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MAGNETIC SPECTRUM ANALYZERS

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The possibility of using paramagnetic and other magnetic resonance effects for studying radiation spectra has been investigated. The attenuation in helical and coaxial lines, sensitivity, saturation, and the limiting spectrum recording speed have been calculated. Descriptions are given of magnetic spectrum analyzers (MSA) of the following types: 1) frequency-compensated MSA, 2) "resonance" MSA, 3) induced-radiation MSA, and 4) induction MSA. The experimental results obtained with a frequency-compensated analyzer are reported; an oscillogram obtained at 3 cm with an MSA filled with the paramagnet $MnSO_4$ and an oscillogram showing the spectrum of an oscillatory spark circuit are also presented.

In plasma investigations and certain other cases it is frequently necessary to measure a radiation spectrum over a wide frequency range in a very short period of time. The region of greatest interest is the wavelength range from 0.01 to 100 cm. In this paper we describe several methods of using paramagnetic and other magnetic resonance effects for studying radiation spectra.

PRINCIPLE OF THE METHOD

The principle of the method is the following: the radiation source is coupled to a broadband system, for example a cable or a helical line, which contains a paramagnetic material and is located in a quasi-static magnetic field. The line is terminated in its characteristic impedance and a detection device. When the frequency of the signal transmitted through the line equals the Larmor frequency of the material, the attenuation of the line increases markedly so that the signal is reduced, causing a corresponding reduction in the detector reading.

If the bandwidth of the input system and the line are both large, the range of frequencies which can be measured is determined only by the quasi-static magnetic field. For example, if the magnetic field can be varied from 10^2 to 10^5 oe, the frequency range for a paramagnetic material with a Lande factor of 2 corresponds to a wavelength range of 100 to 0.1 cm.

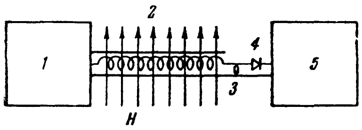

FIG. 1. Diagram of analyzer (the symbols are explained in the text).

A schematic diagram of an analyzer is shown in Fig. 1. Here, 1 is the broadband input device, which provides the coupling between the line 2 and the radiation system, 3 is the loading resistance for the line, which is made equal to the characteristic impedance, 4 is the broadband detector, 5 is the recording device (for example, an oscilloscope), and H is the quasi-static magnetic field. If an ordinary coaxial line is used, the quasi-static magnetic field must be parallel to the line. If the field is perpendicular to the axis of the line the paramagnetic-resonance attenuation factor is reduced by a factor of 2. The line 2 is filled with a paramagnetic material. The sweep in the recording device must be synchronized with the quasi-static magnetic field H.

We now consider the line parameters. The power absorbed by a cubic centimeter of paramagnetic material at paramagnetic resonance is given by the familiar relation:

$$P = \frac{\pi^2\nu^2 g^2\beta^2 H_\sim^2 N\delta f(\nu)}{4kTA}\,\frac{S(S+1)-M(M-1)}{2S+1}, \qquad (1)$$

where ν is the frequency, g is the spectroscopic splitting factor, β is the Bohr magneton, $H_\sim$ is the strength of the high-frequency magnetic field, N is the number of paramagnetic systems in a gram-molecule of the paramagnetic material, δ is the density of the paramagnetic material, k is the Boltzmann constant, T is the temperature, A is the atomic weight, $f(\nu)$ is a function which describes the shape of the paramagnetic resonance line, S is the spin and M is the magnetic quantum number.

We now consider the attenuation due to paramagnetic resonance in a long line. The velocity of propagation in the line is

$$v = c/2\pi r n_1 \sqrt{\varepsilon\mu}, \tag{2}$$

where c is the velocity of light, n_1 is the number of turns per centimeter of line, ϵ is the dielectric constant, μ is the magnetic permeability, and r is the helix radius. For a coaxial line,

$$v = c/\sqrt{\varepsilon\mu}. \tag{3}$$

First we consider a line with no ohmic losses. Dividing Eq. (1) by the energy transmitted per second through unit cross section, we find the attenuation per unit length of helical line:

$$\alpha = \frac{2\pi^4 \nu^2 g^2 \beta^2 N \delta f(\nu) r n_1 \sqrt{\varepsilon\mu}}{kTAc} \frac{S(S+1) - M(M-1)}{2S+1}, \tag{4}$$

whence, for g = 2 and s = ½, we have

$$\alpha = 3\pi^4 \nu^2 \beta^2 N \delta r_0 n_1 \sqrt{\varepsilon\mu} f(\nu)/kTAc. \tag{5}$$

Thus, at paramagnetic resonance the attenuation of the line is proportional to the square of the frequency and inversely proportional to the velocity of propagation.

