SUPERFLUIDITY IN A FERMI SYSTEM IN THE PRESENCE OF PAIRS WITH NONZERO ANGULAR MOMENTUM

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A picture is proposed for the superfluid state in a Fermi system when the Cooper pairs have a nonzero angular momentum. It is shown that if the system does not have a total angular momentum, the ground state will be isotropic. The Fermi excitation spectrum has its usual form with an isotropic gap.

In all the papers on the theory of superconductivity (cf., for example, references 1, 2, etc.) it is assumed that the Cooper pairs[3] are formed in an S state. But it is obvious that one can generalize the Cooper phenomenon in a system of Fermi particles to the case where the attraction between a pair of particles occurs in a state with nonzero relative angular momentum. In particular, Pitaevskii[4] has shown that for a pair of excitations in He^3 there is an attraction in the higher harmonics. It is of interest to study the characteristics of the superfluid state that develops in a Fermi system at sufficiently low temperatures because of this type of interaction. Such an attempt was made in a recent paper by Anderson and Morel,[5] but their results seem to us to be incorrect. In the following we shall investigate this question, using a generalization of the method[6] proposed by one of us in the theory of superconductivity.

Let the interaction between two particles have the following form

$$V(\mathbf{p}-\mathbf{p}') = \sum_l V_l P_l(\theta) \qquad (1)$$

in the momentum representation, i.e., in the neighborhood of the Fermi surface, the interaction depends on the angle between the vectors $\mathbf{p}$ and $\mathbf{p}'$ and is independent of their magnitudes. For simplicity, we assume[1,2] that the interaction is equal to zero* when $|v(p-p_0)|, |v(p'-p_0)| > \tilde{\omega}$ and restrict ourselves to the weak binding approximation.

Let us assume that the relation between the quantities V_l is such that the Cooper pairs which are formed have orbital angular momentum l and spin s.* As was done in reference 6, we then consider the value at absolute zero of the average of a product of four fermion operators $\langle N|\psi_1\psi_2\psi_3^+\psi_4^+|N\rangle$ over the exact ground state for a fixed number of particles. We shall assume that the total angular momentum is zero in the ground state. The operators $\psi_1\psi_2$ which annihilate, and $\psi_3^+\psi_4^+$ which create two particles, contain terms corresponding to the creation and annihilation of bound pairs. At absolute zero the pairs, which obey Bose statistics, are in a state of Bose "condensation," i.e. the number of pairs in states with their momentum equal to zero is comparable to the total number of particles in the system. Therefore the matrix element $\langle N+2|\psi^+\psi^+|N\rangle$ can be replaced by a c-number. In doing this we must remember that, because of the nonzero orbital angular momentum of the pair, the operator $\psi^+\psi^+$ can create a pair with any one of $(2l+1)$ values of the angular momentum projection, and consequently contains $2l+1$ components, corresponding to transitions

$$\langle N+2, l, m|\psi^+\psi^+|N, 0\rangle$$

from the ground state of the N particle system to the ground state of the $(N+2)$-particle system with angular momentum projection m along some direction. In the light of these remarks it is clear that the generalization of the method of reference 6 to the present case consists in introducing $2l+1$ functions $F^+_{m\alpha\beta}(x-x')$ and $F_{m\alpha\beta}(x-x')$:

$$F^+_{m\alpha\beta}(x-x') = \langle N+2, l, m|T(\psi^+_\alpha(x), \psi^+_\beta(x'))|N, 0\rangle,$$

$$F_{m\alpha\beta}(x-x') = \langle N, 0|T(\psi_\alpha(x), \psi_\beta(x'))|N+2, l, m\rangle$$

*In our case, $\tilde{\omega}$ is of the order of the Fermi energy.

*In accordance with the Pauli principle, even l is possible for s = 0, odd l for s = 1. We shall omit the spin in the following discussion, since it can be included in a trivial fashion in the final formula (12).

in place of the functions F and F^+ of reference 6. We shall therefore, in particular, assume that in the expansion of the average of a product of four ψ-operators, the term containing the product of the functions F and F^+ has the form

$$\langle T(\psi_\alpha(1)\psi_\beta(2)\psi_\gamma^+(3)\psi_\delta^+(4))\rangle \to \sum_m F_{m\alpha\beta}(x_1 - x_2)\, F^+_{m\gamma\delta}(x_3 - x_4)$$

As we shall see later, we thus obtain a consistent picture of the superfluid state of the system.

