DEPENDENCE OF THE ANGULAR DISTRIBUTION OF FISSION FRAGMENTS ON THE SPIN OF THE TARGET NUCLEUS

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A simple analytic expression for the angular distribution of fission fragments is derived in the case of small anisotropy. The effect of the initial nuclear spin is discussed.

IN reference 1, a method was suggested for the calculation of the angular distribution of fission fragments and analytic expressions for the angular distribution were obtained for a target nucleus of a given spin. In the case of a Gaussian distribution of the probability of fission with the value of the projection K of the angular momentum J of the compound nucleus in the direction of fission

$$\alpha(K) = \exp(-K^2/2K_0^2) \tag{1}$$

the angular distribution of the fragments depends weakly on the initial spin; the spin-dependent terms in the angular distribution are of the order $J_0^2 l^2/K_0^4$ (where J_0 is the initial spin, l is the orbital angular momentum of the neutron), while the basic term dependent on the angle is of the order l^2/K_0^2. The dependence of the total probability of decay of the compound nucleus Γ_t on the value of the angular momentum of the nucleus also leads to corrections of the second order in the angular distribution. This effect also leads to an additional weak dependence of the anisotropy on the initial spin. In the case of a small anisotropy in the angular distribution and a weak dependence of Γ_t on J, one can obtain an analytic expression for the angular distribution in which the dependence of Γ_t on J is taken into account.

The overall width of the compound nucleus state is the sum of the radiation, neutron, and fission widths. The first is usually negligibly small, while the dependence of Γ_n and Γ_f on the angular momentum can be determined with the aid of statistical theory:

$$\Gamma_n(J) \approx \Gamma_n(0)\exp[-(\alpha_f-\alpha_i)J(J+1)], \tag{2}$$

$$\Gamma_f(J) \approx (2J+1)^{-1}\Gamma_f(0)\exp[(\alpha_i-\alpha_\perp^*)J(J+1)] \times \sum_{K=-J}^{J}\exp[-K^2/2K_0^2]. \tag{3}$$

In (2) and (3), $\alpha = \hbar^2/2\mathcal{J}T$, where $\mathcal{J}$ is the moment of inertia of the nucleus and T is the nuclear temperature. The subscript i indicates quantities referring to the initial compound nucleus and f, to the compound nucleus state after the emission of the neutron; the asterisk indicates quantities referring to the deformed "transition" nucleus. The constant $\mathcal{J}_\perp$ appearing in the expression $\alpha_\perp^* = \hbar^2/2\mathcal{J}_\perp^* T^*$ is the moment of inertia of the transition nucleus with respect to the axis of symmetry. In the remaining cases, the moment of inertia is equal to the moment of inertia of a spherical nucleus $\mathcal{J}_0 = 2AmR^2/5$.

Expanding $\Gamma_n(J)$ and $\Gamma_f(J)$ into a series, we obtain

$$\gamma_f(J) = \Gamma_f(J)/\Gamma_t(J) \approx \gamma_f^0[1+qJ(J+1)+\ldots], \tag{4}$$

where

$$q = \gamma_n^{(0)}(\alpha_f-\alpha_\perp^*)+\gamma_f^{(0)}(1/2K_0^2), \quad \gamma_n^{(0)} = \Gamma_n(0)/\Gamma_f(0),$$

$$\gamma_f^{(0)} = \Gamma_f(0)/\Gamma_t(0).$$

This expression for $\gamma_f(J)$ should be inserted into the general expression for the angular distribution of the fragments, which, for fixed values of l, of the channel spin $S = J_0 \pm 1/2$, and of J has the form[1]

$$W_{lSJ}(\vartheta) = \frac{2J+1}{2(2J_0+1)}\sum_{m\mu K}(C_{S\mu lm}^{JK})^2 a_J(K)|Y_{lm}(\vartheta)|^2, \quad a_J(K) = \gamma_f(J)\alpha(K). \tag{5}$$

The overall angular distribution of the fragments is given by the expression

$$\overline{W}(\vartheta) = \sum_{lSJ}\zeta_l W_{lSJ}(\vartheta) \Big/ \sum_{lSJ}(2l+1)\zeta_l, \tag{6}$$

where ζ_l is the neutron sticking coefficient, which is assumed to be independent of J and S. Inserting Eqs. (5) and (4) into (6), we sum over J, for which we use the following formulas for the summation of the Clebsch-Gordan coefficients:

$$\sum_{J=|l-S|}^{l+S}(C_{S\mu lm}^{JK})^2 = \begin{cases}1, & |K-m| \leqslant S\\ 0, & |K-m| > S\end{cases}$$

and[1]

$$\sum_{J=|l-S|}^{l+S} J(J+1)(C^{JK}_{S\mu l m})^2 = \begin{cases} l(l+1)+S(S+1)+2m\mu, & |\mu|\leqslant S, \quad |m|\leqslant l. \\ 0, & |\mu|>S, \quad |m|>l \end{cases}$$

