EMISSION OF LOW ENERGY γ QUANTA BY ELECTRONS SCATTERED ON PROTONS

S. M. BILEN'KII and R. M. RYNDIN

Joint Institute for Nuclear Research

Submitted to JETP editor September 1, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 819-825 (March, 1961)

The emission of low energy γ quanta in ep scattering is considered. It is shown that the first terms of the expansion of the amplitude in powers of the photon energy can be expressed through the electromagnetic form factors of the proton. The differential cross section for the process is derived in this approximation.

1. INTRODUCTION

THE study of elastic scattering of high-energy electrons by protons and nuclei is, at present, a basic source of information on the electromagnetic structure of nucleons. The experiments performed have made it possible to draw important conclusions on the behavior of the electromagnetic form factors of nucleons $F_1^{p;n}$ and $F_2^{p;n}$ as functions of the square of the four-momentum transfer. Great interest attaches also to other processes, the study of which will make it possible to obtain additional information on the form factors F_1 and F_2.[1,2] In the present article, we consider the emission of low energy bremsstrahlung in ep scattering. It will be shown that the first two terms of the expansion of the differential cross section for this process in powers of the photon energy are expressed through the electromagnetic form factors of the proton and their derivatives with respect to the momentum transfer.

To obtain the amplitude of the process, we employ the general method of considering the radiation of low-energy γ quanta suggested by Low.[3,4] Following this method, we first consider that part of the amplitude of the process which has a pole at $k = 0$ (k is the photon momentum). It is then shown that the product of the exact renormalized vertex operator corresponding to the radiation of a real γ quantum and the exact renormalized propagation function of the nucleon is expressed, according to Ward's identity, to an accuracy of terms of the order $1/k$ and constants, through the charge, mass, and anomalous magnetic moment of a physical nucleon. On the basis of gauge invariance, we then find the part of the amplitude not containing the pole. In the concluding part of the article, we derive an expression for the differential cross section of the process.

2. AMPLITUDE OF THE PROCESS

The process of the radiation of γ quanta in the scattering of electrons by protons in the lowest-order approximation in e, but with allowance for strong interactions of the nucleon, is described by the diagrams of Figs. 1 and 2. In the figures, q and q′ denote the electron 4-momenta before and after the collision; p and p′ denote the proton momenta, and k is the γ-quantum momentum. It is obvious that the nucleon vertex part of Figs. 1a and 1b is determined by the form factors describing the elastic scattering of electrons by protons with a corresponding momentum transfer.

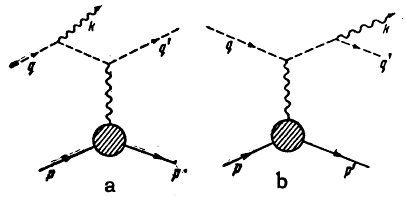

Fig. 1

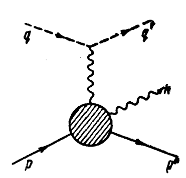

Fig. 2

We write the S matrix in the form

$$S(p', q', k; p, q) = -(2\pi)^4 i \left(M^2 m^2/2k_0 p_0' p_0 q_0' q_0\right)^{1/2} \varepsilon_\mu \times (T_\mu^{\mathrm{I}} + T_\mu^{\mathrm{II}})\, \delta(p' + q' + k - p - q), \quad (1)$$

where M and m are the masses of the proton and electron, ϵ_μ is the polarization vector of the photon, $p_0 = -ip_4$, $q_0 = -iq_4$, etc, and T_μ^{I} and T_μ^{II} represent the respective contributions of the diagrams of Figs. 1a, b and 2.