Repeating the derivation[6] of the equations for the Green's function $G_{\alpha\beta}(x - x') = -i\langle T(\psi_\alpha(x)\psi^+_\beta(x'))\rangle$ and for each of the $2l + 1$ quantities F^+_m, we get the following system of equations for the Fourier components of these functions:

$$(\omega - \xi)\,\hat{G}(p) - i\sum_m \hat{\Delta}_m(p)\,\hat{F}^+_m(p) = 1,$$

$$(\omega + \xi)\,\hat{F}^+_m(p) + i\hat{\Delta}^+_m(p)\,\hat{G}(p) = 0. \qquad (3)$$

Here $\xi = v(p - p_0)$, the symbols $\hat{G}$, etc, denote the matrix form of the spinor indices of the corresponding quantities, and the products $\hat{\Delta}\hat{F}^+$ and $\hat{\Delta}^+\hat{G}$ are the ordinary matrix product. The quantities $\hat{\Delta}_m$ and $\hat{\Delta}^+_m$ have the following meaning:

$$\hat{\Delta}_m(p) = (2\pi)^{-4}\int V(\mathbf{p} - \mathbf{p}')\,\hat{F}_m(p')\,d^3p'\,d\omega,$$

$$\hat{\Delta}^+_m(p) = (2\pi)^{-4}\int V(\mathbf{p} - \mathbf{p}')\,\hat{F}^+_m(p')\,d^3p'\,d\omega. \qquad (4)$$

Since the total angular momentum of the system is zero, $\hat{G}_{\alpha\beta}(p)$ is isotropic, and has the form

$$\hat{G}_{\alpha\beta}(p) = \delta_{\alpha\beta}G(p),$$

where $G(p)$ is independent of the direction of p. As for the matrices $\hat{F}_m$ and $\hat{F}^+_m$, from the commutation relations for Fermi operators we find for $t = t'$:

$$F_{m\alpha\beta}(\mathbf{r} - \mathbf{r}', 0) = -F_{m\beta\alpha}(\mathbf{r}' - \mathbf{r}, 0); \qquad (5)$$

$$F^+_{m\alpha\beta}(\mathbf{r} - \mathbf{r}', 0) = -F^+_{m\beta\alpha}(\mathbf{r}' - \mathbf{r}, 0),$$

$$\{F^+_{m\alpha\beta}(\mathbf{r} - \mathbf{r}', 0)\}^* = -F_{m\alpha\beta}(\mathbf{r} - \mathbf{r}', 0). \qquad (6)$$

According to (2), the functions $\hat{F}^+_m$ and $\hat{F}_m$ correspond to creation and annihilation of pairs of particles in a state with angular momentum l and projection m. It is therefore obvious that $\hat{F}_m(p)$ and $\hat{F}^+_m(p)$ have the structure of the corresponding spherical harmonic, i.e., they are proportional to

$$Y_{lm}(\theta, \varphi) = \left[\frac{2l + 1}{4\pi}\,\frac{(l - |m|)!}{(l + |m|)!}\right]^{1/2} P_l^m(\theta)\,e^{im\varphi},$$

where $P_l^m(\theta)$ are the associated Legendre polynomials. This assumption is justified later; on this basis we can write

$$\hat{F}^+_{m\alpha\beta}(p) = F^*_m(p)\,\hat{I}^s_{\alpha\beta}Y_{lm}(\theta, \varphi),$$

$$\hat{F}_{m\alpha\beta}(p) = -F_m(p)\,\hat{I}^s_{\alpha\beta}Y_{lm}(\theta, \varphi), \qquad (7)$$

where the sign is chosen in agreement with Eq. (6). The matrix $\hat{I}$ reflects the dependence of the wave function of the pair on the spin variables. It is obvious that when the angular momentum l in which the pairing occurs is even, it follows from (6) that $I_{\alpha\beta} = -I_{\beta\alpha}$, i.e., the particles are in a singlet state. In the opposite case, the spin of the pair is unity and $I^s_{\alpha\beta} = I^s_{\beta\alpha}$. In both cases we shall assume that $I^2_{\alpha\beta} = \delta_{\beta\alpha}$