Similarly, the attenuation factor for a coaxial line is

$$\alpha_c = \frac{\pi^3 \nu^2 g^2 \beta^2 N \delta f(\nu) \sqrt{\varepsilon\mu}}{kTAc} \frac{S(S+1) - M(M-1)}{2S+1}. \tag{6}$$

From Eqs. (4) and (6) we find the ratio of attenuation for a helical line and a coaxial line

$$\alpha/\alpha_c = 2\pi r n_1.$$

We now find the line attenuation due to effects other than paramagnetic resonance. The attenuation per unit length of line caused by conductor losses is denoted by α_0 while the nonresonance losses in the paramagnetic material are denoted by α_1; thus, the total attenuation per unit length of line is $\alpha + \alpha_0 + \alpha_1$.

We may estimate the minimum time in which a frequency spectrum can be analyzed in a line filled with diphenyl picril hydrazyl. Taking a relaxation time[1] $T_0 = 3 \times 10^{-8}$ sec with a half-width $\Delta H = 2$ oe, we find that the maximum rate of change of H is 7×10^7 oe/sec. Choosing a frequency range corresponding to the wavelength interval $\lambda = \infty$ to $\lambda = 0.5$ cm, we find the minimum time for a measurement to be 3×10^{-4} sec.

If the spectrum being investigated contains one or more monochromatic lines and the magnetic field is swept at a rate of 7×10^7 oe/sec the recording device 5 (Fig. 1) must have a bandwidth of 3.5×10^7 cps. If the spectrum is smoother, however, or if the paramagnetic resonance line is broader, the bandwidth requirements for the recording device can be relaxed. The conditions under which the spectrum-recording rate can be much greater than that indicated above are considered in a subsequent section.

LIMITING SENSITIVITY

We consider a line along which a traveling wave is propagated (Fig. 1). Let the power at the input of the line be P_0. The line attenuation when there is no paramagnetic resonance is denoted by $\alpha_2 = \alpha_0 + \alpha_1$ while the attenuation due to paramagnetic resonance at frequency ν is denoted by α; thus, the power at the end of a line of length L is

$$P = P_0 e^{-(\alpha_2 + \alpha)L}$$

while the change in power caused by the paramagnetic resonance is

$$\Delta P = P_0 e^{-\alpha_2 L}(1 - e^{-\alpha L}). \tag{7}$$

In order for measurements to be performed ΔP must be greater than the power due to thermal noise; i.e., in the limit

$$P_0 e^{-\alpha_2 L}(1 - e^{-\alpha L}) \approx 4(2 P_0 e^{-\alpha_2 L} kT \Delta\nu)^{1/2}, \tag{8}$$

where $\Delta\nu$ is the frequency interval.

This relation determines the limiting sensitivity of the method: the sensitivity increases linearly with α when $\alpha L \ll 1$ and is independent of α when $\alpha L \gg 1$.

The sensitivity can be reduced considerably if the width of the paramagnetic resonance curve is very small compared with the line width of the radiation source. For this reason, sources with narrow low-intensity spectra must be investigated by means of paramagnetic materials with broad resonance lines. On the other hand, the absorption curve of the MSA at paramagnetic resonance can be broadened by inhomogeneity in the magnetic field over the volume of paramagnetic material. If the magnetic field inhomogeneity is $\Delta H'$, the frequency width of the paramagnetic resonance line in a field H is $\Delta\nu = (g\beta/h)(\Delta H' + \Delta H)$ where ΔH is the half-width of the paramagnetic resonance curve for a uniform field H. When this method of increasing the sensitivity of the MSA is used, account must be taken of the fact that the quantity $f(\nu)$ becomes $f(\nu) = h/g\beta(\Delta H' + \Delta H)$ as follows from Eq. (1); in this case it may be necessary to increase the length of the line L in order to compensate for the reduction of α due to $\Delta H'$ if the condition $\alpha L > 1$ is to be satisfied.

