RELATIVISTICALLY COVARIANT RELATIONS BETWEEN POLARIZATION EFFECTS IN THE SCATTERING OF SPIN ½ PARTICLES

G. V. FROLOV

Radium Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor October 22, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 943-945 (March, 1960)

The relations between all quantities characterizing polarization effects in the scattering of different and identical particles of spin ½ are derived in a relativistically covariant form.

THE relations between polarization effects in the scattering of spin ½ particles were obtained in nonrelativistic form by Puzikov, Ryndin, and Smorodinskii[1] and Baz'.[2] It is of interest to obtain these relations for the relativistic case.

1. We consider the scattering of two different particles of momenta P and p, masses M and m, and polarization 4-vectors Z_μ and ξ_μ, respectively.

The state of polarization of a relativistic particle of spin ½ is described by the matrix of density[3]

$$\rho = \tfrac{1}{2}(1 + \sigma\xi)\,\eta^{(+)}.$$

Here $\sigma\xi = \sigma_\mu\xi_\mu$, $\sigma_\mu = i\gamma_5\gamma_\mu$; ξ_μ is the polarization 4-vector, and $\eta^{(+)}$ is the operator of the projection on a state of positive energy. The formulas for the different polarization effects can be written in the form

$$\sigma d/d\Omega \sim R = \mathrm{Sp}\,L,\quad Z'_\nu R = \mathrm{Sp}\,\{\Sigma_\nu L\},$$

$$\xi'^\alpha R = \mathrm{Sp}\,\{\sigma^\alpha L\},\quad Z'_\nu\xi'^\alpha R = \mathrm{Sp}\,\{\Sigma_\nu\sigma^\alpha L\}, \tag{1}$$

where

$$L = \Lambda'\lambda' A\Lambda(1 + \Sigma Z)\,\Lambda\lambda(1 + \sigma\xi)\,\lambda\bar{A}\lambda'\Lambda'.$$

Here A is determined by the expression for the matrix element

$$M_{if} = \bar{U}(P')\,\bar{u}(p')\,A\,u(p)\,U(P),$$

$$\bar{A} = \Gamma_4\gamma_4 A^+\gamma_4\Gamma_4,\quad \Lambda = M - i\hat{P},\quad \lambda = m - i\hat{p}.$$

The upper case letters and subscripts refer to the first particle, the lower case letters and superscripts refer to the second particle; the final state is indicated by the prime.

We rewrite formulas (1) in somewhat different form, so that the polarization coefficients are separated:

$$R = r + s_\mu Z_\mu + c^\beta\xi^\beta + d^\beta_\mu Z_\mu\xi^\beta,$$

$$Z'_\nu R = l_\nu + m_{\nu\mu} Z_\mu + n^\beta_\nu\xi^\beta + q^\beta_{\nu\mu} Z_\mu\xi^\beta,$$

$$\xi'^\alpha R = f^\alpha + g^\alpha_\mu Z_\mu + h^{\alpha\beta}\xi^\beta + t^{\alpha\beta}_\mu Z_\mu\xi^\beta,$$

$$Z'_\nu\xi'^\alpha R = u^\alpha_\nu + v^\alpha_{\nu\mu} Z_\mu + x^{\alpha\beta}_\nu\xi^\beta + y^{\alpha\beta}_{\nu\mu} Z_\mu\xi^\beta. \tag{2}$$

It is clear in which experiments these polarization coefficients are measured.

Since we have in mind interactions in which parity is conserved, we use, instead of the invariance of the scattering amplitude with respect to time reversal, the equivalent invariance with respect to charge conjugation:

$$A(p,\ p',\ P,\ P') = CcA^T(-p', -p, -P', -P)\,c^{-1}C^{-1}. \tag{3}$$

Here T denotes the transpose with respect to the spinor indices of both particles. Inserting (3) into the explicit expression for the polarization coefficients obtained from (1) and (2), we find the relations

$$l_\mu = \tilde{\tilde{s}}_\mu,\quad c^\beta = \tilde{\tilde{f}}^\beta,\quad d^\beta_\mu = \tilde{\tilde{u}}^\beta_\mu,$$

$$g^\beta_\mu = \tilde{\tilde{n}}^\beta_\mu,\quad q^\beta_{\nu\mu} = \tilde{\tilde{v}}^\beta_{\mu\nu},\quad t^{\alpha\beta}_\mu = \tilde{\tilde{x}}^{\beta\alpha}_\mu. \tag{4}$$