Using the addition theorem for the Legendre polynomials

$$P_l(\gamma) = \sum_m \frac{(l - |m|)!}{(l + |m|)!}\,P_l^m(\theta)\,P_l^m(\theta')\,e^{im(\varphi - \varphi')}, \qquad (8)$$

we find from (1), (4), and (7),

$$\hat{\Delta}^+_{m\alpha\beta}(p) = \Delta^*_m\hat{I}^s_{\alpha\beta}Y_{lm}(\theta, \varphi), \quad \hat{\Delta}_{m\alpha\beta}(p) = -\Delta_m\hat{I}^s_{\alpha\beta}Y_{lm}(\theta, \varphi); \qquad (9)$$

$$\Delta^*_m = \left(\frac{V_l}{2l + 1}\,\frac{mp_0}{2\pi^2}\right)\int\frac{d\omega}{2\pi}\int_{-\tilde{\omega}}^{\tilde{\omega}} d\xi\,F^*_m(p, \omega) \qquad (10)$$

and similarly for Δ_m.

Substituting (7) and (9) in (3), we find

$$F^*_m(p) = -i\Delta^*_m G(p)/(\omega + \xi), \quad G(p) = \frac{\omega + \xi}{\omega^2 - \varepsilon_p^2},$$

$$F^*_m(p) = -i\,\frac{\Delta^*_m}{\omega^2 - \varepsilon_p^2}, \qquad (11)$$

where the spectrum $\epsilon_p = \sqrt{\xi^2 + |\Delta|^2}$ has a gap $|\Delta|$ equal to

$$|\Delta|^2 = \sum_m |\Delta_m|^2\,\frac{2l + 1}{2}\,\frac{(l - |m|)!}{(l + |m|)!}\,(P_l^m(\theta))^2.$$

Since $G(p)$ is isotropic, all the $|\Delta_m|^2$ are equal, and according to Eq. (8),

$$|\Delta|^2 = \tfrac{1}{2}|\Delta_m|^2(2l + 1)(2s + 1)P_l(\theta = 0).$$

Substitution of (11) in (10) gives for the value of the gap:

$$|\Delta| = 2\tilde{\omega}\exp\{-2\pi^2(2l + 1)/mp_0|V_l|\}. \qquad (12)$$

Thus if the pairs are formed in a state with angular momentum l, the gap size is determined only by the interaction component V_l. If an attraction of a pair of particles through the interaction (1) occurs for several harmonics, then as we see from (12), it follows from energy considerations that pairing of the particles will occur with the angular momentum for which the value of $|V_l|/(2l+1)$ is largest.

The Fermi energy spectrum is isotropic and has the usual form.[1,2] In addition, in a system of uncharged fermions in the superfluid state, there is an acoustic vibration branch. Therefore the specific heat of such a system at low temperatures is

$$C_s = \frac{2\sqrt{3}}{5}\frac{T^3}{v^3} + \frac{mp_0}{\pi^2}\sqrt{\frac{2\pi\Delta^3}{T^3}}\,\Delta e^{-\Delta/T},$$

The term in T^3, which comes from the acoustic branch, is important only at the very lowest temperatures, because the range of temperatures within which the superfluid state exists is exponentially small.

In reference 5, the spectrum

$$\varepsilon_p = [\xi^2 + |\Delta_m|^2 (P_l^m(\theta))^2]^{1/2}$$

was found. This is physically absurd since it gives an anisotropic spectrum in an isotropic system, to say nothing of the fact that it gives a zero gap at the points where $P_l^m(\theta) = 0$.

In conclusion we note that, within the framework of the isotropic model, when the interaction has the form (1), it is difficult to think of an experiment for determining the angular momentum value for which the pairing of particles in the superfluid phase occurs.

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