and the mass of the ions. In later stages, after collision of the rarefaction waves, we also have the usual pattern with $c_{acous}^2 = 5T^{(e)}/3m^{(i)}$.

It is easy to find, in the next approximation, the electric fields and the differences in electron and ion density and velocity. For a simple wave we have

$$\frac{\Delta n}{n} = \frac{m^{(i)}}{9\pi e^2 n_0 x_0^2} \left(1 - \frac{1}{5} \frac{x_1}{x_0} \frac{m^{(i)}}{T^{(e)}}\right)^3, \quad (24)$$

$$\frac{\Delta v}{v} = \frac{m^{(i)}}{27\pi e^2 n_0 x_0^2} \left(1 + \frac{3}{5} \frac{x_1}{x_0} \frac{m^{(i)}}{T^{(e)}}\right) \left(1 - \frac{1}{5} \frac{x_1}{x_0} \frac{m^{(i)}}{T^{(e)}}\right)^{-2}. \quad (25)$$

If we neglect the region in direct proximity to the edge of the flowing gas, $x \ll 5x_0 T_4/m$, neutralization starts when $x_0^2 \gg 4m^{(i)}/9m^{(e)} r_0 n_0$ where $r_0 = 4\pi e^2/m$. In the stage described by the general solution, the electric fields decay more rapidly.

c) Motion of a charged layer into a region with constant electric field. We assume that at the initial time $x_0 = 0$ there is a decaying layer of thickness $2l$ which is breaking up with an initial relativistic velocity $u_{in} \gg 1$ into a region occupied by a constant electric field along the x axis, $eE_{in}/m = \chi_{in}(x_1)$. Neglecting temperature effects and making use of gauge invariance, we have

$$\chi + \chi_{in} = \frac{\partial u}{\partial x_0} + \frac{\partial}{\partial x_1}\sqrt{1+u^2}, \tag{26}$$

$$\frac{\partial \chi}{\partial x_0} + \frac{u}{\sqrt{1+u^2}}\frac{\partial \chi}{\partial x_1} = 0. \tag{27}$$

As above, we introduce $x_1 = \psi(x_0, u)$ so that Eqs. (26) and (27) are transformed to

$$\frac{\partial \chi}{\partial x_0} + (\chi + \chi_0)\frac{\partial \chi}{\partial u} = 0, \qquad \frac{\partial \psi}{\partial x_0} + (\chi + \chi_0)\frac{\partial \psi}{\partial u} = \frac{u}{\sqrt{1+u^2}}. \tag{28}$$

In the problem being considered the solution of this system can be obtained in parametric form (the parameter t varies within the following limits: $-l\sqrt{1+u_{in}^2} < t < l\sqrt{1+u_{in}^2}$)

$$x_0 = \int_t^{\psi}\Big[\sqrt{u_{in}^2+1} + \chi_{in}(t)(\psi''-t) - \int_t^{\psi''}\chi_{in}(\psi')\,d\psi'\Big]$$
$$\times\Big\{\Big[\sqrt{u_{in}^2+1} + \chi_{in}(t)(\psi''-t) + \int_t^{\psi''}\chi_{in}(\psi')\,d\psi'\Big]^2 - 1\Big\}^{-1/2} d\psi'', \tag{29}$$

$$\sqrt{1+u^2} - \sqrt{1+u_{in}^2} = \chi_{in}(t)(\psi - t) + \int_t^{\psi}\chi_{in}(\psi')\,d\psi', \tag{30}$$

where $\chi_{in}(t)$ is the initial distribution of the proper electric field inside the layer. For an initial density n_0 constant along the layer, we have $\chi_{in} = n_0 r_0 t$. For an external field which increases linearly the calculation of Eq. (29) leads to elliptic integrals.

d) Motion of a charged layer into a region occupied by a constant magnetic field. A magnetic field causes current to flow, i.e., produces a component u_3 in the velocity of the electron gas. We assume that at the initial time $u_3 = 0$ and that there is no magnetic field within the layer, $A_3 = 0$. Hence, by virtue of what has been shown above, $u_3 = -eA_3/m$ if temperature effects are neglected.

We consider the case in which the shielding of the external field by the produced current is small and we can take $u_3 \approx u_3^0 = -e/m\, A_3^0(x_1)$, where $A_3^0(x_1)$ is the potential of the external field. Finding $x_1 = \psi(x_0, u)$ we obtain

$$\frac{\partial \psi}{\partial x_0} + \left(\chi - \frac{u_3^0}{(1+u^2+u_3^{02})^{1/2}}\frac{eH_2^0}{m}\right)\frac{\partial \psi}{\partial u} = \frac{u}{(1+u^2+u_3^{02})^{1/2}}, \tag{31}$$

$$\frac{\partial \chi}{\partial x_0} + \left(\chi - \frac{u_3^0}{(1+u^2+u_3^{02})^{1/2}}\frac{eH_2^0}{m}\right)\frac{\partial \chi}{\partial u} = 0, \tag{32}$$

where $H_2^0 = -\partial A_3^0/\partial x_1$ is the external magnetic field and $\chi = eE/m$ is the proper electric field.

In the case in which a layer $2l$ moves at the initial time with relativistic velocity $u_{in} \gg 1$ into a region occupied by a magnetic field the solution of (31) and (32) is of the form

$$\sqrt{1+u^2+u_3^{02}(\psi)} - \sqrt{1+u_{in}^2} = (\psi - t)\chi_n(t),$$

$$x_0 = \int_t^{\psi} d\psi'\Big[\sqrt{1+u_{in}^2} + \chi_{in}(t)(\psi'-t)\Big]$$
$$\times\{[\sqrt{1+u_{in}^2} + \chi_{in}(t)(\psi'-t)]^2 - u_3^{02}(\psi') - 1\}^{-1/2}, \tag{33}$$

$$-l\sqrt{1+u_{in}^2} < t < l\sqrt{1+u_{in}^2}. \tag{34}$$

For an external field which increases linearly with distance the calculation of Eq. (34) leads to elliptic integrals. When the magnetic field changes abruptly, the calculation can be carried out in closed form.

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