Moreover, we impose the following conditions on some of the coefficients:

$$m_{\mu\nu} = \tilde{\tilde{m}}_{\nu\mu},\quad h^{\alpha\beta} = \tilde{\tilde{h}}^{\beta\alpha},\quad y^{\alpha\beta}_{\nu\mu} = \tilde{\tilde{y}}^{\beta\alpha}_{\mu\nu}. \tag{5}$$

The sign ≃ over the letter denotes the substitution

$$p \rightleftarrows -p',\quad P \rightleftarrows -P'. \tag{6}$$

In particular, the result obtained earlier by Bilenkii and Ryndin[4] follows from formulas (4):

$$s_\mu = l_\mu,\quad c^\beta = f^\beta. \tag{7}$$

In fact, the most general form of the expression for each of these pseudovectors has the form $N_\mu F$, where

$$N_\mu = i\varepsilon_{\mu\nu\lambda\rho}(p + p')_\nu\,(P + P')_\lambda\,(p - p')_\rho,$$

and F is a function only of invariants. Since neither N_μ nor F change under the substitution (6), we obtain formula (7) from (4).

2. The interaction between particles can sometimes be of the type in which the scattering ampli-

tude is an invariant under charge conjugation with respect to only one of the particles, i.e.,

$$A\,(p,\ p',\ P,\ P') = \beta C A^{t}\,(p,\ p',\ -P',\ -P)\,C^{-1}. \quad (8)$$

Here $|\beta^2| = -1$, t denotes the transpose with respect to the spinor indices of the first particle. A similar situation is encountered, for example, when we consider the scattering of electrons by protons in the first Born approximation for the electromagnetic field,[5] where $\beta = -1$.*

In this case, if we insert (8) in the formulas for the polarization coefficients, we find the following relations:

$$d^{\beta}_{\mu} = \tilde{n}^{\beta}_{\mu}, \quad g^{\beta}_{\mu} = \tilde{u}^{\beta}_{\mu}. \quad (9)$$

Moreover, the demand (8) imposes a number of conditions on the coefficients:

$$r = \tilde{r}, \quad l_{\mu} = \bar{l}_{\mu}, \quad s_{\mu} = \bar{s}_{\mu}, \quad c^{\beta} = \tilde{c}^{\beta},$$
$$m_{\nu\mu} = \bar{m}_{\nu\mu}, \quad q^{\beta}_{\nu\mu} = \tilde{q}^{\beta}_{\mu\nu}, \quad f^{\beta} = \tilde{f}^{\beta},$$
$$h^{\alpha\beta} = \tilde{h}^{\alpha\beta}, \quad x^{\alpha\beta}_{\mu} = \bar{x}^{\beta\alpha}_{\mu}, \quad t^{\alpha\beta}_{\mu} = \bar{t}^{\beta\alpha}_{\mu}, \quad v^{\beta}_{\mu\nu} = \tilde{v}^{\beta}_{\nu\mu}, \quad y^{\alpha\beta}_{\nu\mu} = \tilde{y}^{\alpha\beta}_{\mu\nu}. \quad (10)$$

Here the bar above the letter denotes the substitution p ⇄ − p′; the tilde denotes the substitution P ⇄ − P′.

Relations (9), however, bear a character different from (4). Since the invariants change when we make the substitution P ⇄ − P′, then we can find, say n^{β}_{μ}, only if the analytic expression for d^{β}_{μ} is known.

We note that the expression for the polarization effects in the scattering of electrons by protons[5,7] satisfies the demands (4), (5), (9), and (10).

