σ_r in powers of k (relative momentum of the particles in the initial state) are determined by the value of $(\sigma_r k)$ at $k = 0$, and that for small k the cross section can be written as

$$\sigma_r = (\sigma_r k)_0 \frac{1}{k}\left(1 - \frac{1}{2\pi} k (\sigma_r k)_0 + \ldots\right), \quad (1)$$

where the subscript 0 indicates that the value at $k = 0$ has to be taken. This equation follows from the circumstance that for forces of finite range the expansion of $k \cot \delta$ (δ is the complex s-wave phase shift) in powers of k has the form[1,2]

$$k \operatorname{ctg} \delta = b(k^2) = b(0) + k^2 b'(0) + \ldots . \quad (2)^*$$

We now show that (2), together with unitarity, leads also to a similar form of the expansion of the elastic cross sections in powers of k. Here the first two terms of the series have the form

$$\sigma = \sigma_0 \left(1 - \frac{1}{2\pi} k (\sigma_r k)_0 + \ldots\right), \quad (3)$$

which means that up to terms linear in k the elastic scattering cross section is determined by the quantity $(\sigma_r k)_0$ and by the value of the scattering cross section at $k = 0$.

Since the lowest power with which the partial waves contribute to the power series expansion is k^{4l}, only s waves have to be considered. From (2) we have

$$\sigma = \frac{4\pi}{|b(k^2)|^2 + k^2 - 2k \operatorname{Im} b(k^2)} = \frac{4\pi}{|b(0)|^2}\left(1 + 2k \frac{\operatorname{Im} b(0)}{|b(0)|^2} + \ldots\right). \quad (4)$$

It is easy to see, for example from the optical theorem, that

$$|b(0)|^{-2} \cdot \operatorname{Im} b(0) = -(\sigma_r k)_0 / 4\pi. \quad (5)$$

From this we obtain Eq. (3).

In the case where only elastic scattering occurs, i.e., when $b(k^2)$ is real, the cross section has thus for small k the form

$$\sigma = \sigma_0 + k^2 \sigma_1 + \cdots . \quad (6)$$

If, on the other hand, inelastic processes also occur at $k = 0$ then a term linear in k appears in the power series expansion, and owing to unitarity its coefficient is determined by the total inelastic cross section. The expressions (1) and (3) are applicable for those cases where one can limit oneself to the first term in the expansion (2). We note that when (3) holds the cross section decreases with increasing k. We also note that (3) is correct even if only one of the initial particles has zero spin. Formula (3) can be used in the extrapolation of $\pi^- p$ and $\bar{K}p$ scattering to zero energy.

In conclusion, I express my deep gratefulness to Professor Ya. A. Smorodinskii for interesting discussions.

*ctg = cot

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BEHAVIOR OF THE CROSS SECTION OF ELECTROMAGNETIC PRODUCTION OF PARTICLES AT THE THRESHOLD

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As was shown previously,[1] the total cross section of high energy inelastic and elastic scattering of electrons at large angles in the center-of-mass system (c.m.s.), calculated in a doubly logarithmic approximation, can be represented in the form

$$d\sigma = d\sigma_0 \exp\left\{-\frac{8e^2}{\pi} \ln\frac{E}{m} \ln\frac{E}{\Delta E}\right\}. \quad (1)$$

Here $d\sigma_0$ is the scattering cross section computed in the low-order perturbation theory, E the electron energy, m the electron mass, and ΔE the maximum achievable energy of the quanta radiated in the collision process, which defines the threshold sensitivity of the detector of electrons.

If $\Delta E = E$ (i.e., the detector records all electrons, independently of their energy), then the total cross section coincides with the cross section computed in the low-order perturbation theory approximation ($d\sigma = d\sigma_0$). As is known,[2] this is connected with the fact that in this approximation, the decrease of the cross section of the fundamental process, without emission of additional quanta, due to taking the radiation corrections into account, is entirely compensated by the increase of the cross section of processes with multiple additional radiation of hard photons in the case of an arbitrary radiation ($\Delta E = E$).

The same formula also holds for the scattering of electrons on positrons.

In principle, another situation arises in the conversion of an electron-positron pair into a pair of other charged particles with larger mass (for example, $e^+ + e^- \to \mu^+ + \mu^-$), inasmuch as in this case the maximum energy of the radiated photon cannot be larger than $E - \mu$ (μ is the mass of the particle). Thus, in addition to the limitation imposed by the sensitivity of the apparatus, there arises a physical limitation on the energy of the photon. In this case, the given compensation cannot take place and, independently of the sensitivity of the detector, an account of the radiative corrections seriously changes the character of the dependence of the cross section on energy. It is clear that the effect is greatest close to the threshold of particle production.

As a qualitative example, we consider the production of a pair of fermions with mass μ in the annihilation of an electron-positron pair with energy E in the c.m.s. Calculation can be carried out the same as in reference 1, where it was shown that the contributions of diagrams containing more than one staircase line are mutually compensated, so that it is sufficient to consider only the enveloping virtual lines. This means that we have two "overgrown" vertices, connected by a single virtual photon line. It is evident that for $E \gg \mu$, the difference between fermions with mass μ and electrons becomes insignificant, so that Eq. (1) remains valid, but the maximum $\Delta E \leq E - \mu$. With decrease in energy, the contribution from the virtual quanta with which the heavy fermions are exchanged decreases. As can be shown, at the threshold of the reaction for $E \sim \mu$, their contribution becomes negligible. This is connected with the fact that at threshold the momentum of the fermions produced is very small; therefore, for a description of the interaction between them, the lower orders of perturbation theory are sufficient. Thus, at threshold, only the electron vertex makes a contribution; in this case the cross section of the reaction for arbitrary radiation has the form

$$d\sigma = d\sigma_0 F^2(E^2) \exp\left\{-\frac{4e^2}{\pi}\ln\frac{E}{E-\mu}\ln\frac{E}{m}\right\}, \tag{2}$$

$$d\sigma_0 = \frac{r_0^2}{16\gamma^2}\frac{q}{E}\left[1 + \frac{\mu^2}{E^2} + \frac{q^2}{E^2}\cos^2\theta\right]. \tag{3}$$

Here r_0 is the classical radius of the electron; $\gamma = E/m$; q is the momentum of the fermion; θ is the angle of flight of fermions relative to the direction of motion of the electrons; $F(E^2)$ is the form factor of the electrons.

For nonrelativistic fermions (close to threshold) Eq. (2) can also be written in the form

$$d\sigma = d\sigma_0 F^2(E^2)\,(q/\sqrt{2\mu E})^{(8e^2/\pi)\ln(E/m)}. \tag{4}$$

For μ mesons, the effect under consideration gives (close to threshold) the change in the character of the dependence of the cross section on the momentum of the emerging particles: in place of $d\sigma \sim q$, we have $d\sigma \sim q^{1.1}$ in the case of an arbitrary radiation. The effect will be still greater if we record the energy of the emerging mesons in some energy range.

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