For T_μ^{I} we have

$$T_\mu^{\mathrm{I}} = \bar{u}(q')\left[ie\gamma_\mu \frac{1}{i\gamma(q'+k)+m} ie\gamma_\nu + ie\gamma_\nu \frac{1}{i\gamma(q-k)+m} ie\gamma_\mu\right] u(q)\ \bar{v}(p')\, ie\left[F_1((p'-p)^2)\gamma_\nu - \tfrac{1}{2}\mu_p M^{-1} F_2((p'-p)^2)\,\sigma_{\nu\rho}(p'-p)_\rho\right] v(p)\,(p'-p)^{-2}. \quad (2)$$

Here μ_p, F_1, and F_2 are the anomalous moment (in nuclear magnetons) and the electromagnetic form factors of the proton; $\sigma_{\mu\rho} = (1/2i)(\gamma_\mu\gamma_\rho - \gamma_\rho\gamma_\mu)$. The spinors u and v are normalized by the conditions $\bar{u}u = 1$, $\bar{v}v = 1$.

We shall now consider diagrams of the type in Fig. 2. We separate these diagrams into two classes A and B.[5] In class A we include all diagrams in which the vertex with the emission of a real quantum is connected with the remaining part of the diagram by a nucleon line. In class B we include all other diagrams. We write T_μ^{II} in the following way:

$$T_\mu^{II} = -\bar{u}(q')\, ie\gamma_\nu u(q)\,[T_{\nu\mu}^A + T_{\nu\mu}^B]\,(q'-q)^{-2}. \quad (3)$$

$T_{\nu\mu}^A$ and $T_{\nu\mu}^B$ describe the contribution of diagrams of classes A and B. For $T_{\nu\mu}^A$ we have the following expression:

$$T_{\nu\mu}^A = \bar{v}(p')\,[ie\Gamma_\mu(p', p'+k)\,S(p'+k)\,ie\Gamma_\nu(p'+k, p) + ie\Gamma_\nu(p', p-k)\,s(p-k)\,ie\Gamma_\mu(p-k, p)]\,v(p). \quad (4)$$

Here S and Γ are the exact renormalized propagation function and electromagnetic vertex operator of the proton.

We are interested in the radiation of low-energy γ quanta. We therefore expand the operators occurring in (4) in powers of k and limit ourselves to the first two terms of the expansion. We consider first $S(p-k)\Gamma_\mu(p-k, p)v(p)$. The general expression for the propagation function can be written in the form

$$S(t) = [i\gamma t G(t^2) + MF(t^2)]^{-1}. \quad (5)$$

The values of the functions $G(t^2)$ and $F(t^2)$ and their derivatives $F'(t^2)$ and $G'(t^2)$ at $t^2 = -M^2$ are, as is known,[5] related in the following way:

$$F = G, \qquad F + 2M^2(F' - G') = 1. \quad (6)$$

Expanding $S(p-k)$ in a series in powers of k, using (6), and retaining the first two terms of the expansion, we obtain

$$\begin{aligned} S(p-k) = &-(2pkF)^{-1}\{(-i\gamma p + M)F \\ &+ i\gamma kF - 2pk(-i\gamma pG' + MF') \\ &+ 2pkM^2F(-i\gamma p + M)\,[2G'/M^2 \\ &+ F'' - G'' + (F'^2 - G'^2)/F]\}. \end{aligned} \quad (7)$$

The operator $\Gamma_\mu(t', t)$ has the transformation properties of a 4-vector and, under charge conjugation, transforms like a current vector:

$$C^{-1}\Gamma_\mu(t', t)\,C = -\Gamma_\mu^T(-t, -t'). \quad (8)$$

We thus obtain the expansion of Γ_μ in a series in powers of k:[4,5]

$$\begin{aligned} \Gamma_\mu(p-k, p) = \Gamma_\mu(p, p) &- \tfrac{1}{2}k_\rho\partial\Gamma_\mu(p, p)/\partial p_\rho + pkM^{-3}G_1(p^2)\sigma_{\mu\rho}p_\rho \\ &+ \tfrac{1}{2}M^{-1}G_2(p^2)\,\sigma_{\mu\rho}k_\rho + G_3(p^2)\,M^{-3}\sigma_{\lambda\rho}k_\lambda p_\rho p_\mu \\ &+ \tfrac{1}{2}G_4(p^2)\,M^{-2}\varepsilon_{\mu\lambda\rho\delta}p_\lambda k_\rho\gamma_\delta\gamma_5. \end{aligned} \quad (9)$$