EFFECT OF SATURATING THE PARAMAGNETIC RESONANCE AND LIMITING SPECTRUM RECORDING RATE

When the signal amplitude is very large the paramagnetic resonance may be saturated, causing distortions in the measurements. This effect can become appreciable if the paramagnetic material is at a low temperature or if the strength of the radiation source is high.

If the line contains m grams of paramagnetic material, the maximum energy which can be absorbed within the relaxation time T_0 is

$$U = (N/A)(g\beta^2 H^2/kT)\,m, \tag{9}$$

and if the pulse length $\tau < T_0$, the power of a pulse W which can cause saturation is $W = U/\tau$. If the recording device following the detector in the spectrum analyzer has a bandwidth $\Delta\nu$, the quantity $1/\tau\Delta\nu$ determines the "stretching" of the pulse if $\tau \ll 1/\Delta\nu$. This situation imposes a condition on the minimum power of the radiation pulse, which must yield a pulse at the recording device which is greater than noise. This power can be appreciable if $1/\tau\Delta\nu \gg 1$, and thus can cause saturation of the paramagnetic resonance. If saturation is to be avoided the condition $W < U/\tau$ must be satisfied.

If the spectrum measurement process is to be quasi-stationary, the rate-of-change of the magnetic field must be such that at any instant of time the spin system is in equilibrium with the lattice in the paramagnetic material. This requirement can be written in the form $\Delta H/\gamma > T_0$ where ΔH is the half-width of the paramagnetic resonance curve and $\gamma = dH/dt$ is the rate of change of the magnetic field. On the other hand, it is not always necessary that this requirement be satisfied. The measurement process can be a nonequilibrium process so long as the integrated radiation energy does not cause saturation of the paramagnetic resonance. Saturation effects remain unimportant so long as

$$\iint_{\nu_1\nu_2} w(\nu)\,d\nu dt \ll U_0, \tag{10}$$

where $w(\nu)$ is the radiation energy at frequency ν applied in the measurement line,* ν_1 and ν_2 are the frequency limits of the spectrum being measured, and

$$U_0 = (N/A)(g\beta^2 H_0^2/kT)\,m, \tag{11}$$

where H_0 is the initial magnetic field applied to the paramagnetic material.

The inequality in (10) has a simple meaning: if the total energy absorbed by the paramagnetic material in the measurement line in the frequency region between ν_1 and ν_2 in a time $t < T_0$ does not saturate the spin system, then the spectrum measurement can be carried out in a time shorter than that required to establish equilibrium between the spin system and the lattice in the paramagnetic material. In the limit, this time is comparable with the oscillation period of the spectrum.

It follows from the inequality in (10) that when $H_0 = 0$ the measurement of the spectrum requires that the variation of the magnetic field be quasi-static; on the other hand, as H_0 increases, saturation and stationarity of the field H_0 become less important. It is also apparent from (10) that at lower temperatures the spin system has a higher energy capacity and that the limitations imposed on the rate of change of the magnetic field γ are relaxed.

MILLIMETER AND SUBMILLIMETER WAVES

According to Eq. (1), at wavelengths of 1 mm and below paramagnetic absorption is so large that a thin film of paramagnetic material becomes opaque. However, it is difficult to produce the magnetic fields which correspond to this wavelength range. From the expression $\nu = g\beta H/h$ we see that a magnetic resonance is possible at high ν in weak fields if g is much greater than 2. In certain materials the g factor can be as large as 10 – 15.

We may also note that in certain semiconductors the effective mass differs considerably from unity and the cyclotron resonance is displaced toward weaker fields. For this reason, at very short wavelengths it is interesting to consider the possible utilization of cyclotron resonances. In this case the paramagnetic materials in the measurement scheme described above must be replaced by some other appropriate material.

USE OF FERROMAGNETIC RESONANCES

A ferromagnetic resonance can be used in an MSA in the same way as a paramagnetic or cyclotron resonance. For example, the absorption coefficient in ferrites is very large so that the measurement line can be very short. A shortcoming of ferrites is the nonresonant variation in loss as the fixed magnetic field is varied; this effect tends to complicate calibration of the absorption curve.

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*We neglect line attenuation not due to paramagnetic resonance, while the attenuation due to paramagnetic resonance is assumed to be large.