After summing over J, we obtain

$$W_{lS}(\vartheta)=\sum_J W_{lSJ}(\vartheta) = C\sum_{m=-l}^{l}|Y_{lm}(\vartheta)|^2 \times \sum_{K=m-S}^{m+S}\{1+q[l(l+1)+S(S+1)+2m(K-m)]\}\,\alpha(K), \qquad (7)$$

where the constant C does not depend on l and S. For $\alpha(K)$, we use the expansion

$$\alpha(K)\approx 1-K^2(2K_0^2)^{-1}+\tfrac{1}{2}\eta K^4(2K_0^2)^{-2}+\ldots. \qquad (8)$$

An arbitrary coefficient η is introduced into formula (8) in order to take into account the possibility that the distribution $\alpha(K)$ deviates from a Gaussian one at large values of K. The term with K^4 leads to corrections of the second order. The correction to the angular distribution, as a result of the dependence of γ_f on J is small — of the order $q/2K_0^2$ —, i.e., it is a small quantity of the second order [q and $(2K_0^2)^{-1}$ are of the same order]. Taking this into account, we obtain from (6), (7), and (8), after simple calculations:

$$A(\vartheta)=[\sigma_f(\vartheta)-\sigma_f(90°)]/\sigma_f(90°)=(\overline{l^2}/4K_0^2)\{[1+(\overline{S^2}/6K_0^2)+q(\overline{l^4}/\overline{l^2}-\overline{l^2}+\tfrac{4}{3}\overline{S^2})]\cos^2\vartheta+(\eta/2K_0^2)[\tfrac{3}{8}\overline{l^{4'}}(1-\sin^4\vartheta)+\overline{l^2}(\overline{S^2}+{}^1/_2)\cos^2\vartheta]\}, \qquad (9)$$

where

$$\overline{l^2}=\sum_l(2l+1)\zeta_l l(l+1)\Big/\sum_l(2l+1)\zeta_l \qquad (10)$$

is the mean square angular momentum transferred to the nucleus by the neutron; $\overline{l^4}$ and $\overline{l^{4'}}$ are the analogous mean values of the quantities $l^2(l+1)^2$ and $l(l^2-1)(l+2)$, and $\overline{S^2}=(J_0+\tfrac{1}{2})^2+\tfrac{1}{2}$. In (9), we have discarded third-order terms. In the first order in $\overline{l^2}/2K_0^2$, the angular distribution does not depend on the spin of the target nucleus. The change in the anisotropy of the angular distribution, when the quantity $\overline{S^2}$ is changed to $\Delta\overline{S^2}$ is

$$\Delta_S A(0°)=\tfrac{1}{4}\overline{l^2}K_0^{-2}\{(1-3\eta)/6K_0^2+\tfrac{4}{3}q\}\Delta\overline{S^2}. \qquad (11)$$

According to the experimental data,[2] there is a small systematic difference in the value of the anisotropy for the nuclei U^{235}, U^{233}, and Pu^{239}:

$$A_{U^{235}}(0°)-A_{U^{233}}(0°)\approx A_{U^{233}}(0°)-A_{Pu^{239}}(0°)\approx 0.02-0.03.$$

It is natural to interpret this as an effect of the initial spin. For the three nuclei, the values of $\Delta\overline{S^2}$ are equal to 7 and 8 units. For a theoretical estimate of $\Delta_S A(0^0)$, we set $T_f = T^* = T$ in (4). The quantity $1/\mathcal{J}_f - 1/\mathcal{J}_\perp$ is equal to $\mathcal{J}_0$ $(1.2z + 5.6z^2)$, where $\mathcal{J}_0$ is the moment of inertia for a spherical nucleus, and $z = 1-(Z^2A)/(Z^2/A)_{cr}$.[3] For T = 0.3 Mev, $\gamma_n^{(0)}\approx\gamma_f^{(0)}\approx 0.5$, and a value of $\mathcal{J}_0$ corresponding to a rigid body, we obtain q ≈ 0.01. The parameter $\overline{l^2}/4K_0^2$ is determined directly from the experimental value of the anisotropy, since in a first approximation it is simply the same value. For neutrons of energy 3 — 5 Mev, we have $\overline{l^2}/4K_0^2\approx 0.12$, $K_0^2 = 12-15$.[2,4] From (10) we find that the experimental value of the difference $\Delta A(0°)$ for $\eta = 1$ [i.e. for a Gaussian $\alpha(K)$] would correspond to q four to five times the thermodynamical value, which is unlikely. Another possible effect is a cutoff in the distribution $\alpha(K)$, in comparison with the Gaussian one, for large K, which is natural to expect in view of the finite size of the nucleus (see also ref. 1). The coefficient η should then be less than unity or negative. Then $\eta\approx 0$ would correspond to the experimental value of $\Delta A(0°)$ for the above-mentioned thermodynamical value of q.

A rough estimate of the highest possible value K_{max} can be obtained by equating the rotational energy of the nucleus

$$E_{rot}\approx\hbar^2K^2/2J_{eff}=(K^2/2K_0^2)\,T^*$$

and the excitation energy of the transition nucleus E*. In this way, we obtain

$$K_{max}\approx\{(E^*/T^*)\cdot 2K_0^2\}^{1/2}.$$

For $E^*\approx 2-3$ Mev, we have $K_{max}\approx nK$, where the factor n is several units. This value of K_{max} is in agreement with $\eta\approx 0$, since for $\eta\approx 0$ the distribution $\alpha(K)$ vanishes when $K\approx\sqrt{2K_0^2}$.

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