The first two terms of this expansion are expressed through the functions F and G and their derivatives by means of Ward's identity:

$$\Gamma_\mu(p, p) = -i\partial S^{-1}(p)/\partial p_\mu. \quad (10)$$

The terms in (9) containing G_1 and G_3 do not make any contribution to $S(p-k)\Gamma_\mu(p-k, p)v(p)$ in the required order, since $(-i\gamma p + M)\sigma_{\mu\rho}p_\rho v(p) = 0$. Using also the relation

$$(-i\gamma p + M)\,\varepsilon_{\mu\lambda\rho\delta}p_\lambda k_\rho\gamma_\delta\gamma_5 v(p) = (-i\gamma p + M)(-M)\,\sigma_{\mu\rho}k_\rho v(p) \quad (11)$$

and discarding, by the Lorentz condition, terms proportional to k_μ, we obtain, to an accuracy of terms of the order 1/k and constants, the following expression:

$$\begin{aligned} S(p-k)\,\Gamma_\mu(p-k, p)\,v(p) = &-(2pk)^{-1}[-2ip_\mu + (-i\gamma p + M)\,\sigma_{\mu\rho}k_\rho \\ &\times(F - 1 + G_2 - G_4)/2M + i(\gamma k)\gamma_\mu]\,v(p). \end{aligned} \quad (12)$$

If we consider the scattering of a proton by an external constant magnetic field, then it can be shown, by means of (9), that the quantity $F - 1 + G_2 - G_4$ is equal to the anomalous magnetic moment of the proton μ_p.[5] It is obvious that expression (12) can also be written in the following form:

$$S(p-k)\,ie\Gamma_\mu(p-k, p)\,v(p) = \frac{1}{i\gamma(p-k) + M}\left(ie\gamma_\mu + \mu_p\frac{ie}{2M}\sigma_{\mu\rho}k_\rho\right)v(p). \quad (13)$$

Hence $S(p-k)ie\Gamma_\mu(p-k, p)v(p)$ is expressed through the charge, magnetic moment, and mass of a physical proton if we consider only terms of the order 1/k and constants.

We note that the exact expression (13) obtained by us is of the same form as the corresponding expression obtained in the lowest-order approximation of perturbation theory for a point proton with a Pauli anomalous magnetic moment. Similarly, it can be shown that, in the approximation we are considering,

$$\bar{v}(p')\,ie\Gamma_\mu(p', p'+k)\,S(p'+k) = \bar{v}(p')\left(ie\gamma_\mu + \mu_p\frac{ie}{2M}\sigma_{\mu\rho}k_\rho\right)\frac{1}{i\gamma(p'+k) + M}. \quad (14)$$

For what follows, it will be convenient to introduce the invariants

$$M_1^2 = -(p-k)^2 = M^2 + 2pk,$$
$$M_2^2 = -(p'+k)^2 = M^2 - 2p'k. \qquad (15)$$

With an accuracy to terms of the order 1/k and constants, we have the relations

$$(-i\gamma(p-k) + M)/2pk$$
$$= (-i\gamma(p-k) + M_1)/2pk - 1/2M,$$
$$(-i\gamma(p'+k) + M)/2p'k$$
$$= (-i\gamma(p'+k) + M_2)/2p'k + 1/2M. \qquad (16)$$