Ferrites which are free of this disadvantage and which have narrow resonance lines may successfully replace paramagnetic materials in the region of relatively long wavelengths ($\lambda \geq 3$ cm). At shorter wavelengths, where the absorption coefficient in paramagnetic materials is large, ferrites show no particular advantage, especially if account is taken of the relatively long relaxation time characteristic of ferrites.

MSA SCHEMES

Four possible kinds of magnetic spectrum analyzers have been considered.

1. The frequency-compensated MSA.
2. The discrete-resonance MSA.
3. The induced-radiation MSA.
4. The induction MSA.

The various features of these versions of the MSA will now be described. In order to provide low-frequency compensation it is necessary to have two identical lines filled with paramagnetic material; these lines have a common input and a common load. The detector currents are first balanced. One of the lines is placed in a quasi-static magnetic field H. At paramagnetic resonance the signal applied to the detector in this line is reduced and a voltage pulse appears across the common load. This type of MSA is characterized by high sensitivity, large frequency-measurement range, and the greatest simplicity.

In the discrete-resonance spectrum analyzer a resonant coaxial line (or any other kind of line) is connected to a wideband input device. A detector and indicating device are connected at the other end of the line. The line is filled with paramagnetic material and placed in a field H.

The use of the resonance properties of the line allows us to reduce the required amount of paramagnetic material; by virtue of the Q of the line, its effective length is increased by a factor $Q\lambda/2\pi$. The spectrum which is obtained in this case is a line spectrum with a gain factor

$$\eta = 4Q/\pi n = cQ/\pi\nu L \sqrt{\varepsilon\mu},$$

where n is an odd integer and L is the length of the line. When $n = Q$ the gain approaches unity, $n = 4v/\pi c$, and the advantages of this system are lost accordingly.

An induced-radiation MSA can be used effectively for amplifying weak signals. An analyzer of this kind consists of a broadband input device, connected to the line filled with the paramagnetic material in the magnetic field H, the detector, and the recording device. In addition, we apply a radio-frequency signal to the input, thus increasing the population of the upper energy level of the paramagnetic material by a large factor as compared with the lower level.

Before the signal being studied is applied, the magnetic field is held at the resonance value corresponding to the rf frequency; when the signal is applied the field H is varied rapidly (compared with the relaxation time T_0) in order to traverse the entire frequency range of the spectrum being measured. Under these conditions the input signal is amplified in the line. A large population can also be obtained in the upper energy level without an rf generator if the sign of the magnetic field in the paramagnetic material is reversed rapidly when the input signal is applied.

An induction MSA can be used conveniently in the region of very short wavelengths since it eliminates the need for crystal detectors and the associated inconveniences of these devices. The features of this method are considered briefly below.

The magnetic field H in the frequency region of interest is varied rapidly from H_0 to H_1 (compared with the relaxation time T_0) where $H_0 \neq 0$ and $H_1 \neq 0$; to analyze the radiation spectrum we measure the magnetization vector in the paramagnetic material. The initial field H_0 magnetizes the paramagnetic material and the magnetization vector can be varied only under the effect of a radio-frequency field at a frequency corresponding to the instantaneous value of the resonance magnetic field $H = h\nu/g\beta$. Thus, the spin system is used here as both a memory and integrating device. The energy capacity of the spin system is $U_0 = (N/A)(g\beta^2 H_0^2/kT)\,m$ and can be rather large.

EXPERIMENTS

We have measured line attenuation due to the paramagnetic resonance and to nonresonance losses in a paramagnetic material, the sensitivity of the MSA, the dielectric permittivity of the paramagnetic material, and have verified the principles of operation of a frequency-compensated MSA. We

λ, cm	0.8	3.0	10.0
Paramagnetic resonance attenuation for a line with L = 8.5 cm, %			
measured	100	92	10
calculated	100	96.6	26
Measured dielectric permittivity ε	–	≈1.5	–
Nonresonance absorption for L = 8.5 cm in paramagnet, %	–	<2	–

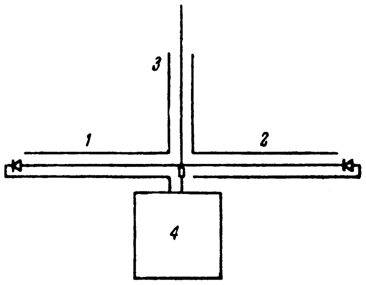

FIG. 2. 1 and 2 are identical lines, filled with paramagnetic material, 3 is a wideband input device and 4 is the detector.

have also obtained a paramagnetic resonance curve for $MnSO_4$ by rapid variation of the magnetic field as well as the spectrum of a spark discharge in an oscillatory circuit; the possibility of using a ferrite in an MSA has also been verified.