3. If the particles are identical, then the scattering amplitude possesses definite symmetry properties, namely:

$$A^{\alpha\beta}_{\mu\nu}\,(p,\ p',\ P,\ P') = -A^{\mu\beta}_{\alpha\nu}\,(p,\ P',\ P,\ p'),$$
$$A^{\alpha\beta}_{\mu\nu}\,(p,\ p',\ P,\ P') = A^{\mu\nu}_{\alpha\beta}\,(P,\ P',\ p,\ p').$$

This leads to the following auxiliary relations between the coefficients:

$$s_{\mu} = \underset{\approx}{c}^{\mu}, \quad l_{\mu} = \underset{\approx}{f}^{\mu}, \quad m_{\nu\mu} = \underset{\mp}{h}^{\nu\mu},$$
$$n^{\beta}_{\mu} = \underset{\approx}{g}^{\mu}_{\beta}, \quad q^{\beta}_{\nu\mu} = \underset{\approx}{t}^{\nu\mu}_{\beta}, \quad v^{\beta}_{\nu\mu} = \underset{\approx}{x}^{\nu\mu}_{\beta}; \quad (11)$$

$$m_{\nu\mu} = \underset{\sim}{g}^{\nu}_{\mu}, \quad n^{\beta}_{\mu} = \underset{\sim}{h}^{\mu\beta}. \quad (12)$$

Here ≈ under the letter denotes the substitution P ⇄ p and P′ ⇄ p′, and ~ denotes the substitution P′ ⇄ p′.

The following conditions are also imposed on the coefficients:

$$r = \underset{\sim}{r} = \underset{\approx}{r}, \quad d^{\beta}_{\mu} = \underset{\sim}{d}^{\beta}_{\mu} = \underset{\approx}{d}^{\mu}_{\beta}, \quad s_{\mu} = \underset{\sim}{s}_{\mu},$$
$$u^{\beta}_{\mu} = \underline{u}^{\beta}_{\mu} = \underset{\approx}{u}^{\mu}_{\beta}, \quad y^{\alpha\beta}_{\nu\mu} = \underset{\sim}{y}^{\nu\beta}_{\alpha\mu} = \underset{\approx}{y}^{\nu\mu}_{\alpha\beta},$$
$$c^{\beta} = \underset{\sim}{c}^{\beta}, \quad f^{\beta} = \underline{f}^{\beta}, \quad l_{\mu} = \underline{l}_{\mu}, \quad q^{\beta}_{\nu\mu} = \underline{q}^{\mu}_{\nu\beta},$$
$$t^{\alpha\beta}_{\mu} = \underline{t}^{\alpha\mu}_{\beta}, \quad v^{\beta}_{\nu\mu} = \underset{\sim}{v}^{\nu}_{\beta\mu}, \quad x^{\alpha\beta}_{\mu} = \underset{\sim}{x}^{\mu\beta}_{\alpha}. \quad (13)$$

Here the bar under the letter denotes the substitution P ⇄ p.

Some of the relations (11) are simply equalities:

$$s_{\mu} = c^{\mu}, \quad l_{\mu} = f^{\mu}.$$

The reasoning is the same as in the derivation of equalities (7).

What was said above concerning relations (9) applies to relations (12), since the invariants change when we make the substitution P′ ⇄ p′.

*Edwards and Matthews[6] call such an invariance property for $\beta = 1$ crossing symmetry for fermions.

[1] Puzikov, Ryndin, and Smorodinskii, JETP **32**, 592 (1957), Soviet Phys. JETP **5**, 489 (1957).

[2] A. I. Baz', JETP **32**, 628 (1957), Soviet Phys. JETP **5**, 521 (1957).

[3] L. Michel and A. Wightman, Phys. Rev. **98**, 1190 (1955).

[4] S. M. Bilen'kii and R. M. Ryndin, JETP **36**, 1609 (1959), Soviet Phys. JETP **9**, 1144 (1959).

[5] Akhiezer, Rozentsveĭg, and Shmushkevich, JETP **33**, 765 (1957), Soviet Phys. JETP **6**, 588 (1958).

[6] S. F. Edwards and P. T. Matthews, Phil. Mag. **2**, 839 (1957).

[7] G. V. Frolov, JETP **34**, 764 (1958), Soviet Phys. JETP **7**, 525 (1958).

Translated by E. Marquit
150