Noting that

$$\gamma(p-k)[-i\gamma(p-k)$$
$$+ M_1] = iM_1[-i\gamma(p-k) + M_1],$$
$$[-i\gamma(p'+k) + M_2]\gamma(p'+k)$$
$$= iM_2[-i\gamma(p'+k) + M_2], \qquad (17)$$

we obtain the following expression for $T^A_{\nu\mu}$:

$$T^A_{\nu\mu} = \bar{v}(p')\Big[ie\Gamma^0_\nu(p', p-k)\frac{1}{i\gamma(p-k)+M}\Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}k_\rho\Big)$$
$$+ \Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}k_\rho\Big)\frac{1}{i\gamma(p'+k)+M}\,ie\Gamma_\nu(p'+k, p)\Big]v(p)$$
$$+ \bar{v}(p')\{[ie\Gamma_\nu(p', p-k)$$
$$- ie\Gamma^0_\nu(p', p-k)]\,ie\gamma_\mu/2M\}v(p)$$
$$+ \bar{v}(p')\{(ie\gamma_\mu/2M)[ie\Gamma_\nu(p'+k, p)$$
$$- ie\Gamma^0_\nu(p'+k, p)]\}v(p), \qquad (18)$$

where

$$\Gamma^0_\nu(t', t) = a\gamma_\nu + b\sigma_{\nu\rho}(t'-t)_\rho + c\sigma_{\nu\rho}(t'+t)_\rho. \qquad (19)$$

This operator is obtained from the general expression for the vertex part $\Gamma_\nu(t', t)$ by putting $i\sqrt{-t^2}$ in place of the operator γt on the right and $i\sqrt{-t'^2}$ in place of the operator $\gamma t'$ on the left. In formula (19), the quantities a, b, and c are functions of $-t^2$, $-t'^2$, and $(t'-t)^2$, where a and b do not change when t is replaced by t′, while c changes sign. We note that, for $t^2 = t'^2 = -M^2$, the functions a and b are equal to $F_1[(t'-t)^2]$ and $-(\mu_p/2M)F_2[(t'-t)^2]$, and the function c vanishes.

We now consider the contribution to the amplitude from diagrams of class B. The contribution of $T^B_{\nu\mu}$ of these diagrams, as $k \to 0$, tends to a constant, while the limit does not depend on the way in which k tends to zero.[3] The last two terms in (18) also have this property. We denote the sum of these terms and $T^B_{\nu\mu}$ by $M^{(2)}_{\nu\mu}$; by $M^{(1)}_{\nu\mu}$ we understand the sum of the first two terms:

$$M^{(1)}_{\nu\mu} = \bar{v}(p')\Big[ie\Gamma^0_\nu(p', p-k)$$
$$\times\frac{1}{i\gamma(p-k)+M}\Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\rho\mu}k_\rho\Big)$$
$$+ \Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}k_\rho\Big)$$
$$\times\frac{1}{i\gamma(p'+k)+M}\,ie\Gamma^0_\nu(p'+k, p)\Big]v(p). \qquad (20)$$

Owing to guage invariance,

$$k_\mu(T^{\mathrm{I}}_\mu + T^{\mathrm{II}}_\mu) = 0. \qquad (21)$$

It is readily seen from formula (2) that $k_\mu T^{\mathrm{I}}_\mu = 0$. Hence it follows from (20) and (21) that

$$k_\mu M^{(2)}_{\nu\mu} = -k_\mu M^{(1)}_{\nu\mu} = e\bar{v}(p')[ie\Gamma^0_\nu(p', p-k)$$
$$- ie\Gamma^0_\nu(p'+k, p)]v(p). \qquad (22)$$

By the law of conservation of energy-momentum $(p'-p+k = q-q')$, the operators Γ^0_ν occurring in (20) and (22) can be written in the form $(\kappa = q-q')$

$$\Gamma^0_\nu(p', p-k) = a(M^2, M^2+2pk, \varkappa^2)\gamma_\nu$$
$$+ b(M^2, M^2+2pk, \varkappa^2)\sigma_{\nu\rho}\varkappa_\rho$$
$$+ c(M^2, M^2+2pk, \varkappa^2)\sigma_{\nu\rho}(p'+p-k)_\rho,$$
$$\Gamma^0_\nu(p'+k, p) = a(M^2-2p'k, M^2, \varkappa^2)\gamma_\nu$$
$$+ b(M^2-2p'k, M^2, \varkappa^2)\sigma_{\nu\rho}\varkappa_\rho$$
$$+ c(M^2-2p'k, M^2, \varkappa^2)\sigma_{\nu\rho}(p'+p+k)_\rho. \qquad (23)$$