In the table we show the principal results of measurements with diphenyl picril hydrazyl [density $\delta = 1.54$ g/cm^3, $f(\nu) = 3.6 \times 10^{-7}$, $T = 300°K$] carried out with a line with $L = 8.5$ cm and a characteristic impedance of 30 ohms. It is apparent from the table that the measured attenuation for the paramagnetic resonances at $\lambda = 0.8$ cm and $\lambda = 3$ cm is in agreement with the calculated value; however, there are discrepancies at $\lambda = 10$ cm.*

The measured sensitivity of MSA is 10^{-9} watts at an amplifier bandwidth of 2 Mc/sec; however, in these experiments the sensitivity was limited by the high noise of the input stage of the amplifier rather than by detector noise. It would appear that the sensitivity can be improved considerably by modification of the input stage. A frequency-compensation scheme has been tried (cf. Fig. 2). This system uses standard demountable DKV-8 detectors; the internal resistance of these detectors can be controlled easily and it is possible to obtain signal cancellation up to 30 db. The demountable detectors have no parasitic resonances.

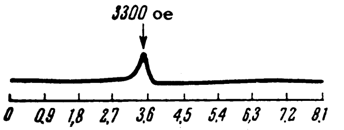

FIG. 3

In Fig. 3 we show an oscillogram obtained with an MSA filled with the paramagnetic material $MnSO_4$ using a 3-cm signal and a rate of rise $dH/dt = 2.5 \times 10^9$ oe/sec for the magnetic field. The measured half-width of the $MnSO_4$ resonance curve in the quasi-static magnetic field is 300 oe. The asymmetry and additional broadening of the curve in Fig. 3 are due to the inadequate bandwidth of the amplifier (5 Mc/sec). The tracing time for the magnetic resonance curve is 1.2×10^{-7} sec. The magnetic field is varied sinusoidally with a period of 20 μsec and an amplitude of 8.1×10^3 oe.

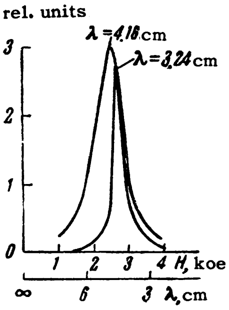

FIG. 4

In Fig. 4 we show the radiation spectrum of a small oscillatory spark circuit with a capacity of 3×10^{-12} farad and an inductance of 1.5×10^{-10} henry at a wavelength of 4.16 cm. For purposes of comparison we show the paramagnetic resonance curve for $MnSO_4$ at 3.24 cm (halfwidth, 300 oe). The halfwidth of the spectrum for the spark circuit indicates that $Q \approx 2.5$, corresponding to an oscillation period of 3.6×10^{-10} sec. This period is in good agreement with the emission time from the discharge, which has been determined earlier.[2] We have also carried out preliminary experiments with ferrites which show that these materials can be used in an MSA.

CONCLUSIONS

Both theory and experiment verify the possibility of the effective utilization of magnetic resonance effects for the static and dynamic analysis of radiation spectra over a broad wavelength region.

As compared with existing microwave devices the MSA has the advantage of extremely wide range, rapid and continuous frequency variation, and simplicity of construction.

A sensitivity of 10^{-9} watts has been obtained in the first experiments with a simple MSA. This sensitivity is far from the optimum which can be obtained and considerable improvement is to be expected.

In principle, the MSA can also be used in the millimeter and submillimeter region.

We wish to express our gratitude to Academician A. E. Arbuzov and Professor F. G. Valitova for preparation of the diphenyl picril hydrazyl and to Professor S. A. Al'tshuler for interesting discussions of this work.

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*The origin of these discrepancies would appear to be the inaccurate determination of $f(\nu)$.

[1] Goldsborough, Mandel, and Pake, Phys. Rev. Letters **4**, 13 (1960).

[2] E. K. Zavoĭskii and S. D. Fanchenko, Doklady Akad. Nauk SSSR **100**, 661 (1955).

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63