The absence of a pole in $M^{(2)}_{\nu\mu}$ as $k \to 0$ allows one to use (22) for a single-valued determination of $M^{(2)}_{\nu\mu}$ accurate to constant terms. Indeed, expanding $M^{(2)}_{\nu\mu}$ in a series of k and retaining the first term of the expansion, we obtain

$$M^{(2)}_{\nu\mu} = -e\bar{v}(p')[ie\partial\Gamma^0_\nu(p', p-k)/\partial k_\mu$$
$$+ ie\partial\Gamma^0_\nu(p'+k, p)/\partial k_\mu]|_{k=0}v(p). \qquad (24)$$

Retaining the corresponding terms in the expansion of $M^{(1)}_{\nu\mu}$ in k, we find, by means of relation (23) that in the sum $M^{(1)}_{\nu\mu} + M^{(2)}_{\nu\mu}$ the derivatives with respect to the masses drop out, and we finally obtain the following expression for T^{II}_μ:

$$T^{\mathrm{II}}_\mu = -\varkappa^{-2}\bar{u}(q')\,ie\gamma_\nu u(q)\,\bar{v}(p')\Big\{\Big[ieF_1(\varkappa^2)\gamma_\nu$$
$$- \frac{ie}{2M}\mu_pF_2(\varkappa^2)\sigma_{\nu\rho}\varkappa_\rho\Big]\frac{1}{i\gamma(p-k)+M}\Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}k_\rho\Big)$$
$$+ \Big(ie\gamma_\mu + \frac{ie}{2M}\mu_p\sigma_{\mu\rho}k_\rho\Big)\frac{1}{i\gamma(p'+k)+M}$$
$$\times\Big[ieF_1(\varkappa^2)\gamma_\nu - \frac{ie}{2M}\mu_p\sigma_{\nu\rho}\varkappa_\rho F_2(\varkappa^2)\Big]\Big\}v(p). \qquad (25)$$

In obtaining this formula, we used the above-mentioned properties of the functions a, b, and c. Hence the amplitude for the process of radiation of low energy γ quanta in ep scattering is entirely expressed through the electromagnetic form factors of the proton with an accuracy of terms of the order $1/k$ and constants. We note that formula (25) agrees with the expression obtained by the dispersion method in the one-nucleon approximation.[6]

CONCLUSION

In conclusion, we derive an expression for the differential cross section for the emission of low energy bremsstrahlung by means of the amplitude we have found [formulas (1), (2), and (25)]. As the independent variables, we choose the energy of the incident electron q_0, the energy of the γ quantum ω, and the following three angles: the angle θ between the directions of the momenta $\mathbf{k}$ and $\mathbf{q}$, the angle θ' between $\mathbf{k}$ and $\mathbf{q}'$, and the angle φ between the directions of the normals to the planes $(\mathbf{k}, \mathbf{q})$ and $(\mathbf{k}, \mathbf{q}')$ (laboratory system). All the remaining variables should be expressed in terms of the independent ones; the cross section should then be expanded in a series in ω.

Since the obtained amplitude is valid only to an accuracy of terms of the order $1/\omega$ and constants, we should retain only the first two terms in this expansion.

Hence

$$d\sigma = d\sigma_0/\omega + d\sigma_1. \tag{26}$$

We obtain the following expression for $d\sigma_0$:

$$d\sigma_0 = \omega d\sigma\Big|_{\omega=0} = \alpha\,(2\pi)^{-2}\omega^2\left[\left(\frac{q'}{q'k} - \frac{q}{qk}\right) - \left(\frac{p'}{p'k} - \frac{p}{pk}\right)\right]^2\Big|_{\omega=0} d\Omega_k d\omega d\sigma_p. \tag{27}$$

Here $d\sigma_p$ is the elastic scattering cross section for electrons of energy q_0 on protons, and $\alpha = e^2/4\pi = 1/137$. The factor in front of $d\sigma_p$ in formula (27) represents the probability of the radiation of a photon when electrons are scattered and goes over, for nonrelativistic particles, into the general expression for the probability of dipole radiation:*

$$\frac{\alpha}{(2\pi)^2}\left\{\frac{1}{m}[\mathbf{q}' - \mathbf{q}, \mathbf{n}] - \frac{1}{M}[\mathbf{p}' - \mathbf{p}, \mathbf{n}]\right\}^2 \frac{d\omega}{\omega} d\Omega_k,$$

where $\mathbf{n}$ is a unit vector in the direction of flight of the photon.

In the case of ultrarelativistic electrons in which we are interested, we have the following well-known expression for $d\sigma_p$:[7]

$$d\sigma_p = [F_1^2 + (\mu_p F_2/2M)^2\varkappa_1^2 + (\varkappa_1^2/2M^2)\,(F_1 + \mu_p F_2)^2\,\mathrm{tg}^2(\theta_1/2)]\,d\sigma_0. \tag{28}^\dagger$$

Here θ_1 is the angle between $\mathbf{q}$ and $\mathbf{q}'$,

$$d\sigma_0 = \alpha^2\cos^2\frac{\theta_1}{2}\left[4q_0^2\left(1 + \frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right)\sin^4\frac{\theta_1}{2}\right]^{-1} d\Omega_{q'},$$

$$\varkappa_1^2 = \varkappa^2\Big|_{\omega=0} = 4q_0^2\sin^2\frac{\theta_1}{2}\left[1 + \frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right]^{-1}.$$

Expression (27) for $d\sigma_0$ is quite obvious; it is determined by the polar term of the amplitude. The method employed by us actually permits one to find the next term of the expansion $d\sigma_1$. After rather lengthy calculations, we obtain (all quantities are taken for $\omega = 0$)

$$\begin{aligned}
d\sigma_1 = \frac{\alpha^3}{(2\pi)^2}\Big\{ & A_q^2\,\varkappa k\left[f_1\left(2M^2 + \frac{1}{2}\varkappa^2\right) - 2f^2\varkappa^2\right] - \frac{1}{2}A_p^2\varkappa k f_1\varkappa^2 \\
& - 2A_qA_p(\varkappa k)\,(f_1M^2 - f^2\varkappa^2) \\
& + f_1\,(pq + pq')\,(A_q - A_p)\left[\left(\frac{q'}{q'k} + \frac{q}{qk}\right)(p'k + pk)\right. \\
& \left. - \left(\frac{p'}{p'k} + \frac{p}{pk}\right)(qk + q'k) - 4\,(p - q')\right] \\
& - 2\,(A_q - A_p)^2 f_1\left[(pq)\,(kq') + (pq')\,(kq) + \frac{1}{2}(pk)\,\varkappa^2\right] \\
& - 2(\varkappa k)\,(A_q - A_p)\,A_q\left[4\left(2F_1F_1' + 2\left(\frac{\mu_p}{2M}\right)^2 F_2F_2'\varkappa^2\right.\right. \\
& \left.\left. + \left(\frac{\mu_p}{2M}F_2\right)^2\right)(pq)\,(pq')\cos^2\frac{\theta_1}{2} + ff'\varkappa^4\right] \\
& + (A_q - A_p)^2\left[4\left(2F_1F_1'\varkappa^2\right.\right. \\
& \left. + 2\left(\frac{\mu_p}{2M}\right)^2 F_2F_2'\varkappa^4 + \left(\frac{\mu_p}{2M}F_2\right)^2\varkappa^2 + f_1\right)(pq)\,(pq')\cos^2\frac{\theta_1}{2} \\
& \left. + f\varkappa^4\,(f'\varkappa^2 + f)\right]\frac{dq_0'}{d\omega}\frac{1}{q_0'}\,\omega\Big\}\, q_0'\omega\left[\varkappa^4 q_0M^2\left(1 + \frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right)\right]^{-1}\Big|_{\omega=0} \\
& \times d\Omega_k d\omega d\Omega_{q'} + \frac{\alpha}{(2\pi)^2}\left[\frac{4\varkappa k}{\varkappa^2}(A_q - A_p)\,A_q\omega + (A_q - A_p)^2\frac{1}{q_0}\right. \\
& \left.\times\left(1 + \frac{2q_0}{M}\sin^2\frac{\theta}{2}\right)\omega^2 + \frac{2}{p'k}\,\omega\right]\Big|_{\omega=0} d\omega\, d\Omega_k\, d\sigma_p + \frac{2\alpha}{(2\pi)^2} \\
& \times\left[\left(1 + \frac{m^2\omega}{q_0'q'k}\cos\theta'\right)\left(\frac{m^2}{(q'k)^2} - \frac{p'q'}{(p'k)^2} + \frac{(p'q)\,(q'k)}{(p'k)^2\,qk}\right) + \frac{m^2\omega\cos\theta'}{(q'k)^2 q_0'}\right. \\
& \times\left(\frac{qq'}{qk} + \frac{p'q'}{p'k} - \frac{pq'}{pk}\right) - \frac{M^2q'k}{(p'k)^3} + \frac{pq'}{(pk)\,(p'k)} - \frac{(pp')\,(q'k)}{(pk)\,(p'k)^2} \\
& \left. + \frac{m^2}{(q'k)\,(p'k)} - \frac{qq'}{(p'k)\,(qk)}\right]\frac{dq_0'}{d\omega}\frac{1}{q_0'}\,\omega^2\Big|_{\omega=0} d\omega d\Omega_k d\sigma_p.
\end{aligned}$$

Here

$$A_q = q'/(q'k) - q/(qk), \quad A_p = p'/(p'k) - p/(pk),$$

$$f = F_1 + \mu_pF_2, \quad f_1 = F_1^2 + (\mu_pF_2/2M)^2\varkappa^2;$$

$$\frac{dq_0'}{d\omega}\frac{1}{q_0'}\Big|_{\omega=0} = 2\left[M\left(1 + \frac{2q_0}{M}\sin^2\frac{\theta_1}{2}\right)\right]^{-1}\sin^2\frac{\theta'}{2} - \frac{1}{q_0}\left(1 + \frac{2q_0}{M}\sin^2\frac{\theta}{2}\right).$$

$$F_1' = \frac{dF_1}{d\varkappa^2}, \quad \text{etc.}$$

Hence the differential cross section for the radiation of γ quanta in ep scattering is fully deter-

*$[\mathbf{q}' - \mathbf{q}, \mathbf{n}] = (\mathbf{q}' - \mathbf{q}') \times \mathbf{n}$. †tg = tan.

mined in our approximation by the electromagnetic form factors of the proton F_1 and F_2 and their derivatives with respect to the 4-momentum. Therefore the experimental investigation of the radiation of low energy γ quanta would allow one to obtain additional information on the behavior of these important quantities.

The authors thank S. S. Gershteĭn, P. S. Isaev, A. A. Logunov, and Ya. A. Smorodinskii for helpful discussions of the problems considered here.

[1] W. K. H. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958).

[2] A. A. Logunov, Doklady Akad. Nauk SSSR **117**, 792 (1957), Soviet Phys.-Doklady **2**, 540 (1957); A. A. Logunov and L. D. Solovyov, Nuclear Phys. **10**, 60 (1959); Fubini, Nambu, and Wataghin, Phys. Rev. **111**, 329 (1958).

[3] F. E. Low, Phys. Rev. **110**, 974 (1958).

[4] E. Kazes, Nuovo cimento **13**, 1226 (1959).

[5] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

[6] P. S. Isaev and I. S. Zlatev, Nuclear Phys. **16**, 608 (1960).

[7] M. Rosenbluth, Phys. Rev. **79**, 615 (1950).

Translated by E. Marquit
133